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Black hole as a wormhole factory

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ABSTRACT

There have been lots of debates about the final fate of an evaporating black hole and the singularity hidden by an event horizon in quantum gravity. However, on general grounds, one may argue that a black hole stops radiation at the Planck mass $(\hbar c/G)^{1/2} \sim 10^{-5}$ g, where the radiated energy is comparable to the black hole's mass. And also, it has been argued that there would be a *wormhole-like* structure, known as "spacetime foam", due to large fluctuations below the Planck length $(\hbar G/c^3)^{1/2} \sim 10^{-33}$ cm. In this paper, as an explicit example, we consider an exact classical solution which represents nicely those two properties in a recently proposed quantum gravity model based on different scaling dimensions between space and time coordinates. The solution, called "Black Wormhole", consists of two different states, depending on its mass parameter M and an IR parameter ω: For the black hole state (with $\omega M^2 > 1/2$), a *non-traversable* wormhole occupies the interior region of the black hole around the singularity at the origin, whereas for the wormhole state (with $\omega M^2 < 1/2$), the interior wormhole is exposed to an outside observer as the black hole horizon is disappearing from evaporation. The black hole state becomes thermodynamically stable as it approaches the merging point where the interior wormhole throat and the black hole horizon merges, and the Hawking temperature vanishes at the exact merge point (with $\omega M^2 = 1/2$). This solution suggests the "Generalized Cosmic Censorship" by the existence of a wormhole-like structure which protects the naked singularity even after the black hole evaporation. One could understand the would-be wormhole inside the black hole horizon as the result of microscopic wormholes created by "negative" energy quanta which have entered the black hole horizon in Hawking radiation process; *the quantum black hole could be a wormhole factory!* It is found that this speculative picture may be consistent with the recent "$ER = EPR$" proposal for resolving the black hole entanglement debates.

It is widely accepted that general relativity (GR) would not be appropriate for describing the small scale structure of spacetime. For example, GR, when combined with quantum mechanics, provides a length scale $l_P = (\hbar G/c^3)^{1/2} \sim 10^{-33}$ cm, which may provide an absolute limitation for the measurements of spacetime distances [1]. Actually, this is the length scale on which quantum fluctuations of the spacetime are expected to be of order of unity. On the other hand, the singularity theorem, stating the necessary existence of singularities, where the classical concept of space and time breaks down, at certain spacetime domains with some reasonable assumptions in GR [2], may be regarded as an indication of the incompleteness of GR.

These circumstances may provide strong motivation to find the quantum theory of gravity which can treat the above mentioned problems of GR. Actually, the necessity of quantizing the gravity has been argued in order to have a consistent interaction with a quantum system [3]. Moreover, it has been also shown that even small quantum gravitational effects dramatically change the characteristic features of a black hole so that it can emit radiation though the causal structure of the classical geometry is unchanged in the semiclassical treatment [4].

However, as the black hole becomes smaller and smaller by losing its mass from emitting particles, the semiclassical treatment becomes inaccurate and one cannot ignore the back reactions of the emitted particles on the metric and the quantum fluctuations on the metric itself anymore. Actually, regarding the back reaction effects, one can argue that a black hole stops radiation at the Planck mass $m_P = (\hbar c/G)^{1/2} \sim 10^{-5}$ g, where the radiated energy is comparable to the black hole's mass, since a black hole cannot

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radiate more energy than it has, via the pair creation process near the black hole horizon. This implies that the black hole should become thermodynamically stable as it becomes smaller and finally has the vanishing Hawking temperature at the smallest black hole mass. It seems that this should be one of verifiable predictions that any theory of quantum gravity make [5]. Moreover, according to large fluctuations of metric below the Planck length l_P [1], the *wormhole-like* structure, known as "spacetime foam", has been proposed by Wheeler [6]. This may be another verifiable prediction of the quantum gravity, also.

The purpose of this paper is to consider an exact classical solution, as an explicit example, which represents nicely those two properties in a recently proposed quantum gravity model, known as Hořava gravity, based on different scaling dimensions between space and time coordinates. The solution, called "Black Wormhole", consists of two different states depending on its mass parameter M and an IR (infrared) parameter ω: For the black hole state (with $\omega M^2 > 1/2$), a *non-traversable* wormhole occupies the interior region of the black hole around the singularity at the origin, whereas for the wormhole state (with $\omega M^2 < 1/2$), the interior wormhole is exposed to an outside observer as the black hole horizon is disappeared from evaporation. The black hole state becomes thermodynamically stable as it approaches to the merge point where the interior wormhole throat and the black hole horizon merges, and the Hawking temperature vanishes at the exact merge point (with $\omega M^2 = 1/2$).

The solution suggests that, in quantum gravity, the 'conventional' cosmic censorship can be generalized even after black hole evaporation by forming a wormhole throat around the used-to-be singularity. In GR, black hole and wormhole are quite distinct objects due to their completely different causal structures. But the claimed "Generalized Cosmic Censorship" suggests that the end state of a black hole is a wormhole, not a naked singularity. This may correspond to a foam-like nature of spacetime at short length scales. Furthermore, one could understand the would-be wormhole inside the black hole horizon as the results of microscopic wormholes created by "negative" energy quanta which have entered the black hole horizon in Hawking radiation processes so that *the quantum black hole could be a wormhole factory*. It is found that this speculative picture may be consistent with the recent "*ER* = *EPR*" proposal for resolving the black hole entanglement debates.

