

Higher dimensional cosmological model with a phantom field

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(Received 3 February 2008; published 4 August 2008)

We consider a higher dimensional gravity theory with a negative kinetic energy scalar field and a cosmological constant. We find that the theory admits an exact cosmological solution for the scale factor of our universe. It has the feature that the universe undergoes a continuous transition from deceleration to acceleration at some finite time. This transition time can be interpreted as that of recent acceleration of our universe.

DOI: [10.1103/PhysRevD.78.047503](https://doi.org/10.1103/PhysRevD.78.047503)

PACS numbers: 04.50.-h, 11.10.Kk, 98.80.-k, 98.80.Es

Since Edwin Hubble discovered the expansion of our universe, the inflationary big bang cosmology has been developed into a precision science by recent cosmological observations including supernova data [1] and measurements of cosmic microwave background radiation [2]. They suggest that our universe is made up of about 4% ordinary matter, about 22% dark matter, and about 74% dark energy. These observations triggered an explosion of recent interests in the origin of dark energy [3]. There are several approaches to understanding the dark energy such as cosmological constant, quintessence [4], k -essence [5–8], and phantom [9]. One of the simple ways to explain it is through an introduction of the cosmological constant, which leads to an exponential expansion of the scale factor of the universe via $a(t) \sim e^{\sqrt{\Lambda/3}t}$ with the four-dimensional cosmological constant Λ . Even though this simple form adequately describes current acceleration with a small value of Λ , it cannot say anything about important issues including coincidence problem. If we could obtain a cosmological solution of the form $a(t) \sim e^{F(t)}$ with $F(t)$ having the property that the second time derivative of $a(t)$ vanishes at some finite time, we may be able to interpret that time as the transition time from deceleration to acceleration of the universe.

We appeal to higher dimensions because it turns out that the 4-dimensional version of our model [See Eq. (2)] does not admit the aforementioned type of cosmological solution. In this paper, we show that this type of solution is

viable in higher dimensional [10,11] phantom cosmology. We find that the theory allows a cosmological solution of the scale factor of our universe in which the standard exponential acceleration is modified by a higher dimensional phantom contribution. The solution exhibits an interesting property that the universe undergoes a continuous transition from deceleration to acceleration at some finite time. This time could be interpreted as the beginning of recent acceleration of our universe.

A phantom energy component is characterized by the equation of state parameter ω less than -1 . Since Caldwell [9] pointed out that the current observations do not rule out the possibility of such an energy component and the universe may end its existence in a finite time by reaching a singularity known as the “big rip” according to some simple models, phantom cosmology has been widely studied. In particular, it was suggested by Scherrer [12] that the finite lifetime of the universe may provide an answer to the cosmic coincidence problem. The simplest phantom model is constructed in terms of a scalar field with negative kinetic energy.

Motivated by the recent interest of the ten-dimensional superstring theories [13] as the unified theories of fundamental interactions and by the finding [14] that there exist interesting dualities between phantom and ordinary matter models which are similar to dualities in superstring cosmologies [15,16], we extend this phantom model to a ten-dimensional spacetime with a (ten-dimensional) cosmological constant $\bar{\Lambda}(>0)$. In this model the equation of state is given by

$$\omega \equiv \frac{p}{\rho} = \frac{-\frac{1}{2}\dot{\sigma}^2 - V(\sigma) - \bar{\Lambda}}{-\frac{1}{2}\dot{\sigma}^2 + V(\sigma) + \bar{\Lambda}}, \quad (1)$$

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where σ is the phantom field. If we insist on the positivity of the energy, then it implies that $\omega \leq -1$, with the equality holding when the scalar field is constant. Unlike in the four-dimensional case, $\omega \leq -1$ does not guarantee the acceleration of our universe due to the presence of scale factor of extra dimensions.

Let us consider the action of the form

$$S = \int d^{10}x \sqrt{-g} \left(-\frac{1}{2}R + \frac{1}{2}g^{MN} \partial_M \sigma \partial_N \sigma - V(\sigma) - \bar{\Lambda} \right). \quad (2)$$

The equations of motion are given by

$$\begin{aligned} R_{MN} - \frac{1}{2}g_{MN}R - g_{MN}\bar{\Lambda} &= -T_{MN}, \\ \frac{1}{\sqrt{-g}} \partial_M (\sqrt{-g} g^{MN} \partial_N \sigma) + \frac{\partial V}{\partial \sigma} &= 0. \end{aligned} \quad (3)$$

Let us assume the metric of the form

$$g_{MN} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & a^2(t) & 0 \\ 0 & 0 & b^2(t) \end{pmatrix}, \quad (4)$$

where $a(t)$ is the scale factor of our three-dimensional universe and $b(t)$ is the scale factor of the extra six dimensions. We assume that the internal space is given by six-dimensional torus [17] and the potential vanishes, $V(\sigma) = 0$.

