# Quenching of orbital momentum by crystalline fields in a multichannel Kondo impurity

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We consider an impurity of spin $S$ interacting via an isotropic spin exchange with conduction electrons of spin $\frac{1}{2}$. The conduction electrons can be in $n$ different orbital channels. We assume that crystalline fields split the orbital degrees of freedom into two multiplets, the one with lower energy consisting of $n^*$ orbitals and the one of higher energy of $n-n^*$ orbitals. The exchange coupling is the same for all channels. We derive the thermodynamic Bethe ansatz equations for this model and discuss the ground-state properties of the impurity as a function of the spin $S$ and the magnetic field. The solution of the ground-state Bethe ansatz equations is obtained numerically. Three situations have to be distinguished when the magnetic field is small compared to the Kondo temperature: (i) If $S=n/2$ or $S=n^*/2$ the conduction electrons exactly compensate the impurity spin into a singlet ground state, (ii) if $S>n/2$ the impurity is undercompensated, i.e., only partially compensated leaving an effective spin $S-n/2$ at low temperatures, and (iii) in all other cases the impurity spin is overcompensated giving rise to critical behavior. The quenching of the orbits by the crystalline field dramatically affects the cases $S<n/2$, i.e., the critical behavior of the overcompensated multichannel Kondo impurity and the singlet ground state with $S=n^*/2$.

## I. INTRODUCTION

Probably the most exciting model of a magnetic impurity embedded in a simple metallic lattice is the $n$-channel Kondo problem. The model for an impurity spin $S$ and an arbitrary number of orbital conduction-electron channels is given by[1]

$$H_K=\sum_{k,m,\sigma}\epsilon_k c^{\dagger}_{km\sigma}c_{km\sigma} +J\sum_{k,k',m,\sigma,\sigma'}\mathbf{S}\cdot c^{\dagger}_{km\sigma}\boldsymbol{\sigma}_{\sigma\sigma'}c_{k'm\sigma'}\,, \tag{1.1}$$

where $\mathbf{S}$ are the spin operators describing the magnetic impurity, $J$ is the exchange coupling, $\sigma$ are Pauli matrices and $m$ labels the $n$ orbital channels. The first exhaustive analysis of the model is due to Nozières and Blandin[1] within a renormalization-group approach. The Hamiltonian was then later exactly diagonalized by means of Bethe's ansatz by Andrei and Destri[2] and Wiegmann and Tsvelick.[3-5] Other treatments of the model include the conformal field theory approach,[6] the bosonization of the conduction electrons[7] and a perturbative $1/n$ expansion.[8]

Although the Hamiltonian is diagonal in $m$ the different orbital channels are not independent of each other. On the contrary, the exact solution,[2,3] confirming earlier renormalization-group results,[1] shows that the orbital channels are strongly correlated close to the impurity and form an orbital singlet. In this way, the spins of the conduction electrons are glued together to form a total spin $s_e=n/2$, which compensates the impurity degrees of freedom partially or totally. As a function of the number of channels $n$ and the impurity spin $S$, we have to distinguish three qualitatively different situations:[1-11]

(i) If $n=2S$ the number of conduction electron channels is exactly sufficient to compensate the impurity spin into a singlet, giving rise to Fermi-liquid behavior at low $T$ ($T\ll T_K$, where $T_K$ is the Kondo temperature). This situation is believed to be realized for Fe and Cr impurities in simple metals,[11,12] like Cu and Ag.

(ii) If $n<2S$ the impurity spin is only partially compensated (undercompensated spin), since there are not enough conduction-electron channels to yield a singlet ground state. This leaves an effective-spin degeneracy (in zero field) at low $T$ of $(2S+1-n)$. The integer-valent limit of impurities with two magnetic configurations like Tm (Refs. 11 and 13) or Tb (Ref. 14) embedded in a metal could be related to this situation.

(iii) If $n>2S$ the number of conduction-electron channels is larger than required to compensate the impurity spin. The impurity is said to be overcompensated and critical behavior is obtained as the temperature and the external field tend to zero.[2,11] Applications for this case are the quadrupolar Kondo effect[15,16] and electron-assisted tunneling of an atom in a double-well potential.[17-19] A very convincing experimental realization of overcompensated behavior is the differential resistance of metal point contacts containing structural disorder,[20] which scales with $T$ with an exponent $\frac{1}{2}$ as expected from conformal field theory for the two-channel Kondo problem.

In particular, the non-Fermi-liquid behavior in the overcompensated case has recently drawn considerable attention. The critical properties can be understood in terms of an essential singularity in the entropy of the impurity at $H=T=0$. For $S<n/2$ and $H=0$ the ground-state entropy corresponds to a fractional spin,[5,9-11]

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$$\mathcal{S}(T=0,H=0)$$
$$=\ln\{\sin[\pi(2S+1)/(n+2)]/\sin[\pi/(n+2)]\} . \quad (1.2)$$

If $H\neq 0$, on the other hand, the ground state is a singlet and the entropy is zero. Under renormalization the model scales into a strong-coupling fixed point with a finite value of interaction constant,[1] leading to power-law dependences as $H$ and $T$ tend to zero. For $n>2$ the low-field zero-temperature magnetization and susceptibility are[2,4,10,11]

$$M_{\rm imp}\propto(H/T_K)^{2/n},\quad \chi_{\rm imp}\propto(H/T_K)^{(2/n)-1} . \quad (1.3)$$

The exponent of $\chi$ vanishes for $n=2$ (and $S=1/2$) and a logarithmic divergence is obtained instead, $M_{\rm imp}\propto(H/T_K)\ln(H/T_K)$ and $\chi_{\rm imp}\propto\ln(H/T_K)$. Similarly for $H=0$ and low $T$ we have with $\tau=4/(n+2)$ (Refs. 5 and 9–11)

$$C_{\rm imp}\propto(T/T_K)^{\tau},\quad \chi_{\rm imp}\propto(T/T_K)^{\tau-1} . \quad (1.4)$$

Again for $n=2$ ($S=1/2$) the exponents of $C_{\rm imp}/T$ and $\chi_{\rm imp}$ vanish, giving rise to a logarithmic dependence. Note that the exponents are independent of the impurity spin and that the limits $H\to 0$ and $T\to 0$ cannot be interchanged.

The instability of the overcompensated fixed point to a magnetic field was confirmed both by numerical renormalization-group calculations[21] and conformal field theory.[22] The question of the stability of the non-Fermi-liquid fixed point to perturbations to the system is of great interest. Besides the magnetic field, two further symmetry-breaking fields have been investigated in detail. (i) For the overscreened impurity, exchange anisotropy is irrelevant at the low-temperature fixed point for $S=\frac{1}{2}$ or $S=(n-1)/2$, but it is relevant for all other values of $S$.[21,22] The model with anisotropic exchange, i.e., the exchange $J\mathbf{S}\cdot\boldsymbol{\sigma}$ is replaced by $J_{\parallel}S_z\sigma_z+J_{\perp}(S_x\sigma_x+S_y\sigma_y)$ in Eq. (1.1), has also been studied with bosonization methods.[7] (ii) The fixed point is also unstable to a channel-symmetry breaking, i.e., an exchange coupling that is different for the various channels. The consequences of channel-symmetry breaking have been investigated using conformal field theory,[22] the numerical renormalization group,[21] bosonization techniques,[23] and the Bethe ansatz.[24]

