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Cosmographic degeneracy

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We examine the dark energy and matter densities allowed by precision measurements of distances out to various redshifts, in the presence of spatial curvature and (near) arbitrary behavior of the dark energy equation of state. Degeneracies among the parameters permit a remarkably large variation in their values when using only distance measurements of the late-time universe and making no assumptions about the dark energy or curvature. Going beyond distance measurements to a lower limit on the growth of structure bounds the allowed region significantly but still leaves considerable freedom to match a flat Λ Cold Dark Matter model with dark energy very different from a cosmological constant. The combination of distances with Hubble parameter, gravitational lensing or other large-scale structure data is essential to determining the cosmological model robustly.

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I. INTRODUCTION

A central goal of modern cosmology is to reveal in detail the energy budget of the universe. In addition to matter (baryonic and dark matter) and small contributions by radiation and neutrinos, there is an (effective) dark energy associated with the accelerated expansion and possibly an (effective) curvature energy associated with deviation from spatial flatness. Cosmological observations, especially over the past decade, have made great strides in constraining the energy-density fractions in each of these, but generally assuming specific behaviors for the dark energy. Since dark energy is an almost total mystery, it behooves us to reexamine the issue and ask to what extent the Λ Cold Dark Matter (ΛCDM) concordance model of $\sim$28% matter density and $\sim$72% cosmological constant energy density is necessarily close to the true energy budget.

Because all the energy densities enter into the Hubble expansion rate, which then determines the distance-redshift relation, degeneracies exist between the components such that more of one can compensate for less of another. Since they evolve differently with redshift, however, each characterized by its own equation of state parameter (0 for matter, $-1/3$ for curvature, $w(z)$ for dark energy), one expects that observations over a sufficiently wide redshift range give leverage to break the degeneracies. This expectation has been explored for restricted scenarios of matter and dark energy densities (e.g. [1–4]) and curvature and dark energy densities (e.g. [5,6]), and nonparametrically from the observations through redshifts bins of dark energy (e.g. [7]). Perhaps closest in philosophy to our approach are the works of [8–10], which look at how the diversity of models translates to dispersion in observables, while we explore the converse of how a tight observable relation can arise from a wide range of models.

In this paper we investigate the freedom around the concordance model caused by degeneracies when we allow for matter, curvature, and dark energy with no *a priori* restriction on its equation of state. The philosophy is to use as direct measurements as possible without making assumptions about the dark energy. Therefore we restrict ourselves to late-time observations since we have no knowledge of dark energy behavior at early times, e.g. is there early dark energy affecting the cosmic microwave background (CMB)? We use purely geometric distance measurements, examining how the degeneracies are broken as the data quality and redshift range improve. That is, suppose we have even exact agreement with a particular flat ΛCDM model in the distance-redshift relation out to some redshift z; how close to the concordance model in the matter density–dark energy density plane does this restrict the cosmological model when allowing for curvature and arbitrary $w(z)$ behavior? We should note that in this paper we are considering the degeneracy between the fundamental cosmological quantities, namely, matter density, curvature, and the effective equation of state of dark energy. There are also other sorts of degeneracies between different cosmological models which are very different by nature, but they result in a similar effective equation of state of dark energy [11,12].

Section II lays out our methodology and explains the constraints imposed by various levels of conditions arising from consistency and from the observations. The influence of measurements apart from distance, such as the age of the universe and the linear growth factor of cosmic structure, are also addressed. In Section III we quantify the constraints in the matter density–dark energy density (Ω_m–Ω_{de}) plane. Section IV summarizes the results about how well we actually know our cosmological model.

II. COSMOGRAPHIC CONSTRAINTS

The method of our analysis is simple and straightforward. We calculate the luminosity distance–redshift relation for a flat ΛCDM model with $\Omega_m = 0.28$ and assume that (future) observations determine that distances agree with this model to a certain precision for all redshifts out to some $z_{\rm max}$. Initially we take the agreement to be exact, to illustrate the level of degeneracies that persist even in this ideal case. For our comparison to theoretical cosmological models we stay within the Friedmann-Robertson-Walker framework, taking noninteracting components of nonrelativistic matter, spatial curvature, and dark energy. Since the dark energy is allowed to behave (nearly) arbitrarily, many elaborations such as interacting components can actually be folded into the dark energy behavior.

