



# Anisotropic pressure in dense neutron matter under the presence of a strong magnetic field

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## ABSTRACT

Dense neutron matter with recently developed BSk19 and BSk21 Skyrme effective forces is considered in magnetic fields up to  $10^{20}$  G at zero temperature. The breaking of the rotational symmetry by the magnetic field leads to the differentiation between the pressures along and perpendicular to the field direction which becomes significant in the fields  $H > H_{th} \sim 10^{18}$  G. The longitudinal pressure vanishes in the critical field  $10^{18} < H_c \lesssim 10^{19}$  G, resulting in the longitudinal instability of neutron matter. For the Skyrme force fitted to the stiffer underlying equation of state (BSk21 vs. BSk19) the threshold  $H_{th}$  and critical  $H_c$  magnetic fields become larger. The longitudinal and transverse pressures as well as the anisotropic equation of state of neutron matter are determined under the conditions relevant for the cores of magnetars.

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## 1. Introduction

Magnetars are strongly magnetized neutron stars [1] with emissions powered by the dissipation of magnetic energy. According to one of the conjectures, magnetars can be the source of the extremely powerful short-duration  $\gamma$ -ray bursts [2–5]. The magnetic field strength at the surface of a magnetar is of about  $10^{14}$ – $10^{15}$  G [6,7]. Such huge magnetic fields can be inferred from observations of magnetar periods and spin-down rates, or from hydrogen spectral lines. In the interior of a magnetar the magnetic field strength may be even larger, reaching values of about  $10^{18}$  G [8,9]. Under such circumstances, the issue of interest is the behavior of neutron star matter in a strong magnetic field [8–15].

In the recent study [11], neutron star matter was approximated by pure neutron matter in a model with the effective nuclear forces. It was shown that the behavior of spin polarization of neutron matter in the high density region in a strong magnetic field crucially depends on whether neutron matter develops spontaneous spin polarization (in the absence of a magnetic field) at

several times nuclear matter saturation density, or the appearance of spontaneous polarization is not allowed at the relevant densities (or delayed to much higher densities). The first case is usual for the Skyrme forces [16–26], while the second one is characteristic for the realistic nucleon–nucleon (NN) interaction [27–34]. In the former case, a ferromagnetic transition to a totally spin polarized state occurs while in the latter case a ferromagnetic transition is excluded at all relevant densities and spin polarization remains quite low even in the high density region.

The scenario for the evolution of spin polarization at high densities in which the spontaneous ferromagnetic transition in neutron matter is absent was considered for the magnetic fields up to  $10^{18}$  G [11]. However, it was argued in the recent study [35] that in the core of a magnetar the local values of the magnetic field strength could be as large as  $10^{20}$  G, if to assume the inhomogeneous distribution of the matter density and magnetic field inside a neutron star, or to allow the formation of a quark core in the high-density interior of a neutron star (concerning the last point, see also Ref. [36]). Under such circumstances, a different scenario is possible in which a field-induced ferromagnetic phase transition occurs in the magnetar core. This idea was explored in the recent research [37], where it was shown within the framework of a lowest constrained variational approach with the Argonne  $V_{18}$  NN potential that a fully spin polarized state in neutron matter could be formed in the magnetic field  $H \gtrsim 10^{19}$  G. Note, however,

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that, as was pointed out in the works [35,38], in such ultrastrong magnetic fields the breaking of the  $\mathcal{O}(3)$  rotational symmetry by the magnetic field results in the anisotropy of the total pressure, having a smaller value along than perpendicular to the field direction. The possible outcome could be the gravitational collapse of a magnetar along the magnetic field, if the magnetic field strength is large enough. Thus, exploring the possibility of a field-induced ferromagnetic phase transition in neutron matter in a strong magnetic field, the effect of the pressure anisotropy has to be taken into account because this kind of instability could prevent the formation of a fully polarized state in neutron matter. This effect was not considered in Ref. [37], thus, leaving the possibility of the formation of a fully polarized state of neutron spins in a strong magnetic field open. In the given study, we provide a fully self-consistent calculation of the thermodynamic quantities of spin polarized neutron matter taking into account the appearance of the pressure anisotropy in a strong magnetic field. We consider spin polarization phenomena in a degenerate magnetized system of strongly interacting neutrons within the framework of a Fermi liquid approach [39–42], unlike to the previous works [35,38], where interparticle interactions were switched off.