Exploiting the above equations, we have four differential equations

$$\begin{aligned} 3\frac{\ddot{a}}{a} + 6\frac{\ddot{b}}{b} &= \frac{1}{4}\bar{\Lambda} + \dot{\sigma}^2, & \frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + 6\frac{\dot{a}}{a} \cdot \frac{\dot{b}}{b} &= \frac{1}{4}\bar{\Lambda}, \\ \frac{\ddot{b}}{b} + 5\frac{\dot{b}^2}{b^2} + 3\frac{\dot{a}}{a} \cdot \frac{\dot{b}}{b} &= \frac{1}{4}\bar{\Lambda}, & \ddot{\sigma} + \left(3\frac{\dot{a}}{a} + 6\frac{\dot{b}}{b}\right)\dot{\sigma} &= 0, \end{aligned} \quad (5)$$

where the overdots denote the derivatives with respect to time.

Now, we define variables α and β as

$$\alpha = H - h, \quad \beta = H + h. \quad (6)$$

with $H = \dot{a}/a$ and $h = \dot{b}/b$. After some manipulations, we rewrite (5) in terms of α and β as

$$\begin{aligned} 3\alpha^2 - \frac{4}{3} \frac{d}{dt} \left(\frac{\dot{\alpha}}{\alpha} \right) &= \frac{3}{2} \dot{\sigma}^2, & \frac{3}{2} (3\beta - \alpha) + \left(\frac{\dot{\alpha}}{\alpha} \right) &= 0, \\ \ddot{\sigma} - \left(\frac{\dot{\alpha}}{\alpha} \right) \dot{\sigma} &= 0. \end{aligned} \quad (7)$$

For very late time ($t \rightarrow \infty$) we assume that $\dot{\sigma} \sim 0$ asymptotically so that the solution approaches the maximally symmetric ten-dimensional spacetime with cosmological constant $\bar{\Lambda}$, $a \propto e^{1/6\sqrt{\bar{\Lambda}}t}$ and $b \propto e^{1/6\sqrt{\bar{\Lambda}}t}$. In this case one can show that $\frac{\dot{\alpha}}{\alpha} \sim \text{constant}$. In order to find an

exact solution¹ we take the following ansatz

$$\frac{\dot{\alpha}}{\alpha} = \nu = \text{const}, \quad (8)$$

which is consistent with the asymptotic behavior. From the physical point of view, this ansatz corresponds to the case in which the volume of the universe as a ten-dimensional spacetime increases at a constant rate.

With the above ansatz the solutions to (7) are then given by

$$\begin{aligned} \alpha &= \alpha_0 e^{\nu t}, & \beta &= -\frac{2}{9}\nu + \frac{1}{3}\alpha_0 e^{\nu t}, \\ \sigma &= -\frac{\sqrt{2}}{\nu} \alpha_0 e^{\nu t}, \end{aligned} \quad (9)$$

where α_0 is the initial value of α at $t = 0$ and ν is given by

$$\nu = -\frac{3}{2}\sqrt{\bar{\Lambda}}. \quad (10)$$

We define α_0 in terms of a dimensionless variable n

$$\alpha_0 = n\sqrt{\bar{\Lambda}}. \quad (11)$$

We then arrive at the solutions to the differential equations (5) as follows

$$\begin{aligned} a &= a_0 e^{1/6\sqrt{\bar{\Lambda}}t} e^{-(4n/9)(e^{-(3/2)\sqrt{\bar{\Lambda}}t} - 1)}, \\ b &= b_0 e^{1/6\sqrt{\bar{\Lambda}}t} e^{2n/9(e^{-(3/2)\sqrt{\bar{\Lambda}}t} - 1)}, & \sigma &= \sigma_0 e^{-(3/2)\sqrt{\bar{\Lambda}}t}, \end{aligned} \quad (12)$$

where a_0 and b_0 are the initial values of a and b at $t = 0$, respectively, and

$$\sigma_0 = \frac{2\sqrt{2}n}{3}. \quad (13)$$

Moreover, ω in (1) is then given by

$$\omega = \frac{n^2 e^{-3\sqrt{\bar{\Lambda}}t} + 1}{n^2 e^{-3\sqrt{\bar{\Lambda}}t} - 1}. \quad (14)$$