The Hubble expansion parameter, giving the logarithmic time variation of the scale factor $a = 1/(1 + z)$, is

$$\begin{aligned} h(z)^2 &\equiv [H(z)/H_0]^2 \\ &= \Omega_m(1+z)^3 + (1 - \Omega_m - \Omega_{de})(1+z)^2 \\ &\quad + \Omega_{de} \exp\left[3\int_0^z \frac{dz'}{1+z'}[1 + w(z')]\right], \end{aligned} \tag{2.1}$$

where the last term on the second line represents the spatial curvature energy density. This leads to the luminosity distance through

$$\begin{aligned} d_l(z) &= \frac{1+z}{\sqrt{1 - \Omega_m - \Omega_{de}}} \\ &\quad \times \sinh\left(\sqrt{1 - \Omega_m - \Omega_{de}} \int_0^z \frac{dz'}{h(z')}\right). \end{aligned} \tag{2.2}$$

Note that sinh is an analytic function valid for positive, zero, or negative curvature. Although we write the luminosity distance, all the results still hold if one uses angular diameter distance $d_a = d_l/(1 + z)^2$ instead.

We concentrate on what values of the matter density Ω_m and dark energy density Ω_{de} are allowed, without assuming a specific equation of state $w(z)$, given the distances $d_l(z)$. Note that only late-time parameters enter, i.e. we do not have to make any assumptions about conditions at high redshift in the early universe.

A. Radius of curvature condition

The first constraint on the Ω_m–Ω_{de} plane is the requirement that, for a positive-curvature universe, the radius of curvature is large enough to allow the luminosity distance to match that of the input ΛCDM model out to $z_{\rm max}$. Mathematically, this corresponds to the sine [analytic continuation of sinh in Eq. (2.2)] function having an amplitude bounded by 1. Basically, $\Omega_m + \Omega_{de} - 1 \le [(1 + z)/d_l(z)]^2$ for all $z \le z_{\rm max}$. We call the region of Ω_m–Ω_{de} space that violates this constraint "Forbidden Region 1."

Note that this is a more restrictive condition than the "no-bounce" condition, which requires that a transition from contraction to expansion is avoided out to the $z_{\rm max}$ considered. A bounce—that is, the Hubble parameter going to zero—imposes a minimum scale factor and hence maximum redshift; note that, when $H \to 0$, then the argument of the sine function goes to infinity. Avoiding a bounce out to infinite redshift is necessary for having a big bang, but again our radius of curvature condition does not require any extrapolation to early-universe conditions.

B. Positive density condition

The next constraint on the densities comes from requiring that the dark energy density be positive. One can rewrite Eq. (2.1) as

$$\Omega_{de}(z) = h(z)^2 - \Omega_m(1+z)^3 - (1 - \Omega_m - \Omega_{de})(1+z)^2, \tag{2.3}$$

and so the condition that $\Omega_{de}(z) \ge 0$ for all $z \le z_{\rm max}$ restricts the allowed region of the density plane. We call this constraint "Forbidden Region 2." Note that we do not restrict w from going to $-\infty$, needed to allow the dark energy density to go to zero. A negative dark energy density could also violate the radius of curvature condition, so we expect overlap between the forbidden regions.

C. Age condition

The above two conditions are basically consistency restrictions within the framework of the distance matching. We could ask for further observational conditions that are general enough not to require assumptions about the nature of the components.

The lookback time–redshift relation is one possibility. It is solely a function of the Hubble parameter,

$$T(z) = t_0 - t(z) = H_0^{-1} \int_0^z \frac{dz'}{(1+z')h(z')}. \tag{2.4}$$

However, we currently do not have any robust, model-independent, and accurate limits on the lookback time–redshift relation. One could use the age of the universe, the limit of lookback time as redshift gets large. This might seem to contradict our approach of not requiring any knowledge of early-universe conditions but this can be worked around. The observational constraint itself can be taken from the age of globular clusters [13] or white dwarf stars [14]. We consider these as placing a lower bound on the age of the universe of 12.5 Gigayears (Gyr). As far as the theoretical models, we simply take the maximum possible age, i.e. we maximize the contribution for redshifts above our $z_{\rm max}$ for each Ω_m–Ω_{de} point, and if the total age still falls below the observational bound, then that region of the density plane is ruled out.