The purpose of this paper is to study the response of the multichannel Kondo model to a crystalline field. We assume here that the exchange coupling is isotropic and remains the same for all orbital channels. A crystalline field also breaks the symmetry between the different channels by changing their electron population. Different orbitals then contribute differently to the screening of the impurity spin. The properties of the low-temperature fixed point are only dramatically affected by the crystalline field in the overcompensated case. To simplify we assume that the $n$ channels are split into two manifolds, the one of lower energy consisting of $n^*$ orbitals and the one of higher energy is $(n-n^*)$-fold degenerate. The ground-state Bethe ansatz equations for the metal then consist of two rapidity bands that are totally filled in zero magnetic field and partially filled if the field is nonzero. All other rapidity bands are empty. This contrasts to the situation without crystalline fields where only one rapidity band contributes. As a consequence now for $S=n/2$ or $S=n^*/2$ the ground state of the impurity is a singlet (Fermi-liquid behavior), and it remains overcompensated for all other values of $S$ ($S<n/2$), but with modified critical exponents and residual entropy. We present numerical results for the ground-state magnetization and susceptibility as a function of magnetic field for $n=4, n^*=2$ and $S\leq 5/2$, which is the simplest situation representing all possible cases.

It is not very surprising that crystalline fields lead to similar results as an exchange coupling that is different for the various channels,[24] as long as the asymmetry in the channel coupling is small. In both cases the symmetry between the channels is broken, although the underlying mechanisms are physically different.

The rest of the paper is organized as follows. In Sec. II we briefly restate Andrei and Destri's[2] Bethe ansatz solution of the multichannel Kondo model pointing out the changes introduced by the crystalline field. The modifications affect the driving terms of the thermodynamic Bethe ansatz equations satisfied by the energy potentials of the rapidity bands. In Sec. III we obtain the ground-state integral equations by considering the limit $T\to 0$. The numerical results (and some analytical ones) are presented in Sec. IV. Conclusions and some remarks on analogies to Heisenberg chains with impurity spins follow in Sec. V.

## II. BETHE ANSATZ EQUATIONS

Model (1.1) is integrable only if the kinetic energy of the electrons is linear in the momentum, i.e., all electrons move with Fermi velocity. A straight application of Bethe's ansatz then yields that the scattering matrix is diagonal in the orbital channel, i.e., proportional to $\delta_{m,m'}$.[2,3,25] The spin and orbital channels, however, cannot be completely independent in a multiparticle process, as can be verified from the perturbative second-order corrections to the vertex function (or the invariant coupling).[1] The $\delta_{m,m'}$ in the Bethe ansatz scattering matrix is an artifact of the linearization (and hence lack of energy cutoff) of the kinetic energy. Andrei and Destri[2] overcame this drawback by adopting an artificial cutoff procedure involving a quadratic term in the momentum, i.e., $\epsilon_k=k-k_F+\Upsilon^{-1}(k-k_F)^2$. To preserve the integrability local counterterms must be added, which become irrelevant as $\Upsilon\to\infty$. This regularization scheme applies to general $S$ and arbitrary $n$.

Tsvelick and Wiegmann, on the other hand, considered two alternative models. (a) An integrable variant of the Anderson model[3–5,11] with an exchange interaction $U$ (attractive within the orbital channel and repulsive in spin space) that is equivalent to Hamiltonian (1.1) in the spin-compensated situation (i). The exchange $U$ maximizes the total spin of the $f$ shell, leading to an orbital singlet ground state for the shell, i.e., $\mathcal{L}=0$, $S=n/2$, for the appropriate choice of parameters. (b) An exchange model consisting of electrons of effective spin $s_e=n/2$ interacting with an impurity of spin $S_i$ has the same Bethe

ansatz solution as model (1.1) for the orbital singlet.[4] The interaction is a special polynomial of $(\mathbf{S}_i\cdot\mathbf{s}_e)$ of order $\min(2s_e,2S_i)$ defined so that the model is integrable and SU(2) invariant by construction for arbitrary $n$ and $S_i$. It should be pointed out that the equivalence of these alternative models to Hamiltonian (1.1) holds only in the absence of crystalline fields. In the variant of Anderson's model (a) the impurity spin continuously varies from $S=n/2$ to $S=n^*/2$ as a function of the crystalline-field splitting (but the ground state is always spin compensated, see, e.g., Ref. 26). In the case of model (b) crystalline fields would require two classes of fermions, one of spin $s_e=n/2$ and the other one with spin $s_e^*=n^*/2$, hence considerably complicating the construction of an effective model.

In this paper we follow the procedure developed by Andrei and Destri,[2] since it corresponds to the direct diagonalization of the $n$-channel problem. The crystalline-field splitting is incorporated into (1.1) by adding the term

$$H_{\rm cf}=\frac{\Delta}{2}\sum_{k,m,\sigma}\text{sign}(m-n^*-1/2)\,c^\dagger_{km\sigma}c_{km\sigma}\ , \tag{2.1}$$

which lowers the energy of the first $n^*$ bands by $\Delta/2$ and raises the energy of the remaining $n-n^*$ bands by the same amount. Since $H_{\rm cf}$ commutes with Hamiltonian (1.1), the crystalline field does not affect the integrability of the model. The scattering matrices, the wave functions and the discrete Bethe ansatz equations (but not the energy) remain invariant. The model is diagonalized in terms of $n+1$ sets of rapidities:[2] one set of charge rapidities $\{k_j\}, j=1,\ldots,N_e$, where $N_e$ is the total number of electrons, one set of spin rapidities $\{\chi_\gamma\}$, $\gamma=1,\ldots,M$, where $M$ is the number of electrons with down spin, and $n-1$ sets for the orbital (flavor) degrees of freedom $\{\omega_\alpha^{(r)}\}, r=1,\ldots,n-1$, $\alpha=1,\ldots,m_r$. Within the orbital space this solution corresponds to the Young tableau

$$(N_e-m_1)\geq(m_1-m_2)\geq\cdots\geq(m_{n-2}-m_{n-1})\geq m_{n-1}\ .$$

Andrei and Destri[2] have shown that in the thermodynamic limit $L\to\infty$, where $L$ is the length of the box, the orbital rapidities are arranged in strings of the form ($\Delta=0$)

$$\omega_\gamma^{(r)}=\xi_\alpha^n+i(J/2)[n-r+1-2q],\quad q=1,\ldots,n-r\ ,$$
$$\gamma=1,\ldots,m_r,\quad \alpha=1,\ldots,N_e/n,\quad r=0,\ldots,n-1\ , \tag{2.2}$$

where we assumed that $N_e$ is a multiple of $n$ and $\omega_\gamma^{(0)}=k_\gamma/\Upsilon$. These string arrangements assume the equal population of all orbital bands and correspond to orbital singlet states. The crystalline field changes the population of the bands and hence the string structure of the orbital rapidities. With our choice of splitting the bands into two multiplets of degeneracy $n^*$ and $n-n^*$, respectively, a second set of string states has to be incorporated,

$$\omega_\gamma^{(r)}=\xi_\alpha^{n^*}+i(J/2)[n^*-r+1-2q]\ ,$$
$$q=1,\ldots,n^*-r,\quad r=0,\ldots,n^*-1\ . \tag{2.3}$$