Here one notes that exploiting the relation $\bar{\Lambda} = 12\Lambda$ between the ten-dimensional cosmological constant $\bar{\Lambda}$ and the four-dimensional one Λ , in the vanishing phantom field limit with $n = 0$, the solution for a in (12) is reduced to the standard form $a(t) \sim e^{\sqrt{\Lambda/3}t}$ of the exponential expansion of the scale factor of the universe. In order to investigate the transition time t_{tr} from deceleration to acceleration, we consider the differential equation

$$\frac{\ddot{a}}{a} = \left(\frac{\dot{a}}{a} \right)^2 - n\bar{\Lambda} e^{-(3/2)\sqrt{\bar{\Lambda}}t} = 0, \quad (15)$$

whose solution is given by

¹In recent time ($t \ll \infty$), both the phantom field and extra dimensions are necessary to have a nontrivial solution of the form Eq. (12).

$$t_{\text{tr}} = \frac{2}{3\sqrt{\Lambda}} \left(\ln n - \ln \frac{7 - \sqrt{45}}{8} \right). \quad (16)$$

In order to analyze the solutions (12) further near the present time, we define a dimensionless variable x as

$$t = xt_* \quad (17)$$

where the age of the universe is given by $t_* = 13.58 \text{ Gyr} = 4.28 \times 10^{17} \text{ sec}$ [18]. The vacuum energy density is given by $\rho_\Lambda = 10^{-8} \text{ erg} \cdot \text{cm}^{-3}$ [19] to yield $\Lambda = 8\pi G\rho_\Lambda = 2.07 \times 10^{-56} \text{ cm}^{-2}$. Taking the ansatz that the transition time from deceleration to acceleration in (16) is equal to $t_{\text{tr}} = 5.04 \text{ Gyr} = 0.37t_*$ [18], we fix the value of n to be $n = 1.277$. We then have a and b in terms of a_* and b_* as follows

$$\begin{aligned} a &= a_* e^{1.07(x-1) - 0.568(e^{-9.63x} - e^{-9.63})}, \\ b &= b_* e^{1.07(x-1) + 0.284(e^{-9.63x} - e^{-9.63})}, \end{aligned} \quad (18)$$

where $a_* = 4.60 \times 10^{10} \text{ light-years} = 4.35 \times 10^{28} \text{ cm}$ [20] is the size of the our present universe and b_* is the size of the present extra dimensional space.

We depict in Fig. 1 the graph of $\ddot{a}/(\Lambda a)$ and in Fig. 2 the graph of ω in (14) in terms of the dimensionless variable x . The circle in Fig. 2 denotes the transition point from deceleration to acceleration at $t_{\text{tr}} = 0.37t_*$ to yield the value of ω at that moment,

$$\omega = \frac{79 - 21\sqrt{5}}{15 - 21\sqrt{5}} = -1.0026, \quad (19)$$

which is not ruled out by the current observation [9]. We

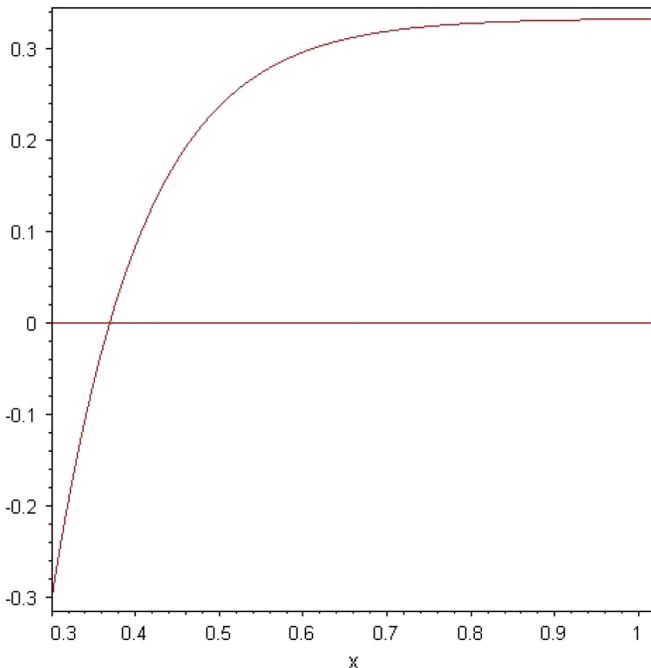


FIG. 1 (color online). $\ddot{a}/(\Lambda a)$ in terms of $x = t/t_*$ where t_* is the age of our universe.

assume that our solution describes the recent acceleration of our universe starting at time $t = t_{\text{tr}}$ and b_* is very small compared to a_* so that the exponential increase of the size of the extra dimensions does not contradict with the experimental observations.