Note that recently new parametrizations of Skyrme forces were suggested, BSk19–BSk21 [43], aimed to avoid the spontaneous spin instability of nuclear matter at densities beyond the nuclear saturation density for vanishing temperature. This is achieved by adding new density-dependent terms to the standard Skyrme interaction. The BSk19 parametrization was constrained to reproduce the equation of state (EoS) of nonpolarized neutron matter [44] obtained in variational calculation with the use of the realistic Urbana  $v_{14}$  nucleon–nucleon potential and the three-body force called there TNI. The BSk20 force corresponds to the stiffer EoS [45], obtained in variational calculation with the use of the realistic Argonne  $V_{18}$  two-body potential and the semiphenomenological UIX\* three-body force which includes also a relativistic boost correction  $\delta v$ . Even a stiffer neutron matter EoS was suggested in the Brueckner–Hartree–Fock calculation of Ref. [46] based on the same  $V_{18}$  two-body potential and a more realistic three-body force containing different meson-exchange contributions. This EoS is the underlying one for the BSk21 Skyrme interaction. Further we would like to contrast the results obtained with the Skyrme forces constrained to soft [44] and most stiff [46] underlying EoS, and, by this reason, choose the BSk19 and BSk21 parametrizations in the subsequent analysis.

At this point, it is worthy to note that we consider thermodynamic properties of spin polarized states in neutron matter in a strong magnetic field up to the high density region relevant for astrophysics. Nevertheless, we take into account the nucleon degrees of freedom only, although other degrees of freedom, such as pions, hyperons, kaons, or quarks could be important at such high densities.

## 2. Basic equations

The normal (nonsuperfluid) states of neutron matter are described by the normal distribution function of neutrons  $f_{\kappa_1\kappa_2} = \text{Tr} \varrho a_{\kappa_2}^\dagger a_{\kappa_1}$ , where  $\kappa \equiv (\mathbf{p}, \sigma)$ ,  $\mathbf{p}$  is momentum,  $\sigma$  is the projection of spin on the third axis, and  $\varrho$  is the density matrix of the system [24–26]. The energy of the system is specified as a functional of the distribution function  $f$ ,  $E = E(f)$ , and determines the single particle energy

$$\varepsilon_{\kappa_1\kappa_2}(f) = \frac{\partial E(f)}{\partial f_{\kappa_2\kappa_1}}. \quad (1)$$

The self-consistent matrix equation for determining the distribution function  $f$  follows from the minimum condition of the thermodynamic potential [39,40] and is

$$f = \left\{ \exp(Y_0\varepsilon + Y_i \cdot \mu_n \sigma_i + Y_4) + 1 \right\}^{-1} \equiv \left\{ \exp(Y_0\xi) + 1 \right\}^{-1}. \quad (2)$$

Here the quantities  $\varepsilon$ ,  $Y_i$  and  $Y_4$  are matrices in the space of  $\kappa$  variables, with  $(Y_{i,4})_{\kappa_1\kappa_2} = Y_{i,4}\delta_{\kappa_1\kappa_2}$ ,  $Y_0 = 1/T$ ,  $Y_i = -H_i/T$  and  $Y_4 = -\mu_0/T$  being the Lagrange multipliers,  $\mu_0$  being the chemical potential of neutrons, and  $T$  the temperature. In Eq. (2),  $\mu_n = -1.9130427(5)\mu_N \approx -6.031 \times 10^{-18}$  MeV/G is the neutron magnetic moment [47] ( $\mu_N$  being the nuclear magneton),  $\sigma_i$  are the Pauli matrices. Note that, unlike to Refs. [14,15], the term with the external magnetic field  $\mathbf{H}$  is not included in the single particle energy  $\varepsilon$  but is separately introduced in the exponent of the Fermi distribution (2).