The index $\alpha$ in (2.2) now runs from 1 to $m_{n-1}$, while in (2.3) it runs from 1 to $m_1-m_{n-1}$. Below we denote $m_{n-1}$ by $m_e$ (excited band) and $m_1$ by $m_g$ (ground band). $m_g$ and $m_e$ are related to the total number of electrons via

$$N_e=n^*m_g+(n-n^*)m_e\ . \tag{2.4}$$

Inserting the above string solutions into the discrete Bethe ansatz equations one obtains, in the limit of the regularizing cutoff $\Upsilon$ tending to infinity, the following "fused" Bethe ansatz equations:

$$\frac{\chi_\gamma+1+iJS}{\chi_\gamma+1-iJS}\left[\frac{\chi_\gamma+iJn/2}{\chi_\gamma-iJn/2}\right]^{m_e}\left[\frac{\chi_\gamma+iJn^*/2}{\chi_\gamma-iJn^*/2}\right]^{m_g-m_e}=-\prod_{\alpha=1}^{M}\frac{\chi_\gamma-\chi_\alpha+iJ}{\chi_\gamma-\chi_\alpha-iJ}\ , \tag{2.5}$$

for $\gamma=1,\ldots,M$. The fused equations are the consequence of the strong interaction among the orbital channels forming composite spin operators of effective spin $s_e=n/2$ and $s_e^*=n^*/2$, in analogy to the alternate effective-spin model (b) (Refs. 3 and 4) mentioned above. For $m_e=m_g$ ($n$ channels, no crystalline fields) or $m_e=0$ ($n^*$ channels) Eqs. (2.5) are identical to those of Andrei and Destri.[2] The first factor on the left-hand side of (2.5) represents the impurity. The other two parametrize the noninteracting gas of electrons. It is convenient to rewrite (2.5) so that the electron gas factors do not depend on $J$ by rescaling the spin rapidities $\chi_\gamma$ by $J$, i.e., $\Lambda_\gamma=\chi_\gamma/J$. The energy of the system is then given by

$$E=-\frac{1}{L}(m_g-m_e)\sum_{\gamma=1}^{M}[\pi+2\arctan(2\Lambda_\gamma/n^*)]-\frac{1}{L}m_e\sum_{\gamma=1}^{M}[\pi+2\arctan(2\Lambda_\gamma/n)] \tag{2.6}$$

and the magnetization is $S_z=N_e/2-M+S$.

In the thermodynamic limit the solutions to Eqs. (2.5) are strings of arbitrary length,

$$\Lambda_\alpha^{(l),q}=\Lambda_\alpha^{(l)}+i(l+1-2q)/2,\quad q=1,\ldots,l\ , \tag{2.7}$$

where $\Lambda_\alpha^{(l)}$ is the rapidity for the center of mass of a string involving $l$ spin rapidities and $\alpha=1,\ldots,\zeta_l$ is the running index of the set. The number of strings in each set $\zeta_l$ is constrained by the total number of spin rapidities

$$M=\sum_{l=1}^{\infty}l\zeta_l\ . \tag{2.8}$$

These string solutions are inserted into (2.5), the resulting equations logarithmized and differentiated with respect to $\Lambda^{(l)}$. In the thermodynamic limit we introduce distribution functions for the string rapidities, $\rho_l(\Lambda)$, and similarly for the corresponding "hole" distributions, $\rho_{lh}(\Lambda)$, $l=1,2,\ldots$. The density functions then satisfy the following integral equations:

$$\rho_{lh}(\Lambda)+\sum_{k=1}^{\infty}\int d\Lambda' A_{lk}(\Lambda-\Lambda')\rho_k(\Lambda')=\frac{1}{\pi N_e}\sum_{k=1}^{\min(l,2S)}\frac{(l+2S+1-2k)/2}{(\Lambda+1/J)^2+[(l+2S+1-2k)/2]^2}$$
$$+\frac{m_e}{\pi N_e}\sum_{k=1}^{\min(l,n)}\frac{(l+n+1-2k)/2}{\Lambda^2+[(l+n+1-2k)/2]^2}$$
$$+\frac{m_g-m_e}{\pi N_e}\sum_{k=1}^{\min(l,n^*)}\frac{(l+n^*+1-2k)/2}{\Lambda^2+[(l+n^*+1-2k)/2]^2}\ , \tag{2.9}$$

where $A_{lk}(\Lambda)$ is the Fourier transform of

$$\hat{A}_{lk}(\omega)=\coth(|\omega|/2)[\exp(-|l-k||\omega|/2)-\exp(-(l+k)|\omega|/2)]\ . \tag{2.10}$$

The first term on the right-hand side of (2.9) represents the impurity, while the other two are the driving terms for the gas of electrons. In terms of the string states the energy and the $\zeta_l$ are given by

$$E=-\frac{(m_g-m_e)}{L}\sum_{l=1}^{\infty}\sum_{k=1}^{\min(l,n^*)}\int d\Lambda\,\rho_l(\Lambda)\left[\pi+2\arctan\left[\frac{2\Lambda}{l+n^*+1-2k}\right]\right]$$
$$-\frac{m_e}{L}\sum_{l=1}^{\infty}\sum_{k=1}^{\min(l,n)}\int d\Lambda\,\rho_l(\Lambda)\left[\pi+2\arctan\left[\frac{2\Lambda}{l+n+1-2k}\right]\right]\ , \tag{2.11}$$
$$\zeta_l=N_e\int d\Lambda\,\rho_l(\Lambda)\ .$$

The above relations are valid without restrictions in the thermodynamic limit. Thermal equilibrium is imposed by minimizing the free-energy functional with respect to the particle and hole density functions, subject to the constraints (2.9) and a constant magnetization. The entropy is given by the Fermi statistics of the rapidities. It is usual to introduce an energy potential for each class of strings

$$\epsilon_l(\Lambda)=T\ln(\rho_{lh}/\rho_l)=T\ln(\eta_l)\ , \tag{2.12}$$

where $T$ is the temperature. The thermodynamic Bethe ansatz equations are then given by

$$\ln[1+\eta_l]=\frac{g_l(\Lambda)}{T}+\sum_{k=1}^{\infty}\int d\Lambda' A_{lk}(\Lambda-\Lambda')\ln[1+(\eta_k(\Lambda'))^{-1}]\ ,$$
$$g_l(\Lambda)=lH-\frac{(m_g-m_e)}{L}\sum_{k=1}^{\min(l,n^*)}\left[\pi+2\arctan\left[\frac{2\Lambda}{l+n^*+1-2k}\right]\right]$$
$$-\frac{m_e}{L}\sum_{k=1}^{\min(l,n)}\left[\pi+2\arctan\left[\frac{2\Lambda}{l+n+1-2k}\right]\right]\ , \tag{2.13}$$

where $H$ is the magnetic field. These equations can be inverted yielding the following recursion relations:

$$\ln[\eta_l(\Lambda)]=\int d\Lambda'\{2\cosh[\pi(\Lambda-\Lambda')]\}^{-1}$$
$$\times\ln\{[1+\eta_{l-1}(\Lambda')][1+\eta_{l+1}(\Lambda')]\}$$
$$-\frac{2}{LT}[m_e\delta_{l,n}+(m_g-m_e)\delta_{l,n^*}]$$
$$\times\arctan(e^{\pi\Lambda})\ , \tag{2.14}$$

where $\eta_0\equiv 0$. Since equations (2.14) do not explicitly depend on the magnetic field, they are completed by the asymptotic condition

$$\lim_{l\to\infty}\frac{1}{l}\ln[\eta_l(\Lambda)]=\frac{H}{T}=2X_0\ . \tag{2.15}$$