To maximize the age for a given Ω_m–Ω_{de}, we minimize $H(z)$ at the high redshifts. The minimum occurs by driving the dark energy density contribution to zero immediately after $z_{\rm max}$ (we do not allow negative energy density), thus leaving only the matter density and curvature contributions set by the low redshift behavior. In this way we obtain a robust bound that does not actually require knowledge of the high-redshift-universe conditions. We call regions of the density plane that even so do not achieve the minimum age of 12.5 Gyr "Forbidden Region 3."

Note that, because this is a conservative bound, many dark energy models in the still-allowed region of the density plane will have a low age, but we only rule out the region where *no* possible dark energy behavior permits a universe consistent with the age constraint. Also note that to estimate age we need to know the value of H_0; the region shown uses the 1σ lower limit (so as to maximize the age) of the measurement $H_0 = 73.8 \pm 2.4$ km/s/Mpc [15]. If we use the 2σ lower limit of 69, however, then no region beyond that of Forbidden Region 2 is ruled out for the $z_{\rm max}$ we consider.

D. Growth condition

While the positive dark energy density condition is very effective at ruling out high matter densities (since this gives a large Hubble parameter and hence too-small distances relative to ΛCDM, and further dark energy density only exacerbates the situation), very low matter densities are allowed. We could impose a condition that the matter density must be at least as large as the baryon density implied by primordial nucleosynthesis or the CMB, i.e. $\Omega_m \gtrsim 0.04$, but this would use early-universe conditions. Instead, we look at growth of structure. Growth is particularly important because we allow the dark energy to behave as matter with equation of state $w = 0$, so it enters equivalently to matter in the Hubble parameter (thus allowing low Ω_m), but growth is also sensitive to the clustered density while we take dark energy to be smooth. Growth thus allows distinction between them and breaking of the degeneracy.

Components other than clustered matter have two effects on growth of matter structures: they change the expansion rate, and hence the friction term in the linear density perturbation growth equation, and they change the source term (basically, the matter density). We can write the growth equation as

$$\delta'' + (2 - q)a^{-1}\delta' - (3/2)a^{-2}\Omega_m(a)\delta = 0, \qquad (2.5)$$

where q is the deceleration parameter and a prime denotes a derivative with respect to scale factor a. We will be interested in how much the growth is enhanced or suppressed relative to a pure-matter universe, where δ grows linearly with a. A component with negative equation of state both increases the friction and decreases the source [through reducing $\Omega_m(a)$] relative to this case. One might think that a component with $w > 0$ could enhance growth since it reduces the friction, but we will find that the reduction in the source term is more important.

The presence of dark energy and curvature reduces $\Omega_m(a)$ below the pure matter value of 1 and changes the friction term

$$2 - q = (3/2)(1 - w_{\rm mix}(1 - \Omega_m(a)), \qquad (2.6)$$

where $w_{\rm mix}$ is the effective equation of state of the combination of curvature and dark energy. The question is whether the reduced friction can make up for the reduced source term.

Putting the function $\delta \propto a^m$ into the growth equation (2.5), we get the characteristic equation

$$2m^2 + [1 - 3w_{\rm mix}(1 - \Omega_m(a))]m - 3\Omega_m(a) = 0. \qquad (2.7)$$

For any allowed value of $\Omega_m(a)$, there is no root with $m > 1$, i.e. growth can never be greater than in the pure matter case, when $w_{\rm mix} \le 1$. Taking $m = 1 + \epsilon$, one finds $\epsilon = -3(1 - w_{\rm mix})(1 - \Omega_m(a))/(5 - 3w_{\rm mix}(1 - \Omega_m(a)))$, which is always negative for $w_{\rm mix} < 1$, since additional high-redshift components can only reduce $\Omega_m(a)$ below the pure matter value of 1, our baseline.