Further it will be assumed that the third axis is directed along the external magnetic field  $\mathbf{H}$ . Given the possibility for alignment of neutron spins along or opposite to the magnetic field  $\mathbf{H}$ , the normal distribution function of neutrons and the matrix quantity  $\xi$  (which we will also call a single particle energy) can be expanded in the Pauli matrices  $\sigma_i$  in spin space

$$f(\mathbf{p}) = f_0(\mathbf{p})\sigma_0 + f_3(\mathbf{p})\sigma_3, \quad (3)$$

$$\xi(\mathbf{p}) = \xi_0(\mathbf{p})\sigma_0 + \xi_3(\mathbf{p})\sigma_3. \quad (4)$$

The distribution functions  $f_0$ ,  $f_3$  satisfy the normalization conditions

$$\frac{2}{V} \sum_{\mathbf{p}} f_0(\mathbf{p}) = \varrho, \quad (5)$$

$$\frac{2}{V} \sum_{\mathbf{p}} f_3(\mathbf{p}) = \varrho_\uparrow - \varrho_\downarrow \equiv \Delta\varrho. \quad (6)$$

Here  $\varrho = \varrho_\uparrow + \varrho_\downarrow$  is the total density of neutron matter,  $\varrho_\uparrow$  and  $\varrho_\downarrow$  are the neutron number densities with spin up and spin down, respectively. The quantity  $\Delta\varrho$  may be regarded as the neutron spin order parameter which determines the magnetization of the system  $M = \mu_n \Delta\varrho$ . The spin ordering of neutrons can also be characterized by the spin polarization parameter

$$\Pi = \frac{\Delta\varrho}{\varrho}.$$

The magnetization may contribute to the internal magnetic field  $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$ . However, we will assume, analogously to the previous studies [9,11,14], that, because of the tiny value of the neutron magnetic moment, the contribution of the magnetization to the inner magnetic field  $\mathbf{B}$  remains small for all relevant densities and magnetic field strengths, and, hence,

$$\mathbf{B} \approx \mathbf{H}. \quad (7)$$

Indeed, e.g., the magnetic field necessary to produce a fully polarized spin state is, at least, greater than  $10^{19}$  G at the densities relevant for the cores of magnetars (as will be shown later), while for totally spin polarized neutron matter with the density  $\varrho = 1 \text{ fm}^{-3}$  the contribution of the term with the magnetization to the inner magnetic field amounts at  $4\pi M \simeq 1.2 \times 10^{17}$  G.

In order to get the self-consistent equations for the components of the single particle energy, one has to set the energy functional of the system. It represents the sum of the matter and field energy contributions

$$E(f, H) = E_m(f) + \frac{H^2}{8\pi} V. \quad (8)$$

The matter energy is the sum of the kinetic and Fermi-liquid interaction energy terms [25,26]

$$\begin{aligned} E_m(f) &= E_0(f) + E_{int}(f), \\ E_0(f) &= 2 \sum_{\mathbf{p}} \varepsilon_0(\mathbf{p}) f_0(\mathbf{p}), \\ E_{int}(f) &= \sum_{\mathbf{p}} \{ \tilde{\varepsilon}_0(\mathbf{p}) f_0(\mathbf{p}) + \tilde{\varepsilon}_3(\mathbf{p}) f_3(\mathbf{p}) \}, \end{aligned} \quad (9)$$

where

$$\tilde{\varepsilon}_0(\mathbf{p}) = \frac{1}{2V} \sum_{\mathbf{q}} U_0^n(\mathbf{k}) f_0(\mathbf{q}), \quad \mathbf{k} = \frac{\mathbf{p} - \mathbf{q}}{2}, \quad (10)$$

$$\tilde{\varepsilon}_3(\mathbf{p}) = \frac{1}{2V} \sum_{\mathbf{q}} U_1^n(\mathbf{k}) f_3(\mathbf{q}). \quad (11)$$

Here  $\varepsilon_0(\mathbf{p}) = \frac{p^2}{2m_0}$  is the free single particle spectrum,  $m_0$  is the bare mass of a neutron,  $U_0^n(\mathbf{k}), U_1^n(\mathbf{k})$  are the normal Fermi liquid (FL) amplitudes, and  $\tilde{\varepsilon}_0, \tilde{\varepsilon}_3$  are the FL corrections to the free single particle spectrum. Taking into account Eqs. (1), (2) and (9), expressions for the components of the single particle energy read

$$\xi_0(\mathbf{p}) = \varepsilon_0(\mathbf{p}) + \tilde{\varepsilon}_0(\mathbf{p}) - \mu_0, \quad \xi_3(\mathbf{p}) = -\mu_n H + \tilde{\varepsilon}_3(\mathbf{p}). \quad (12)$$