The free energy of the system can be written as the sum of three terms, $F=F_n+F_{n^*}+F_{2S}$, which are defined as

$$F_n=-m_e\int d\Lambda[2\cosh(\pi\Lambda)]^{-1}$$
$$\times\{T\ln[1+\eta_n(\Lambda)]-g_n(\Lambda)\}\ ,$$
$$F_{n^*}=-(m_g-m_e)\int d\Lambda[2\cosh(\pi\Lambda)]^{-1}$$
$$\times\{T\ln[1+\eta_{n^*}(\Lambda)]-g_{n^*}(\Lambda)\}\ , \tag{2.16}$$
$$F_{2S}=-\int d\Lambda[2\cosh(\pi\Lambda+\pi/J)]^{-1}$$
$$\times\{T\ln[1+\eta_{2S}(\Lambda)]-g_{2S}(\Lambda)\}\ .$$

The first two terms represent the free energy of the noninteracting electron gas, while the last term is the impurity free energy. The free energy of the host can be rewritten as

$$F_{\rm host}=-\frac{\pi}{2}n^*\frac{m_g^2}{L}-\frac{\pi}{2}n\frac{m_e^2}{L}$$
$$-T\int d\Lambda[2\cosh(\pi\Lambda)]^{-1}$$
$$\times\{(m_g-m_e)\ln[1+\eta_{n^*}(\Lambda)]$$
$$+m_e\ln[1+\eta_n(\Lambda)]\}\ , \qquad (2.17)$$

where the first terms represent the ground-state energy in zero magnetic field and the integrals contain all the temperature and field dependence.

In a similar way we may rewrite the free energy of the impurity as

$$F_{\rm imp}=-\frac{(m_g-m_e)}{L}\sum_{k=1}^{\min(2S,n^*)}I_{n^*+2S+1-2k}$$
$$-\frac{m_e}{L}\sum_{k=1}^{\min(2S,n)}I_{n+2S+1-2k}$$
$$-T\int d\Lambda[2\cosh(\pi\Lambda+\pi/J)]^{-1}$$
$$\times\ln[1+\eta_{2S}(\Lambda)]\ , \qquad (2.18)$$

where

$$I_k=\int d\Lambda\frac{\pi+2\arctan(2\Lambda/k)}{2\cosh(\pi\Lambda+\pi/J)}$$
$$=\frac{\pi}{2}+i\ln\left\{\frac{\Gamma[(k+3)/4+i/(2J)]}{\Gamma[(k+3)/4-i/(2J)]}\right.$$
$$\left.\times\frac{\Gamma[(k+1)/4-i/(2J)]}{\Gamma[(k+1)/4+i/(2J)]}\right\}\ . \qquad (2.19)$$

In the following section we show that the first two terms in (2.18) represent the ground-state energy for the impurity in zero field for all spin values $S$.

Note that Eqs. (2.14)–(2.16) are similar to those derived by Andrei and Jerez[24] for exchange channel-symmetry breaking.

## III. GROUND-STATE INTEGRAL EQUATIONS

To consider the $T\to 0$ limit of Eqs. (2.13) and (2.14) we define $\epsilon_l^+$ and $\epsilon_l^-$ as the positive and negative parts of the energy potentials $\epsilon_l$. According to the Fermi statistics of the rapidities, states with positive potential are empty and those with negative potential are occupied in the ground state. It follows from (2.14) and (2.15) that all $\epsilon_l$ are positive for all $\Lambda$ except $\epsilon_n(\Lambda)$ and $\epsilon_{n^*}(\Lambda)$, which can become negative. Hence, the only populated rapidity bands in the ground state correspond to $n$ strings and $n^*$ strings of spin rapidities. In particular, if $H=0$ the string rapidities fill the entire real axis and a solution can straightforwardly be obtained by Fourier transformation

$$\epsilon_n(\Lambda)=-2\frac{m_e}{L}\arctan(e^{\pi\Lambda})\ ,$$
$$\epsilon_{n^*}(\Lambda)=-2\frac{(m_g-m_e)}{L}\arctan(e^{\pi\Lambda})\ . \qquad (3.1)$$

Similarly the distribution density for the string states can be obtained from (2.9)

$$\rho_n(\Lambda)=\frac{m_e}{N_e}[2\cosh(\pi\Lambda)]^{-1}\ ,$$
$$\rho_{n^*}(\Lambda)=\frac{(m_g-m_e)}{N_e}[2\cosh(\pi\Lambda)]^{-1}\ . \qquad (3.2)$$

All other particle densities are identically zero. Integrating over the densities we have that $\zeta_n=m_e$ and $\zeta_{n^*}=m_g-m_e$, so that the zero-field ground-state magnetization of the electron gas is zero. The results (3.1) and (3.2) hold for $m_g-m_e>0$ and $m_e>0$. In the limits $m_e=m_g$ or $m_e=0$ one of the two bands has zero spectral weight, and (3.1) and (3.2) reproduce the driving energy band and density distribution of the traditional multichannel Kondo model (no crystalline-field splittings) for $n$ and $n^*$ orbital channels, respectively.[2]

The magnetic field acts as the chemical potential for the occupation of the bands. A finite magnetic field raises the energies of the two bands, so that states in the intervals $[-\infty,B_n]$ for $\epsilon_n$ and $[-\infty,B_{n^*}]$ for $\epsilon_{n^*}$ are empty. The two energy potentials now satisfy the following coupled Wiener-Hopf integral equations:

$$\epsilon_n^+(\Lambda)+\int_{B_n}^{\infty}d\Lambda' A_{nn}(\Lambda-\Lambda')\epsilon_n^-(\Lambda')$$
$$+\int_{B_{n^*}}^{\infty}d\Lambda' A_{nn^*}(\Lambda-\Lambda')\epsilon_{n^*}^-(\Lambda')=g_n(\Lambda)\ ,$$
$$\epsilon_{n^*}^+(\Lambda)+\int_{B_{n^*}}^{\infty}d\Lambda' A_{n^*n^*}(\Lambda-\Lambda')\epsilon_{n^*}^-(\Lambda') \qquad (3.3)$$
$$+\int_{B_n}^{\infty}d\Lambda' A_{n^*n}(\Lambda-\Lambda')\epsilon_n^-(\Lambda')=g_{n^*}(\Lambda)\ ,$$

where the integration limits are determined by the zeroes of the energy potentials, $\epsilon_n(B_n)=0$ and $\epsilon_{n^*}(B_{n^*})=0$. Obviously $B_n=B_{n^*}=-\infty$ if the field is zero. The integration limits monotonically increase with the field. The relative values of $B_n$ and $B_{n^*}$ depend on $m_e$ and $m_g$, i.e., on the crystalline-field splitting.