Thus, the reduced source term always suppresses growth more than the easing from the less positive friction term. If we allow $w_{\rm mix} > 1$, however, then the friction goes negative and growth can actually be enhanced. So our growth condition will apply, unlike the other conditions, only to those dark energy models with $w \le 1$ at $z > z_{\rm max}$ (as all canonical scalar fields obey). We emphasize that we still allow w to take any value at $z \le z_{\rm max}$.

Now we apply an argument similar to the age condition. Since growth can only be suppressed upon allowing for dark energy (with $w \le 1$ at $z > z_{\rm max}$) or curvature, we take the growth factor at $z > z_{\rm max}$ to be the maximal (Einstein-de Sitter pure matter) value $g = \delta/a = 1$. Then we calculate the growth factor for today, using the expansion history for any particular point in the density plane derived to match the distance relation to $z_{\rm max}$. This procedure maximizes the total growth to the present. (In fact, it is overly conservative, but it has the virtue of model independence.) If the growth factor today is still too small to be acceptable observationally, which we take as $g_0 < 0.65$, or, basically, $\sigma_8 < 0.7$ (cf. [16]), then we consider that region as ruled out. We call such an area "Forbidden Region 4." As expected, this rules out very-low-matter densities.

III. BREAKING DEGENERACIES

Having established four conditions on acceptable values for the density parameters despite spatial curvature and (nearly) arbitrary dark energy behavior, we now apply them individually and jointly to the density plane of Ω_m–Ω_{de}.

Figure 1 shows the Ω_m–Ω_{de} parameter space and each of the forbidden regions. Forbidden Region 1 from the radius of curvature condition constrains high positive curvature, where the sum $\Omega_m + \Omega_{de}$ is large. As $z_{\rm max}$ increases, the maximum $d_l/(1 + z)$ in the inverse of the

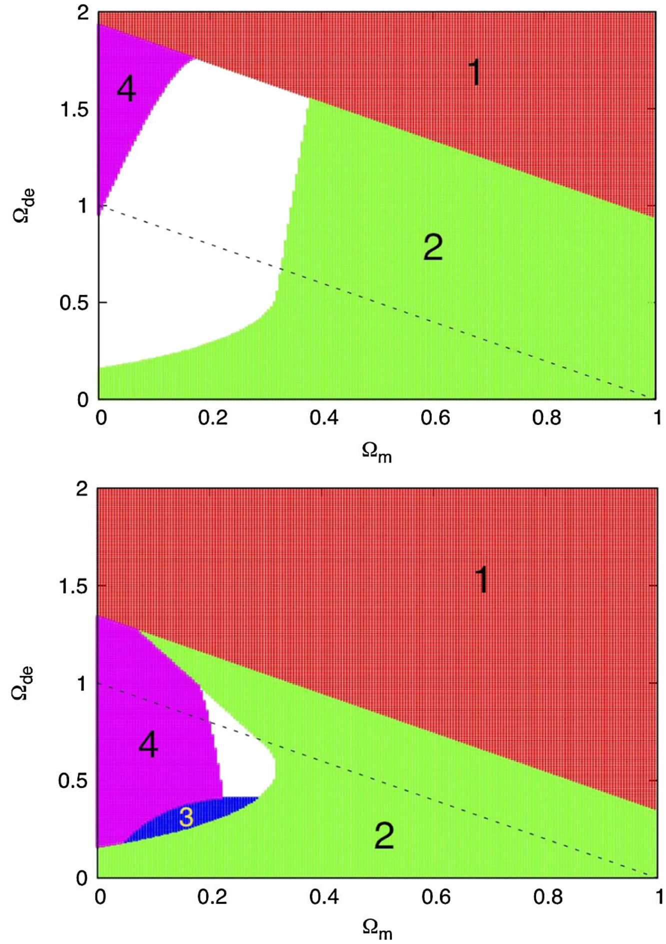

FIG. 1 (color online). The density parameter space Ω_m–Ω_{de} has forbidden regions from (1) minimum radius of curvature, (2) positivity of dark energy density, (3) minimum age of the universe, and (4) minimum total growth of structure. The dotted line passes through the points where the spatial curvature is zero. Parameter values lying in the unshaded areas are capable of exactly matching the distance-redshift relation of flat ΛCDM with $\Omega_m = 0.28$ out to $z_{\max} = 1.5$ (top panel) or $z_{\max} = 4$ (bottom panel).