In Eq. (12), the quantities  $\tilde{\varepsilon}_0, \tilde{\varepsilon}_3$  are the functionals of the distribution functions  $f_0, f_3$  which, using Eqs. (2) and (3), can be expressed, in turn, through the quantities  $\xi$ :

$$f_0 = \frac{1}{2} \{ n(\omega_+) + n(\omega_-) \}, \quad (13)$$

$$f_3 = \frac{1}{2} \{ n(\omega_+) - n(\omega_-) \}, \quad (14)$$

where

$$n(\omega_{\pm}) = \{ \exp(Y_0 \omega_{\pm}) + 1 \}^{-1}, \quad \omega_{\pm} = \xi_0 \pm \xi_3.$$

The quantity  $\omega_{\pm}$ , being the exponent in the Fermi distribution function  $n$ , plays the role of the quasiparticle spectrum. The branches  $\omega_{\pm}$  correspond to neutrons with spin up and spin down.

Thus, Eqs. (12)–(14) form the self-consistency equations for the components of the single particle energy, which should be solved jointly with the normalization conditions (5), (6).

The pressures (longitudinal and transverse with respect to the direction of the magnetic field) in the system are related to the diagonal elements of the stress tensor whose explicit expression reads [48]

$$\sigma_{ik} = \left[ \tilde{f} - \varrho \left( \frac{\partial \tilde{f}}{\partial \varrho} \right)_{\mathbf{H}, T} \right] \delta_{ik} + \frac{H_i B_k}{4\pi}. \quad (15)$$

Here

$$\tilde{f} = f_H - \frac{H^2}{4\pi}, \quad (16)$$

$f_H = \frac{1}{V} (E - TS) - \mathbf{H} \mathbf{M}$  is the Helmholtz free energy density. For the isotropic medium, the stress tensor (15) is symmetric.

The transverse  $p_t$  and longitudinal  $p_l$  pressures are determined from the formulas

$$p_t = -\sigma_{11} = -\sigma_{22}, \quad p_l = -\sigma_{33}.$$

At zero temperature, using Eqs. (8), (15), one can get the approximate expressions

$$p_t = \varrho \left( \frac{\partial e_m}{\partial \varrho} \right)_H - e_m + \frac{H^2}{8\pi}, \quad (17)$$

$$p_l = \varrho \left( \frac{\partial e_m}{\partial \varrho} \right)_H - e_m - \frac{H^2}{8\pi}, \quad (18)$$

where  $e_m$  is the matter energy density, and we disregarded the higher order terms containing  $M$ . The structure of the pressures  $p_t$  and  $p_l$  is different that reflects the breaking of the rotational symmetry by the magnetic field. In ultrastrong magnetic fields, the quadratic on the magnetic field term (the Maxwell term) will be dominating, leading to increasing the transverse pressure and to decreasing the longitudinal pressure. Hence, at some critical magnetic field, the longitudinal pressure will vanish, resulting in the longitudinal instability of neutron matter. The question then arises: What is the magnitude of the critical field and the corresponding maximum degree of spin polarization in neutron matter?

### 3. Longitudinal and transverse pressures. Anisotropic EoS at zero temperature

In order to solve the self-consistent equations, we utilize the BSk19 and BSk21 parametrizations of the Skyrme interaction, developed in Ref. [43] and generalizing the conventional Skyrme parametrizations. By choosing these Skyrme forces, we would like to study the influence of the underlying EoS, to which these Skyrme forces were constrained, on thermodynamic quantities of strongly magnetized dense neutron matter.

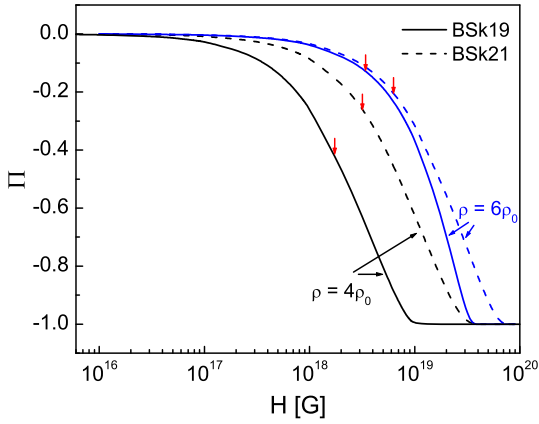
The normal FL amplitudes in Eqs. (10), (11) can be related to the parameters of the Skyrme interaction by formulas [49]