The distribution densities satisfy similar integral equations which are straightforwardly obtained from (2.9). The integration limits are the same as in (3.3). The driving terms of the integral equations for the density functions consist of terms for the host and for the impurity. Since the equations are linear, it is convenient to separate $\rho_l$ into host and impurity contributions, $\rho_l(\Lambda)=\rho_l^{\rm host}(\Lambda)+(1/N_e)\rho_l^{\rm imp}(\Lambda)$. The magnetization of the electron gas and the impurity are then given by

$$M_{\rm host}=\frac{N_e}{2}-N_e n^*\int_{B_{n^*}}^{\infty}d\Lambda\,\rho_{n^*}^{\rm host}(\Lambda)$$
$$-N_e n\int_{B_n}^{\infty}d\Lambda\,\rho_n^{\rm host}(\Lambda)$$
$$=\frac{N_e}{2}\int_{-\infty}^{B_n}d\Lambda\,\rho_{nh}^{\rm host}(\Lambda)\ ,$$
$$M_{\rm imp}=S-n^*\int_{B_{n^*}}^{\infty}d\Lambda\,\rho_{n^*}^{\rm imp}(\Lambda)-n\int_{B_n}^{\infty}d\Lambda\,\rho_n^{\rm imp}(\Lambda) \qquad (3.4)$$
$$=S-\frac{1}{2}\min(2S,n)+\frac{1}{2}\int_{-\infty}^{B_n}d\Lambda\,\rho_{nh}^{\rm imp}(\Lambda)\ .$$

In the absence of a magnetic field the "hole" densities for the impurity and the host are identically zero, so that the magnetization of the electron gas vanishes. The magnetization of the impurity is zero for $S \leq n/2$, but $M_{\rm imp} = S - n/2$ if $S > n/2$. In the latter case the impurity spin is only partially compensated, because there are not enough conduction-electron channels to produce a singlet (undercompensated impurity).

Finally, we show that the ground-state energy of the impurity in zero field is given by the first two terms of (2.18). It is equivalent to show that the last term in (2.18) vanishes as $T \to 0$, i.e.,

$$\int_{-\infty}^{\infty} d\Lambda [2\cosh(\pi\Lambda + \pi/J)]^{-1} \epsilon_{2S}^{+}(\Lambda) = 0 \ . \tag{3.5}$$

This is obviously the case for $S = n^*/2$ and $S = n/2$, since the corresponding rapidity bands are completely filled. For other spin values we use that

$$\epsilon_{2S}^{+}(\Lambda) = g_{2S}(\Lambda) - \int_{-\infty}^{\infty} d\Lambda' A_{2S\,n^*}(\Lambda - \Lambda') \epsilon_{n^*}^{-}(\Lambda') \ ,$$
$$- \int_{-\infty}^{\infty} d\Lambda' A_{2S\,n}(\Lambda - \Lambda') \epsilon_{n}^{-}(\Lambda') = 0 \ , \tag{3.6}$$

which is proven from the zero-temperature limit of (2.13), the $T = H = 0$ solution (3.1) and the asymptotic field condition (2.15). Hence, the zero-field ground-state energy of the impurity is

$$E_G = -\frac{(m_g - m_e)}{L} \sum_{k=1}^{\min(2S, n^*)} I_{n^* + 2S + 1 - 2k}$$
$$- \frac{m_e}{L} \sum_{k=1}^{\min(2S, n)} I_{n + 2S + 1 - 2k} \ . \tag{3.7}$$

## IV. RESULTS

In this section we present results extracted from the integral equations derived in the previous sections. We first discuss the ground-state entropy of the impurity for $H = 0$, which is nonzero except for $S = n^*/2$ and $S = n/2$. Then we relate the integration limits $B_{n^*}$ and $B_n$ to the magnetic and crystalline fields. Some general conclusions about the ground state of the impurity are given. Finally, the impurity magnetization and susceptibility obtained numerically for $n = 4, n^* = 2$ and several values of the impurity spin are presented.

### A. Zero-field ground-state entropy

In order to obtain the ground-state entropy it is necessary to solve Eqs. (2.14), subject to the asymptotic condition (2.15) for $\Lambda \to \infty$. For sufficiently large $\Lambda$ the driving terms become independent of $\Lambda$. The $\Lambda'$ integration can be carried out and the following recursion relation is obtained:

$$\ln[\eta_l] = \frac{1}{2} \ln[(1 + \eta_{l+1})(1 + \eta_{l-1})]$$
$$- \frac{\pi}{LT} [m_e \delta_{l,n} + (m_g - m_e) \delta_{l,n^*}] \ . \tag{4.1}$$

At very low $T$ the driving terms are large and negative, so that necessarily $\eta_{n^*} = \eta_n = 0$. The general solution of (4.1) is then

$$\ln(1 + \eta_l) = \begin{cases} 2 \ln \left[ \dfrac{\sinh[(l + 1 - n) X_0]}{\sinh(X_0)} \right], & \text{for } l \geq n \ ; \\ 2 \ln \left[ \dfrac{\sin[\pi(l + 1 - n^*)/(n + 2 - n^*)]}{\sin[\pi/(n + 2 - n^*)]} \right], & \text{for } n \geq l \geq n^* \ ; \\ 2 \ln \left[ \dfrac{\sin[\pi(l + 1)/(n^* + 2)]}{\sin[\pi/(n^* + 2)]} \right], & \text{for } n^* \geq l \geq 1 \ . \end{cases} \tag{4.2}$$

Using this result in the last term of (2.18) we obtain that the zero-temperature zero-field entropy is

$$\mathcal{S}(T = 0, H = 0) = \frac{1}{2} \ln(1 + \eta_{2S}) \tag{4.3}$$

with $\eta_{2S}$ given by (4.2). It is usual to perform the replacement $\pi(\Lambda + 1/J) \to \pi\Lambda - \ln(T_K/T)$ before integrating to absorb the temperature in the driving terms. In this way $\Lambda \to -\infty$ corresponds to the high-temperature limit. The solution of (4.1) in the high-temperature limit is

$$\ln(1 + \eta_l) = 2 \ln \left[ \frac{\sinh[(l + 1) X_0]}{\sinh(X_0)} \right] , \tag{4.4a}$$

and the high-$T$ free energy is the one of a free spin $S$,

$$F_{\rm imp} = -T \ln \left[ \frac{\sinh[(2S + 1) X_0]}{\sinh(X_0)} \right] . \tag{4.4b}$$

Note that (4.3) is only valid for $H = T = 0$. Three situations must be distinguished. (i) As already mentioned the ground state of the impurity is a singlet if $S = n/2$ or $S = n^*/2$. In either case there is a band of rapidities of fused spins to exactly compensate the impurity spin. The second band is there as a consequence of the crystalline-field splitting. (ii) If $S > n/2$ the number of orbital channels is insufficient to yield a singlet ground state and according to (3.4) there is a remaining spin of magnitude $(S - n/2)$. The entropy of the impurity at $T = H = 0$ is $\ln[(2S - n) + 1]$ and an arbitrarily small magnetic field quenches the remaining spin, so that the $T = 0$ entropy is zero (singlet). (iii) All other cases correspond to an over-

compensated impurity, i.e., the magnetization vanishes in zero field, but the entropy is nonzero corresponding to a fractional spin. The entropy has an essential singularity at $H=T=0$ giving rise to critical behavior.

### B. Integration limits as a function of $H$ and $\Delta$

The integration limits $B_{n^*}$ and $B_n$ are determined by the magnetic and crystalline fields through Eqs. (3.3). In general these Wiener-Hopf integral equations have to be solved numerically by discretizing the integrals. Here we consider $n=4$ and $n^*=2$, which is the simplest case containing all possibilities discussed in Sec. IV A. It is useful to introduce the occupation densities of the two lower lying (ground) bands, $n_g$, and the excited bands, $n_e$,

$$n_g=\frac{2m_g}{N_e}=0.5(1+x), \quad n_e=\frac{2m_e}{N_e}=0.5(1-x) . \tag{4.5}$$

Note that $n_g+n_e=1$ and $n_g-n_e=x$. Hence $x$ is just twice the density of states per orbital times the crystalline-field splitting $\Delta$.