sine function increases monotonically, and so the sum of the energy densities in the curvature factor is more tightly restricted, causing the forbidden region boundary to sweep down through the plane.

Forbidden Region 2 from the dark energy density positivity condition rules out high values of Ω_m, since then the Hubble parameter is so large (and any dark energy can only add to it) that the distances are too small to match the required ΛCDM values. Similarly, in the negative curvature region, the curvature contribution to the Hubble parameter is too large, especially for small values of the dark energy that increase the curvature contribution (as do small values of the matter density, but those also give a smaller matter contribution), so again the distances are too small to be viable. Thus Region 2 is roughly (reverse) L-shaped. As $z_{\max}$ increases, the positive curvature region becomes more restricted as the distance is further reduced by the curvature, i.e. $\sin x < x$, so the L-shape begins to fold over on itself.

The minimum age of the universe, defining Forbidden Region 3, does not have a strong effect. Some of the higher values of Ω_{de}, which would only help to avoid Forbidden Region 2, here increase H enough to decrease the age of the universe below the required limit. However, for low $z_{\max}$, this does not occur since the dark energy density can vanish at higher redshifts, and even for $z_{\max} = 4$, the values $H_0 < 69$ km/s/Mpc cause this constraint to recede within Forbidden Region 2.

Finally, the growth condition, as predicted, gives Forbidden Region 4, cutting off the low matter densities and high dark energy densities (which aid in suppressing growth). As $z_{\max}$ increases, the period of helpful, maximal (Einstein-de Sitter) growth is shortened, and so even low dark energy densities do not allow low matter densities to deliver sufficient growth.

We reiterate the meaning of the remaining allowed region: it illustrates the cosmographic degeneracy, that for such combinations of Ω_m and Ω_{de} there exists a combination of curvature and an evolving dark energy model (where the exact necessary $w(z)$ can be reconstructed) that results in a distance-redshift relation out to $z_{\max}$ exactly equal to that of the flat ΛCDM model with $\Omega_m = 0.28$. So no matter how precise our measurements of $d_l(z)$, these models cannot be distinguished, and, in particular, one cannot conclude that the dark energy arises from a cosmological constant. Furthermore, such a cosmology is viable in the sense that it obeys the lower limits from the growth factor and the age of the universe.

As $z_{\max}$ increases, the parameter region capable of matching exactly the distance-redshift relation of flat ΛCDM with $\Omega_m = 0.28$ out to $z_{\max}$, by using the presence of curvature and arbitrary dark energy equation of state behavior, diminishes. Forbidden Region 1 sweeps down, Region 2 closes in from the right and above, and Region 4 squeezes from the left and below. The top panel of Fig. 1 using $z_{\max} = 1.5$ is roughly an idealized version of near-future cosmological observations, and $z_{\max} = 4$ might represent further future observations using, for example, neutral hydrogen surveys. Only as $z_{\max}$ gets very large does the allowed parameter space zero in on the input flat ΛCDM, $\Omega_m = 0.28$ model, completely breaking the cosmographic degeneracy.