$$U_0^n(\mathbf{k}) = 2t_0(1 - x_0) + \frac{t_3}{3} \varrho^\alpha (1 - x_3) + \frac{2}{\hbar^2} [t_1(1 - x_1) + t_4(1 - x_4) \varrho^\beta + 3t_2(1 + x_2) + 3t_5(1 + x_5) \varrho^\gamma] \mathbf{k}^2, \quad (19)$$

$$U_1^n(\mathbf{k}) = -2t_0(1 - x_0) - \frac{t_3}{3} \varrho^\alpha (1 - x_3) + \frac{2}{\hbar^2} [t_2(1 + x_2) + t_5(1 + x_5) \varrho^\gamma - t_1(1 - x_1) - t_4(1 - x_4) \varrho^\beta] \mathbf{k}^2. \quad (20)$$

In these equations, the terms with the factors  $t_4$  and  $t_5$  are the additional density-dependent terms generalizing the  $t_1$  and  $t_2$  terms in the conventional form of the Skyrme interaction [50]. They were added to the usual form with the aim to avoid the appearance of spontaneous spin instabilities in nuclear and neutron matter at high densities. Specific values of the parameters  $t_i$ ,  $x_i$ ,  $\alpha$ ,  $\beta$  and  $\gamma$  for each parametrization are given in Ref. [43].

Now we present the results of the numerical solution of the self-consistency equations. Fig. 1 shows the spin polarization parameter of neutron matter as a function of the magnetic field  $H$  at two different values of the neutron matter density,  $\varrho = 4\varrho_0$  and  $\varrho = 6\varrho_0$ , which can be relevant for the central regions of a magnetar ( $\rho_0 = 0.1596 \text{ fm}^{-3}$  for the BSk19 force and  $\rho_0 = 0.1582 \text{ fm}^{-3}$  for the BSk21 force). It is seen that the impact of the magnetic field remains small up to the field strength  $10^{17} \text{ G}$ . For the BSk21 force (stiff underlying EoS), the magnitude of the spin polarization parameter is smaller than for the BSk19 force (soft underlying EoS). For both parametrizations, the larger the density is, the smaller the effect produced by the magnetic field on spin polarization of neutron matter. At the magnetic field  $H = 10^{18} \text{ G}$ , usually considered as the maximum magnetic field strength in the core of a magnetar (according to a scalar virial theorem [51]), the magnitude of the spin polarization parameter doesn't exceed 25% for the BSk19 force and 8% for the BSk21 force (for the densities under consideration). However, the situation changes if the larger magnetic fields are allowable: With further increasing the magnetic field strength, the magnitude of the spin polarization parameter increases till it

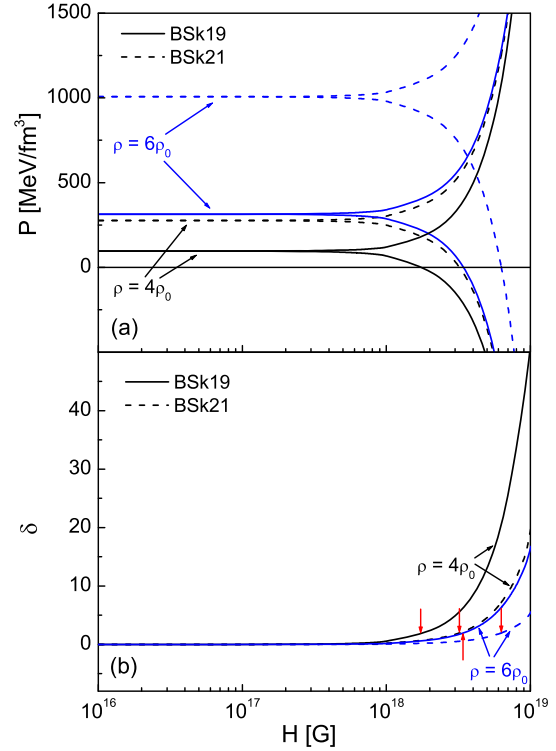


**Fig. 1.** (Color online.) Neutron spin polarization parameter as a function of the magnetic field  $H$  for the Skyrme forces BSk19 (solid lines) and BSk21 (dashed lines) at zero temperature and fixed values of the density,  $\rho = 4\rho_0$  (two lower branches) and  $\rho = 6\rho_0$  (two upper branches). The vertical arrows indicate the maximum magnitude of spin polarization attainable for the corresponding Skyrme force at the given density, see further details in the text.

reaches the limiting value  $\Pi = -1$ , corresponding to a fully spin polarized state. For example, this happens at  $H \approx 1.3 \times 10^{19}$  G for the BSk19 force and at  $H \approx 4.3 \times 10^{19}$  G for the BSk21 force at  $\rho = 4\rho_0$ , i.e., certainly, for magnetic fields  $H \gtrsim 10^{19}$  G. Nevertheless, we should check whether the formation of a fully spin polarized state in a strong magnetic field is actually possible by calculating the anisotropic pressure in dense neutron matter. The meaning of the vertical arrows in Fig. 1 is explained later in the text.