For sufficiently small magnetic fields (fields much smaller than the electron cutoff or the inverse of the density of states) and constant crystalline-field splitting $x$ the difference $B_2-B_4$ is constant and the magnetic field is proportional to $\exp(\pi B_4)$. In other words, the difference between the integration limits parametrizes the crystalline-field splitting as shown in Fig. 1. $B_2-B_4$ decreases monotonically with $x$. For $x\to 0$ $B_2$ diverges since the two-string band is depopulated. For sufficiently small $x$ we obtain numerically

$$B_2-B_4=-0.2164\ln(x)-0.4545 .$$

The relation $H\propto\exp(\pi B_4)$ can be made plausible invoking (3.1) (valid only in zero magnetic field) and the fact that the field acts as the chemical potential for the spin strings. For sufficiently small $H$, $B_2$ and $B_4$ are sufficiently negative so that $\arctan(e^{\pi B})\approx e^{\pi B}$, so that $H$ and $e^{\pi B}$ are proportional. The proportionality constant has to be determined numerically for each $x$. The magnetization of the host is proportional to $H$ and (within the numerical accuracy) the susceptibility is independent of $x$. This independence of the magnetic susceptibility on $x$ is not surprising because the density of states is constant and the same for all orbital bands. The above considerations remain valid for arbitrary $n$ and $n^*$, i.e., $B_{n^*}-B_n$ parametrizes the crystalline-field splitting and $H\propto\exp(\pi B_n)$.

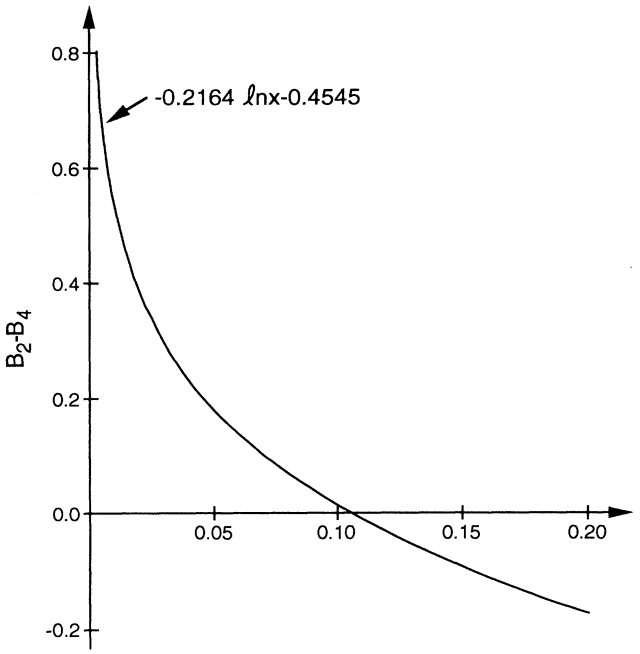


FIG. 1. The integration limits $B_2$ and $B_4$ are determined by the magnetic and crystalline fields through Eqs. (3.3). The magnetic field can be considered proportional to $\exp(\pi B_4)$. The difference between the integration limits $B_2-B_4$ parametrizes the crystalline-field splitting. $x$ is twice the density of states per orbital times $\Delta$. $B_2-B_4$ decreases monotonically with $x$ and diverges for $x\to 0$ since the two-string band has to get depopulated when the splitting vanishes.

### C. Impurity magnetization and susceptibility

To discuss the impurity magnetization and susceptibility we first state the coupled integral equations to be solved. Defining $\sigma_l(\Lambda)=\rho_l^{\rm imp}(\Lambda+B_n)$, we have from (2.9)

$$\sigma_{nh}(\Lambda)+\int_0^\infty d\Lambda' A_{nn}(\Lambda-\Lambda')\sigma_n(\Lambda')+\int_{(B_{n^*}-B_n)}^\infty d\Lambda' A_{nn^*}(\Lambda-\Lambda')\sigma_{n^*}(\Lambda')$$
$$=\frac{1}{\pi}\sum_{k=1}^{\min(n,2S)}\frac{(n+2S+1-2k)/2}{[\Lambda+(1/\pi)\ln(H/T_H)]^2+[(n+2S+1-2k)/2]^2} ,$$
$$\tag{4.6}$$

$$\sigma_{n^*h}(\Lambda)+\int_{(B_{n^*}-B_n)}^\infty d\Lambda' A_{n^*n^*}(\Lambda-\Lambda')\sigma_{n^*}(\Lambda')+\int_0^\infty d\Lambda' A_{n^*n}(\Lambda-\Lambda')\sigma_n(\Lambda')$$
$$=\frac{1}{\pi}\sum_{k=1}^{\min(n^*,2S)}\frac{(n^*+2S+1-2k)/2}{[\Lambda+(1/\pi)\ln(H/T_H)]^2+[(n^*+2S+1-2k)/2]^2} ,$$

where $T_H$ is an energy scale proportional to the Kondo temperature. As $H \to 0$ the Lorentzian driving terms have their maximum at large positive $\Lambda$, so that

$$\xi_n^{\rm imp} = \int_0^\infty d\Lambda\, \sigma_n(\Lambda) = \frac{\min(2S,n) - \min(2S,n^*)}{2(n-n^*)} ,$$
$$\xi_{n^*}^{\rm imp} = \int_{(B_{n^*} - B_n)}^\infty d\Lambda\, \sigma_{n^*}(\Lambda) \tag{4.7}$$
$$= -\frac{\min(2S,n)}{2(n-n^*)} + \frac{\min(2S,n^*)}{2(n-n^*)} \frac{n}{n^*} .$$

As learned from the zero-field zero-temperature entropy we have to distinguish five cases. (a) For $S < n^*/2$, Eq. (4.7) yields $\xi_n^{\rm imp} = 0$ so that the $n$ strings do not contribute to the screening of the impurity spin. The impurity spin is then overcompensated with an effective number of channels $n^*$. (b) If $S = n^*/2$ the ground-state entropy is zero, the $n$ strings still do not contribute, and the impurity spin is compensated into a singlet by the $n^*$ strings. (c) For $n^*/2 < S < n/2$ the impurity spin is screened by both sets of strings. The impurity spin is first screened to an effective spin $S - n^*/2$, which is then overcompensated by an effective number of channels $n - n^*$. (d) If $S = n/2$ the $n$ strings compensate the impurity into a singlet ground state. Here $\xi_{n^*}^{\rm imp} = 0$ so that the $n^*$ strings do not contribute. (e) For $S > n/2$ the impurity spin is undercompensated by the $n$ channels, which are not sufficient to produce a singlet ground state. Again, the $n^*$ strings are irrelevant in this case.

If we assume that the crystalline field is small compared to the bandwidth (i.e., $x$ is small), the number of $n$ strings is overwhelmingly larger than the number of $n^*$ strings. Hence, we expect the cases where $n^*$ strings contribute to the screening, i.e., cases (a), (b), and (c), to strongly depend on the crystalline field, but not the situations (d) and (e), where the spin compensation is only due to the $n$ strings. For the overcompensated situations (a) and (c) the leading critical behavior is again given by (1.3) and (1.4) with the exponents determined by the effective number of channels, i.e., $n^*$ [case (a)] and $(n - n^*)$ [case (c)].

To illustrate the above considerations we now present numerical results for the magnetization and the susceptibility for $n^* = 2$ and $n = 4$. To cover all five qualitatively different cases we consider five spin values, $S = 1/2, 1, \ldots, 5/2$.