As we have already seen from use of the growth factor, other observations besides the distance to some redshift z can prove effective at breaking the cosmographic degeneracy. For example, distance intervals such as those entering in gravitational lensing, the Alcock-Paczynski effect, or Hubble parameter measurements involve the curvature in a distinct way [17–19]. Further measures of growth, such as relative growth between two redshifts or the integrated Sachs-Wolfe effect, could prove useful, as would direct

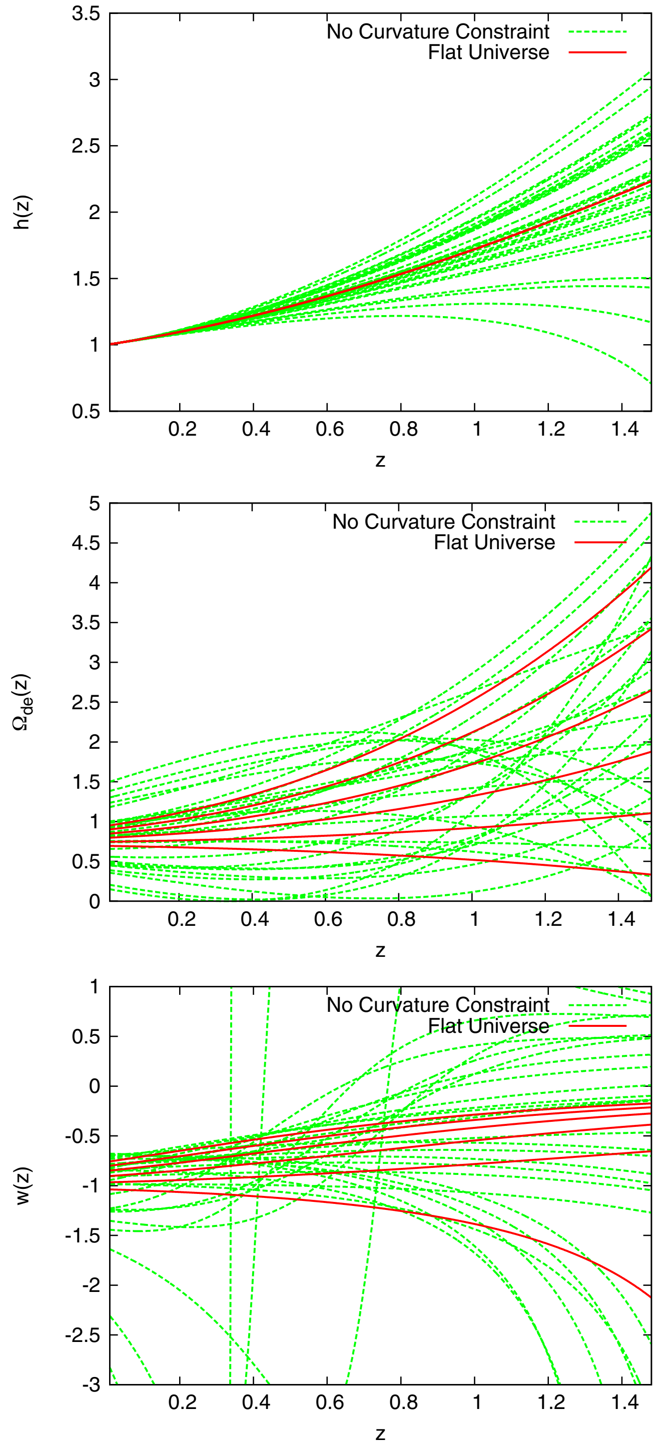

FIG. 2 (color online). Nonexhaustive sample of the Hubble parameter $h(z)$, dark energy density $\Omega_{de}(z)$, and $w(z)$ for different points in the Ω_m–Ω_{de} parameter space that match the ΛCDM distances exactly out to $z_{\max} = 1.5$ and satisfy our four conditions. Light (green) lines represent the results with curvature allowed to be nonflat. Dark (red) lines restrict to the zero-curvature case. The assumed true model is a spatially flat ΛCDM model with $\Omega_m = 0.28$.

estimate of the matter density through the clustering statistics of large-scale structure, although these would have to allow for arbitrary dark energy evolution. If we are willing to restrict to models where dark energy has no role at high redshifts, then the cosmic microwave background provides a rich source of constraints, but that is not the philosophy adopted here.

Figure 2 illustrates nonexhaustively the variety of Hubble parameters $h(z)$, dark energy density $\Omega_{de}(z)$, and dark energy equations of state $w(z)$ that exist at different points in the Ω_m–Ω_{de} density space that satisfies all our mathematical and cosmological constraints, out to $z_{\max} = 1.5$. Light (green) dashed lines represent results when allowing for curvature as well as dark energy behavior, while dark (red) solid lines fix the curvature to be zero and only employ the freedom in the dark energy equation of state.