Fig. 2a shows the pressures (longitudinal and transverse) in neutron matter as functions of the magnetic field  $H$  at the same densities,  $\rho = 4\rho_0$  and  $\rho = 6\rho_0$ . The upper branches in the branching curves correspond to the transverse pressure, the lower ones to the longitudinal pressure. First, it is clearly seen that up to some threshold magnetic field the difference between transverse and longitudinal pressures is unessential that corresponds to the isotropic regime. Beyond this threshold magnetic field strength, the anisotropic regime holds for which the transverse pressure increases with  $H$  while the longitudinal pressure decreases. The stiffer the underlying EoS is (BSk21 vs. BSk19), the larger the pressure, transverse  $p_t$  or longitudinal  $p_l$ . Also, the increase of the density has the same effect on the pressures  $p_t$  and  $p_l$  as stiffening of the underlying EoS. The most important feature is that the longitudinal pressure vanishes at some critical magnetic field  $H_c$  marking the onset of the longitudinal collapse of a neutron star. For example,  $H_c \approx 1.7 \times 10^{18}$  G for BSk19 force and  $H_c \approx 3.2 \times 10^{18}$  G for BSk21 force at  $\rho = 4\rho_0$ , and  $H_c \approx 3.4 \times 10^{18}$  G for BSk19 force and  $H_c \approx 6.3 \times 10^{18}$  G for BSk21 force at  $\rho = 6\rho_0$ . In all cases under consideration, this critical value doesn't exceed  $10^{19}$  G. In Ref. [35], the critical field for a relativistic dense gas of free charged fermions was found to be close to  $10^{19}$  G.

The magnitude of the spin polarization parameter  $\Pi$  cannot also exceed some limiting value corresponding to the critical field  $H_c$ . These maximum values of the  $\Pi$ 's magnitude are shown in Fig. 1 by the vertical arrows. In particular,  $\Pi_c \approx -0.42$  for BSk19 force and  $\Pi_c \approx -0.26$  for BSk21 force at  $\rho = 4\rho_0$ , and  $\Pi_c \approx -0.13$  for BSk19 force and  $\Pi_c \approx -0.20$  for BSk21 force at  $\rho = 6\rho_0$ . As can be inferred from these values, the appearance of the negative longitudinal pressure in an ultrastrong magnetic field prevents the formation of a fully spin polarized state in the core of a magnetar. Therefore, only the onset of a field-induced ferromagnetic phase transition, or its close vicinity, can be caught under increasing the magnetic field strength in dense neutron matter. A complete spin polarization in the magnetar core is not allowed by the appear-



**Fig. 2.** (Color online.) (a) Pressures, longitudinal (descending branches) and transverse (ascending branches), as functions of the magnetic field  $H$  for the Skyrme forces BSk19 (solid lines) and BSk21 (dashed lines) at zero temperature and fixed values of the density,  $\rho = 4\rho_0$  (two lower branching curves) and  $\rho = 6\rho_0$  (two upper branching curves). (b) Same as in the top panel but for the normalized difference between the transverse and longitudinal pressures. The vertical arrows in the lower panel indicate the points corresponding to the onset of the longitudinal instability in neutron matter.

ance of the negative pressure along the direction of the magnetic field, contrary to the conclusion of Ref. [37] where the pressure anisotropy in a strong magnetic field was disregarded.