For $S = \frac{1}{2}$ the residual entropy [see (4.3)] is $\mathcal{S}(T = H = 0) = \frac{1}{2}\ln(2)$, and the effective number of channels is $n^* = 2$. This effectively corresponds to the two-channel Kondo problem, or equivalently to the quadrupolar Kondo effect or to electron-assisted tunneling of an atom between two sites with two orbital channels. Hence, for $x \neq 0$, the susceptibility has a logarithmic field-dependence, diverging as $H \to 0$ as seen in Fig. 2(a). The slope of the logarithmic term increases as $x$ decreases, and for $x = 0$ the power law (the exponent is $-0.5$) corresponding to the four-channel Kondo problem is recovered. In Fig. 2(b) the magnetization as a function of field is shown for three crystalline-field splittings. Note the different field dependence at low fields, while at high fields the free spin $S = \frac{1}{2}$ is reached asymptotically on a logarithmic scale.

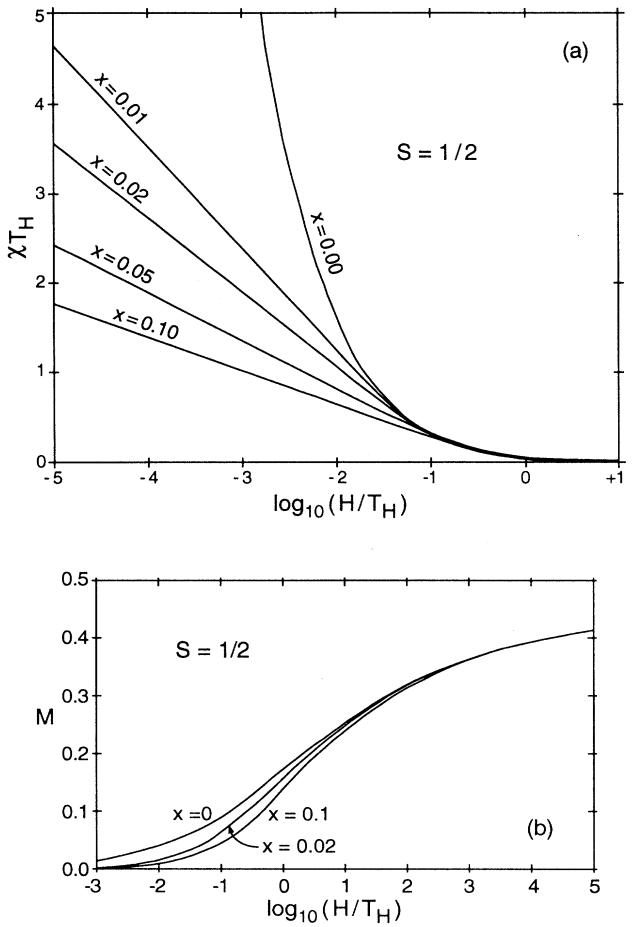


FIG. 2. (a) Susceptibility and (b) magnetization of a spin-$\frac{1}{2}$ impurity for $n^* = 2$ and $n = 4$ as a function of magnetic field for several crystalline-field splittings. For $x \neq 0$ the low-field dependence of the susceptibility is logarithmic, since the effective number of channels available for the overcompensation of the spin is two. For $x = 0$ the susceptibility diverges according to (1.3) with $n = 4$ as $H^{-1/2}$. The magnetization has a marked $x$ dependence at low fields and asymptotically approaches the free-spin value for large fields on a logarithmic scale.

For $S = 1$ and $x \neq 0$ the ground state is a singlet as shown in Fig. 3(a). The zero-field susceptibility strongly depends on the crystalline-field splitting, i.e., on the relative number of two strings. Numerially we obtain $\chi \propto x^{-1/2}$. As $x \to 0$ the $H^{-1/2}$ dependence characteristic of the four-channel Kondo problem is recovered. The magnetization as a function of field for three representative values of $x$ is displayed in Fig. 3(b). Again for small fields the magnetization is $x$ dependent and it reaches the free-spin value asymptotically on a logarithmic scale.

If $S = 3/2$ the effective spin for small fields is $S' = S - n^*/2 = \frac{1}{2}$, and the effective number of channels available for the overcompensation of $S'$ is $n - n^* = 2$.

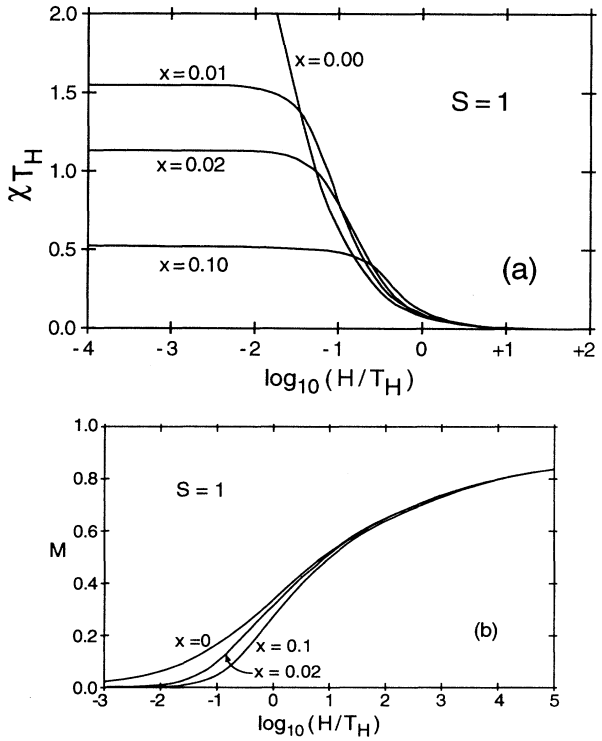


FIG. 3. (a) Susceptibility and (b) magnetization of a spin-1 impurity for $n^*=2$ and $n=4$ as a function of magnetic field for several crystalline-field splittings. For $x\neq 0$ the susceptibility is finite as $H\rightarrow 0$ as a consequence of the spin compensation into a singlet state by the $n^*$ strings. For $x=0$ the susceptibility diverges according to (1.3) with $n=4$ as $H^{-1/2}$. The magnetization has a marked $x$ dependence at low fields and asymptotically approaches the free-spin value for large fields on a logarithmic scale.

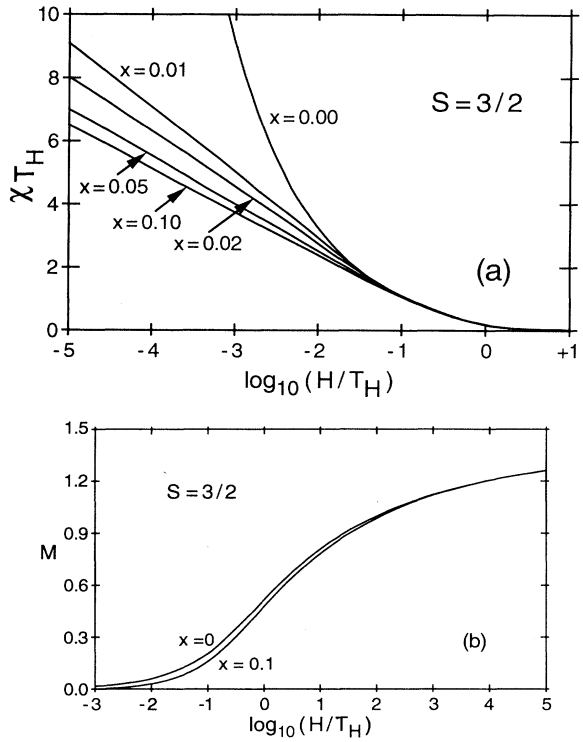


FIG. 4. (a) Susceptibility and (b) magnetization of a spin-$\frac{3}{2}$ impurity for $n^*=2$ and $n=4$ as a function of magnetic field for several crystalline-field splittings. For $x\neq 0$ the low-field dependence of the susceptibility is logarithmic, corresponding to an effective spin $S-n^*/2=1/2$ and an effective number of channels available for the overcompensation of the spin $(n-n^*)=2$. The $x$ dependence of the slope is less pronounced than the one for $S=\frac{1}{2}$. For $x=0$ the susceptibility diverges according to (1.3) with $n=4$ as $H^{-1/2}$. The magnetization has a weak $x$ dependence at low fields and asymptotically approaches the free-spin value for large fields on a logarithmic scale.