For the case of $h(z)$, models with the same curvature, and forced to have the same distances, necessarily have the same $h(z)$. (Think of it as a derivative of the distance.) For $\Omega_{de}(z)$, however, there is additional freedom coming from changing the matter contribution even if the curvature is fixed, and this carries through to $w(z)$ as well. Without fixing curvature, an even greater variety of dark energy behaviors is exhibited—despite the distance relation agreeing perfectly with a cosmological-constant universe.

To this point we have required exact distance matching, i.e. perfect precision on the observations that agree with the ΛCDM model. Even so, the cosmographic degeneracy region is substantial. We now consider the effect of some

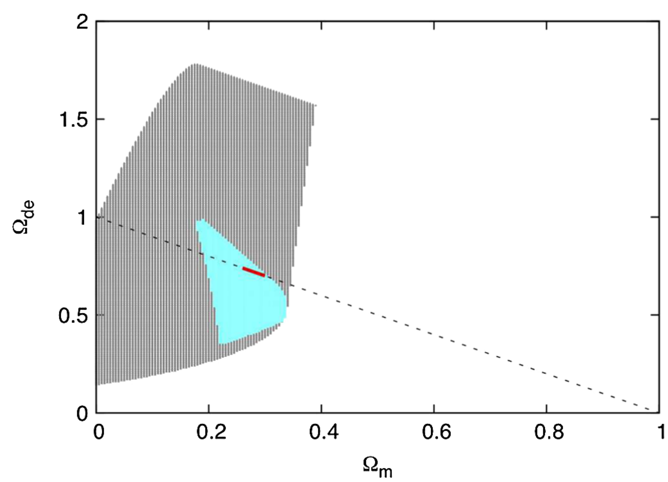

FIG. 3 (color online). Allowed region in the density parameter space if the true model can be any flat ΛCDM model with $0.26 \leq \Omega_m \leq 0.3$ and the four forbidden regions are excluded. The shaded grey area applies the distance matching up to $z_{\max} = 1.5$ and the light (blue) area to $z_{\max} = 4$. The black dashed line shows the flatness line, and the short (red) solid line along it represents the range of true flat ΛCDM models allowed.

uncertainty (statistical or systematic) in the distance measurements by accepting as an allowed region in the density plane any values that fit a flat ΛCDM model with matter densities $0.26 \leq \Omega_m \leq 0.3$. This roughly corresponds to a 1.6% (2.3%) distance uncertainty out to $z_{\max} = 1.5$ (4).

Figure 3 shows the allowed region in the density parameter space now allowing for this uncertainty in the true distances, and applying the forbidden regions as before. The larger, grey shaded area is for $z_{\max} = 1.5$ and the smaller, light (blue) area is for $z_{\max} = 4$. The black dashed line shows the flatness line and the short solid (red) line shows the range of true flat ΛCDM models. We see that the regions allowed with such a variation in true distances does not expand greatly over the white, unshaded regions allowed in Fig. 1, so this level of measurement uncertainty does not change our conclusions.

IV. CONCLUSIONS

We have quantified the cosmographic degeneracy present when matter, spatial curvature, and unrestricted dark energy models can contribute to the distance-redshift relation. Even when perfectly matching the distances out to $z_{\max} = 1.5$ for a flat ΛCDM model with a given matter density, a substantial region of the density parameter space remains degenerate with the true model. This implies that we *cannot* assume that the cosmological constant describes the dark energy through such distance measurements alone.

Imposing other low-redshift constraints, such as basic consistency conditions on the radius of curvature of closed universes and positivity of the dark energy density, and observational criteria such as a minimum age of the universe and a simple lower bound on the total growth factor for large-scale structure, still leaves considerable freedom for the curvature and dark energy contributions. One would zero in on a ΛCDM model only when restricting the form of the dark energy evolution or bringing in early-universe constraints (e.g. CMB and more detailed large-scale structure characteristics)—assuming dark energy has negligible contribution at high redshift. Distances involving curvature differently, e.g. parallax [20] or distance intervals, including direct measurement of the Hubble parameter or gravitational lensing, of galaxies or the CMB, may offer another path (see e.g. [17–19,21]).