Fig. 2b shows the difference between the transverse and longitudinal pressures normalized to the value of the pressure  $p_0$  in the isotropic regime (which corresponds to the weak field limit with  $p_l = p_t = p_0$ ):

$$\delta = \frac{p_t - p_l}{p_0}.$$

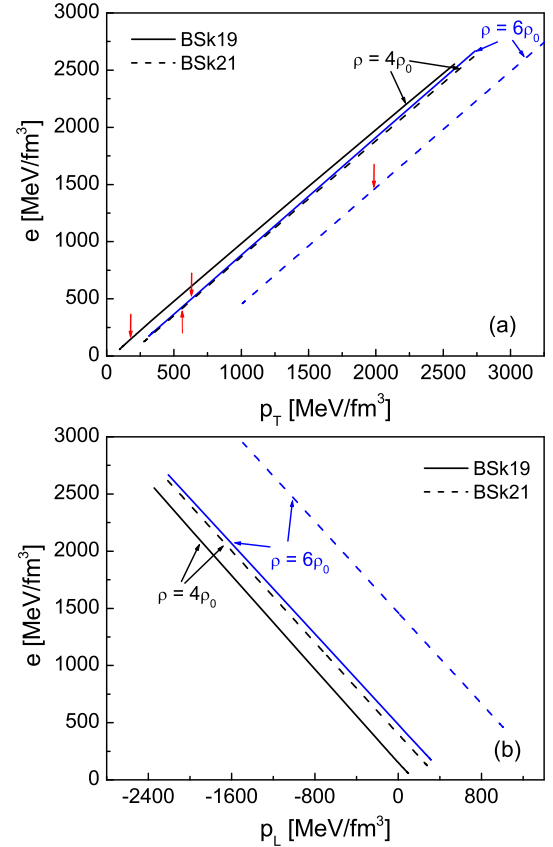
It is quite reasonable to admit that, when the anisotropic regime sets in, the splitting between the transverse and longitudinal pressures becomes comparable with the value of the pressure in the isotropic regime [35]. Applying for the transition from the isotropic regime to the anisotropic one the approximate criterion  $\delta \simeq 1$ , the transition occurs at the threshold field  $H_{th} \approx 1.2 \times 10^{18}$  G for BSk19 force and  $H_{th} \approx 2.2 \times 10^{18}$  G for BSk21 force at  $\rho = 4\rho_0$ , and at  $H_{th} \approx 2.3 \times 10^{18}$  G for BSk19 force and  $H_{th} \approx 4.6 \times 10^{18}$  G for BSk21 force at  $\rho = 6\rho_0$ . In all cases under consideration, the threshold field  $H_{th}$  is larger than  $10^{18}$  G, and, hence, the isotropic regime holds for the fields up to  $10^{18}$  G. For comparison, the threshold field for a relativistic dense gas of free charged fermions was found to be about  $10^{17}$  G [35] (without including the anomalous magnetic moments of fermions). For a degenerate gas of free neutrons the model-dependent estimate gives  $H_{th} \simeq 4.5 \times 10^{18}$  G [38] (including the neutron anomalous magnetic moment). The normalized splitting of the transverse and longitudinal pressures increases more rapidly with the magnetic field at the smaller density and/or for the Skyrme force BSk19 with the softer underlying EoS. The vertical arrows in Fig. 2b indicate the points



corresponding to the onset of the longitudinal instability in neutron matter. Since the threshold field  $H_{th}$  is less than the critical field  $H_c$  for the appearance of the longitudinal instability, the anisotropic regime can be relevant for the core of a magnetar. The maximum allowable normalized splitting of the pressures corresponding to the critical field  $H_c$  is  $\delta \sim 2$ . If the anisotropic regime sets in, a neutron star has the oblate form. Thus, as follows from the preceding discussions, in the anisotropic regime the pressure anisotropy plays an important role in determining the spin structure and configuration of a neutron star.

Because of the pressure anisotropy, the EoS of neutron matter in a strong magnetic field is also anisotropic. Fig. 3 shows the dependence of the energy density of the system on the transverse pressure (top panel) and on the longitudinal pressure (bottom panel) at the same densities considered above. Since in an ultrastrong magnetic field the dominant Maxwell term enters the pressure  $p_t$  and energy density with positive sign and the pressure  $p_l$  with negative sign, the energy density is the increasing function of  $p_t$  and decreasing function of  $p_l$ . In the case of  $e(p_t)$  dependence, at the given density, the same  $p_t$  corresponds to the larger magnetic field  $H$  for the BSk19 force compared with the BSk21 force (see Fig. 2a). The overall effect of two factors (the stiffness/softness of the underlying EoS and magnetic field) will be the larger value of the energy density at the given  $p_t$  and density for the BSk19 force compared with the BSk21 force (see Fig. 3a). The analogous arguments show that, for the given Skyrme parametrization and at the given  $p_t$ , the energy density is larger for the smaller density. In the case of  $e(p_l)$  dependence, at the given density, the same  $p_l$  corresponds to the smaller magnetic field  $H$  for the BSk19 force compared with the BSk21 force (see Fig. 2a). Hence, the energy density at the given  $p_l$  and density is larger for the BSk21 force than that for the BSk19 force (see Fig. 3b). Analogously, for the given Skyrme parametrization and at the given  $p_l$ , the energy density is larger for the larger density. In the bottom panel, the physical region corresponds to the positive values of the longitudinal pressure. Note that, because of the validity of the approximation given by Eq. (7), the energy density containing the field energy contribution is practically indistinguishable from the Helmholtz free energy density.