In summary, we found that ten-dimensional phantom model provides a new cosmological solution of the type $a(t) \sim e^{\lambda(t)}$ with $\lambda(t) = \frac{1}{6}\sqrt{\Lambda}t - \frac{4n}{9}(e^{-(3/2)\sqrt{\Lambda}t} - 1)$. This function has the feature that it has a vanishing second time derivative at time $t_{\text{tr}} = 5.04 \text{ Gyr}$, at which the recent acceleration of the universe starts. This leads to the interesting possibility that this model can give one way of understanding the coincidence problem [21].

A couple of comments are in order. In the standard cosmology with a cosmological constant Λ , there is a transition from the matter-dominated era to the Λ -dominated era. It is shown that there exists the non-smooth curvature associated with the multiple discontinuities at the transition [22]. In our case with a phantom field in higher dimensional gravity theory, the solution belongs to the class of generalized exponential acceleration where $a(t) \sim e^{F(t)}$ with $\ddot{a}(t) = 0$ for some finite time t . In this case, the cosmic transition occurs continuously at time $t = t_{\text{tr}}$. This kind of approach appeared in applying intermediate inflation models [23] to the late-time acceleration with $F = At^f$ where $A > 0$ and $0 < f < 1$. It has the transition time given by $t = (\frac{1-f}{Af})^{1/f}$. This also appears in a scalar phantom-nonphantom transition model [8] to unify phantom inflation with late-time acceleration. We have ne-

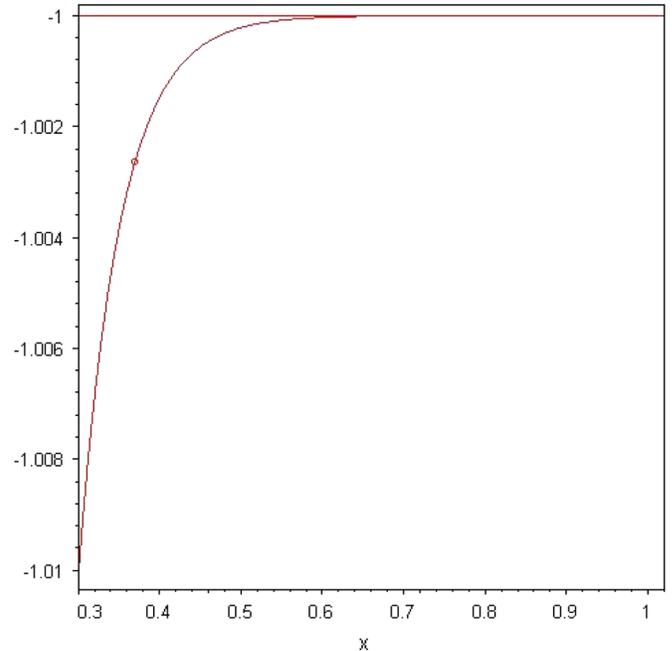


FIG. 2 (color online). ω in terms of $x = t/t_*$. The circle denotes the transition point from deceleration to acceleration at $t_{\text{tr}} = 0.37t_*$.

glected the matter contribution in our analysis. Since \ddot{a} is always negative for matter contribution and it is negligible after $t = t_{\text{tr}}$, the inclusion of matter part in our model will not change the characteristic feature of the accelerating universe except that it could shift the transition time slightly.

Our solution is obtained in ten-dimensional spacetime and crucially depends on the condition (8). It would be interesting to check whether relaxing these conditions could lead to some more general solutions.

We would like to thank an anonymous referee for helpful comments. The work of S. T. H. was supported by the Korea Research Foundation (MOEHRD), Grant

No. KRF-2006-331-C00071, and by the Korea Research Council of Fundamental Science and Technology (KRCF), Grant No. C-RESEARCH-2006-11-NIMS. J. L. was supported in part by the University of Seoul through 2007 UOS Research Fund. The work of T. H. L. was supported by the Soongsil University Research Fund, and by the Korea Research Foundation Grant funded by the Korean Government (MOEHRD, Basic Research Promotion Fund) (KRF-2007-313-C00165). P. O. was supported by the Science Research Center Program of the Korea Science and Engineering Foundation through the Center for Quantum Spacetime(CQUeST) of Sogang University with Grant No. R11-2005-021.

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