We emphasize that we have made essentially no assumptions about the dark energy behavior, allowing its equation of state parameter w to range from $-\infty$ to $+\infty$ (with the exception of restricting $w \leq 1$ above $z_{\max}$ when using the minimum growth condition). A constraint on w to lie within $[-1, +1]$, say, the values for a canonical scalar field, would limit the allowed region to a much smaller area around the true values in the density space. When the assumed Ω_m is larger than the actual matter density, at some redshifts we need a very large negative equation of state of dark energy to suppress the contribution of dark energy in the total energy density of the universe. Conversely, when the assumed Ω_m is smaller than the actual matter density, we need a dark energy with equation of state of greater than -1 and in some cases even greater than 1 to compensate the lack of contribution from the matter part. Thus, imposing the limit $w < +1$, say, rules out an area with low matter and dark energy densities. A constraint on spatial curvature (while still allowing for arbitrary dark energy behavior) would cut a diagonal swath in the density plane parallel to the flatness line, considerably restricting the allowed region.

It is interesting to see how our ignorance of the nature of dark energy and the geometric curvature of space diffuses the strength of evidence for the cosmological constant model from distance measurements. The true universe may be much more complicated, and yet perfectly consistent with cosmography, than this highly restricted model. Combining distance measurements with gravitational lensing data and mapping of large-scale structure will greatly reduce the degeneracies exhibited, and such a suite of future observations offers true hope to understanding our universe in a less model-dependent manner.

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[1] G. Huey, L. Wang, R. R. Caldwell, and P. J. Steinhardt, Phys. Rev. D **59**, 063005 (1999).
[2] A. Shafieloo, U. Alam, V. Sahni, and A. A. Starobinsky, Mon. Not. R. Astron. Soc. **366**, 1081 (2006).
[3] A. Shafieloo, Mon. Not. R. Astron. Soc. **380**, 1573 (2007).
[4] V. Sahni, A. Shafieloo, and A. A. Starobinsky, Phys. Rev. D **78**, 103502 (2008).
[5] E. V. Linder, Astropart. Phys. **24**, 391 (2005).
[6] R. Hlozek, M. Cortês, C. Clarkson, and B. Bassett, Gen. Relativ. Gravit. **40**, 285 (2008).
[7] N. Suzuki *et al.*, arXiv:1105.3470 (Astrophys. J., to be published).
[8] M. J. Mortonson, W. Hu, and D. Huterer, Phys. Rev. D **79**, 023004 (2009).

[9] M. J. Mortonson, Phys. Rev. D **80**, 123504 (2009).
[10] M. J. Mortonson, W. Hu, and D. Huterer, Phys. Rev. D **81**, 063007 (2010).
[11] V. Sahni, Y. Shtanov, and A. Viznyuk, J. Cosmol. Astropart. Phys. 12 (2005) 005.
[12] V. Sahni and Y. Shtanov, Phys. Rev. D **71**, 084018 (2005).
[13] L. M. Krauss and B. Chaboyer, Science **299**, 65 (2003).
[14] B. M. Hansen *et al.*, Astrophys. J. **574**, L155 (2002).
[15] A. G. Riess *et al.*, Astrophys. J. **730**, 119 (2011).
[16] R. Mandelbaum and U. Seljak, J. Cosmol. Astropart. Phys. 06 (2007) 024.
[17] G. Bernstein, Astrophys. J. **637**, 598 (2006).
[18] L. Knox, Phys. Rev. D **73**, 023503 (2006).
[19] E. A. Kazin, A. G. Sanchez, and M. R. Blanton, arXiv:1105.2037.
[20] S. Weinberg, Astrophys. J. **161**, L233 (1970).
[21] S. H. Suyu *et al.*, Astrophys. J. **711**, 201 (2010).