It is worthy to notice that the occurrence of the longitudinal instability in a strong magnetic field will lead to the compression of a neutron star along the magnetic field with a subsequent increase of the density. Such an increase can eventually cause the appearance of new particle species. Already for the conditions of high density considered in this study one can assume that a deconfined phase of quarks exists in the interior of a neutron star. Then a hybrid star consisting of deconfined quark matter in the core and nuclear matter in the outer layers can be regarded as a relevant astrophysical object. As follows from the general arguments presented in our study, the critical magnetic field strength at which the longitudinal pressure vanishes inevitably exists for a hybrid star as well. The determination of the corresponding critical value needs a separate investigation. As a possible guess for this value, one can refer to the results of the study of quark matter within the MIT bag model [35], where it was estimated to be close to  $10^{19}$  G (with the bag constant put to zero in the final expressions). Also, the mass–radius relationship is a relevant characteristic of neutron stars. Usually it is found by solving the Tolman–Oppenheimer–Volkoff (TOV) equations [52] for a spherically symmetric and static neutron star. In an ultrastrong magnetic field, the EoS becomes essentially anisotropic. Unlike to the standard scheme, the mass–radius relationship should be found by the self-consistent treatment of the anisotropic EoS and axisymmetric TOV equations substituting the conventional TOV equations in the case of an axisymmetric neutron star.



**Fig. 3.** (Color online.) The energy density of the system as a function of: (a) the transverse pressure  $p_t$ , (b) the longitudinal pressure  $p_l$  for the Skyrme forces BSk19 (solid lines) and BSk21 (dashed lines) at zero temperature and fixed values of the density,  $\rho = 4\rho_0$  and  $\rho = 6\rho_0$ . The meaning of the vertical arrows in the top panel is the same as in Fig. 2. In the bottom panel, the physical region corresponds to  $p_l > 0$ .

Note that in this research we have studied the impact of a strong magnetic field on thermodynamic properties of dense neutron matter at zero temperature. It would be also of interest to extend this research to finite temperatures relevant for proto-neutron stars which can lead to a number of interesting effects, such as, e.g., an unusual behavior of the entropy of a spin polarized state [53,54].

In summary, we have considered spin polarized states in dense neutron matter in the model with the Skyrme effective NN interaction (BSk19 and BSk21 parametrizations) under the presence of strong magnetic fields up to  $10^{20}$  G. It has been shown that in the magnetic field  $H > H_{th} \sim 10^{18}$  G the pressure anisotropy has a significant impact on thermodynamic properties of neutron matter. In particular, vanishing of the pressure along the direction of the magnetic field in the critical field  $H_c > H_{th}$  leads to the appearance of the longitudinal instability in neutron matter. For the Skyrme force with the stiffer underlying EoS (BSk21 vs. BSk19), the threshold  $H_{th}$  and critical  $H_c$  magnetic fields become larger. The increase of the density of neutron matter also leads to increasing the fields  $H_{th}$  and  $H_c$ . Even in the extreme scenario with the most stiff underlying EoS considered in this work and at the densities about several times nuclear saturation density, the critical field  $H_c$  doesn't exceed  $10^{19}$  G which can be considered as the upper bound on the magnetic field strength inside a magnetar. Our calculations show that the appearance of the longitudinal instability prevents the formation of a fully spin polarized state in neutron matter, and only the states with mild spin polarization can be developed. The longitudinal and transverse pressures and anisotropic

EoS of neutron matter in a strong magnetic field have been determined at the densities relevant for the cores of magnetars. The obtained results can be of importance in the structure studies of strongly magnetized neutron stars.

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