Hence, again a logarithmic field dependence is obtained as $H\rightarrow 0$ as shown in Fig. 4(a). The slope of the logarithmic field dependence increases with decreasing $x$. This $x$ dependence is, however, much weaker than for the $S=\frac{1}{2}$ case. For $x=0$ we recover the $H^{-1/2}$ dependence, characteristic of the four-channel problem. The crystalline-field dependence of the magnetization [see Fig. 4(b)] is also much weaker than for smaller spins.

The magnetization for the $S=2$ and $S=\frac{5}{2}$ cases is shown in Fig. 5. The curves are essentially $x$ independent within our numerical accuracy. This is not unexpected, since for these spin values (and higher ones) the screening is driven by the $n$ strings, rather than by the $n^*$ strings. The ground state for $S=2$ is a singlet (the susceptibility is finite), while for $S=\frac{5}{2}$, the spin is undercompensated, leaving an effective low-$T$ spin of $S-n/2=\frac{1}{2}$. The asymptotic free spin is reached on a logarithmic scale in both cases.

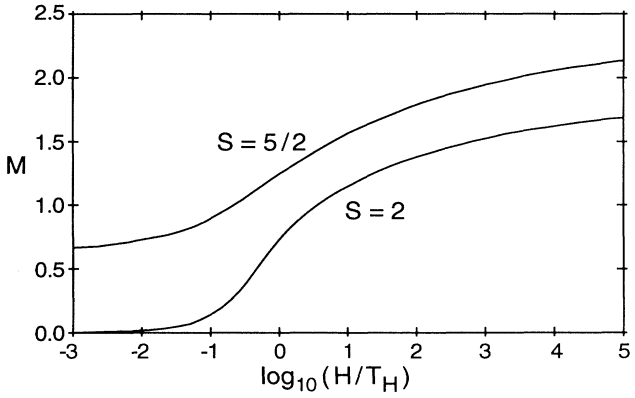


FIG. 5. Magnetization of a spin-2 and a spin-$\frac{5}{2}$ impurity as a function of magnetic field for $n^*=2$ and $n=4$. The curves are essentially independent of $x$, since the $n$ strings rather than the $n^*$ strings compensate the impurity spin. For $S=2$ the zero-field ground state is a singlet (finite susceptibility, completely compensated spin), while for $S=\frac{5}{2}$ the impurity is undercompensated leaving an effective spin $S-n/2=\frac{1}{2}$ at low $T$ and $H$. The magnetization asymptotically approaches the free-spin value for large fields on a logarithmic scale.

## V. CONCLUSIONS

We considered an impurity of spin $S$ embedded into an electron gas of $n$ orbital channels interacting via spin ex-

change ($n$-channel Kondo problem of spin $S$). The $n$ orbitals are split by a crystalline field into two multiplets of degeneracy $n^*$ and $(n-n^*)$, respectively. We derived the thermodynamic Bethe ansatz equations for this system and discussed them in detail as a function of magnetic field in the $T\to 0$ limit. In general we have to consider three different situations for the multichannel Kondo problem: (i) the completely compensated spin case, (ii) the overcompensated impurity, and (iii) the undercompensated impurity spin. We assumed the crystalline-field splitting is small compared to the bandwidth.

In the undercompensated situation the impurity spin is partially screened into an effective spin $(S-n/2)$ at low temperatures and small magnetic fields. All orbitals participate in this partial screening, so that the impurity properties are essentially independent of the crystalline-field splitting.

Two completely compensated cases have to be distinguished. For $S=n/2$ the ground state is a singlet and the zero-field susceptibility is finite. Again, all the orbital channels contribute to the screening, and hence the dependence on the crystalline-field splitting is very small (negligible). For $S=n^*/2$ the ground state is also a singlet, which now is caused by the crystalline-field splitting. The electrons compensating the impurity spin correspond to the population excess of the crystalline-field ground multiplet over the excited multiplet ($n^*$ strings). The zero-field susceptibility at low $T$ is then strongly dependent on the crystalline-field splitting.

We also have to distinguish among two overcompensated cases. Critical behavior is obtained in both cases as a consequence of the essential singularity of the impurity entropy at $T=H=0$. For $S<n^*/2$ only the population excess of the crystalline-field ground multiplet over the excited multiplet plays a role. The critical exponents are given by (1.3) and (1.4) with an effective number of orbital channels $n^*$. The other situation arises when $n^*/2<S<n/2$. Here, the impurity is first screened due to the crystalline field to an effective (undercompensated) spin $S'=S-n^*/2$, which is then overcompensated by an effective number of channels $n'=n-n^*$. The critical exponents are again given by (1.3) and (1.4) for $n'$ channels.

The most relevant applications of the $n$-channel Kondo model are the quadrupolar Kondo effect and electron-assisted tunneling of an atom in a potential well. Experimental realizations are believed to occur for $S=\frac{1}{2}$ and two channels. A crystalline field lifting the degeneracy of the channels then suppresses the marginal critical behavior (logarithmic singularities) and the susceptibility and $C_{\rm imp}/T$ are finite as $H$ and $T$ tend to zero. This result is similar if the channel symmetry is broken in the exchange coupling[24] rather than by crystalline fields.

A system with related properties to the multichannel Kondo impurity is the antiferromagnetic Babujian-Takhtajan Heisenberg chain of spin $S$ with an impurity of spin $S'$.[11,27–30] The Babujian-Takhtajan model[31] is an integrable SU(2)-invariant generalization of the Heisenberg chain to higher spins. The interaction between impurity and chain is constructed so that the integrability is preserved. The properties for low $T$ and small fields are very similar to those of the $n$-channel Kondo model (with $n=2S$ and impurity spin $S'$). Again, we distinguish three cases:[11,29] (i) $S'=S$ the spin-compensated singlet, (ii) $S'>S$, the undercompensated spin, and (iii) $S'<S$, the overcompensated impurity displaying critical behavior.

If instead of an isolated impurity of spin $S'$ in the Babujian-Takhtajan Heisenberg chain of spin-$\frac{1}{2}$, we consider a finite concentration $x$ of impurities the properties of the impurities change dramatically.[32] Two Fermi seas play a role in this case in a similar fashion as for the multichannel Kondo problem with crystalline fields. The ground state in this case becomes a singlet, i.e., the nonanalytic behavior is quenched and two critical fields appear as the magnetic field gradually depletes the rapidity bands. The limit $x\to 0$, $H\to 0$ and $T\to 0$ is singular.

## ACKNOWLEDGMENTS

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