To see how this picture can arise explicitly, we consider the Hořava gravity which has been proposed as a four-dimensional, renormalizable, higher-derivative quantum gravity without ghost problems, by adopting different scaling dimensions for space and time coordinates in UV (ultraviolet) energy regime, $[t] = -1$, $[\mathbf{x}] = -z$ with the dynamical critical exponents $z \geq 3$, "at the expense of Lorentz invariance" [7]. We formally define the quantum gravity by a path integral

$$Z = \int [\mathcal{D}g_{ij}][\mathcal{D}N_i][\mathcal{D}N]e^{iS/\hbar} \tag{1}$$

with the proposed action ($z = 3$ is considered, for simplicity), up to the surface terms,

$$\begin{aligned} S = \int dtd^3x\sqrt{g}N \Bigg[&\frac{2}{\kappa^2}\left(K_{ij}K^{ij} - \lambda K^2\right) - \frac{\kappa^2}{2\nu^4}C_{ij}C^{ij} \\ &+ \frac{\kappa^2\mu}{2\nu^2}\epsilon^{ijk}R^{(3)}_{i\ell}\nabla_j R^{(3)\ell}{}_k - \frac{\kappa^2\mu^2}{8}R^{(3)}_{ij}R^{(3)ij} \\ &+ \frac{\kappa^2\mu^2}{8(3\lambda-1)}\left(\frac{4\lambda-1}{4}(R^{(3)})^2 - \Lambda_W R^{(3)} + 3\Lambda_W^2\right) \\ &+ \frac{\kappa^2\mu^2\omega}{8(3\lambda-1)}R^{(3)}\Bigg], \end{aligned} \tag{2}$$

and with the ADM decomposition of the metric,

$$ds^2 = -N^2c^2dt^2 + g_{ij}\left(dx^i + N^idt\right)\left(dx^j + N^jdt\right), \tag{3}$$

the extrinsic curvature,

$$K_{ij} = \frac{1}{2N}\left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i\right), \tag{4}$$

the Cotton tensor,

$$C^{ij} = \epsilon^{ik\ell}\nabla_k\left(R^{(3)j}{}_\ell - \frac{1}{4}R^{(3)}\delta^j_\ell\right), \tag{5}$$

and coupling constants, $\kappa, \lambda, \nu, \mu, \Lambda_W, \omega$. The last term in the action (2) represents a "soft" violation, with the IR parameter ω, of the "detailed balance" condition in [7] and this modifies the IR behaviors so that Newtonian gravity limit exists [8–10].

The proposed action is not the most general form for a power-counting renormalizable gravity, compatible with the assumed foliation preserving *Diff* but it is general enough to contain all the known GR solutions, and the qualitative features of the solutions are expected to be similar [8–11]. Here, originally, the *non-relativistic* higher-derivative deformations were introduced from the technical reason of the necessity of renormalizable interactions without the ghost problem which exists in relativistic higher-derivative theories [7]. But we further remark that, which has not been well emphasized before, the (UV) Lorentz violation *might* have a more fundamental reason in our quantum gravity setup since this may be consistent with the existence of the absolute minimum length l_P which does not depend on the reference frames, violating the usual *relativistic* length contraction.[1]

For the simplest case of static, *i.e.*, non-rotating, uncharged black holes, where only the last three terms in the action (2) are relevant, the exact solutions have been found completely for arbitrary values of coupling constants, λ, Λ_W, and ω [8–11]. However, for the present purpose we only consider a simple example of $\lambda = 1$, $\Lambda_W = 0$,

$$ds^2 = -N(r)^2c^2dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{6}$$

with

$$N^2 = f = 1 + \omega r^2 - \sqrt{r[\omega^2r^3 + 4\omega M]} \tag{7}$$

so that the standard Einstein–Hilbert action and the asymptotically flat, Schwarzschild black hole solution are recovered in the IR limit, *i.e.*, $N^2 = f = 1 - 2M/r + \mathcal{O}(r^{-4})$ with $c^2 = \kappa^4\mu^2\omega/32$, $G = \kappa^2c^2/32\pi$ [8,9]. Here M denotes the '$(G/c^2) \times$ *ADM mass*' and the positive IR parameter ω controls the strength of higher-derivative corrections so that the limit $\omega \to \infty$, $\mu \to 0$ with '$\mu^2\omega =$ *fixed*' corresponds to GR limit.[2]

One remarkable property of the solution is that there is an inner horizon r_- as well as the outer horizon r_+, which solves $f(r_\pm) = 0$, at

$$r_\pm = M\left(1 \pm \sqrt{1 - \frac{1}{2\omega M^2}}\right) \tag{8}$$

as the result of higher (spatial) derivatives (Fig. 1): The higher derivative terms act like some (non-relativistic) effective matters

[1] It could be also possible that the Lorentz violation occurs only "dynamically" at some level in quantum gravity. For some extensive discussions about this possibility, see [12].

[2] The importance of this limit in a more extensive context will be discussed elsewhere [13].

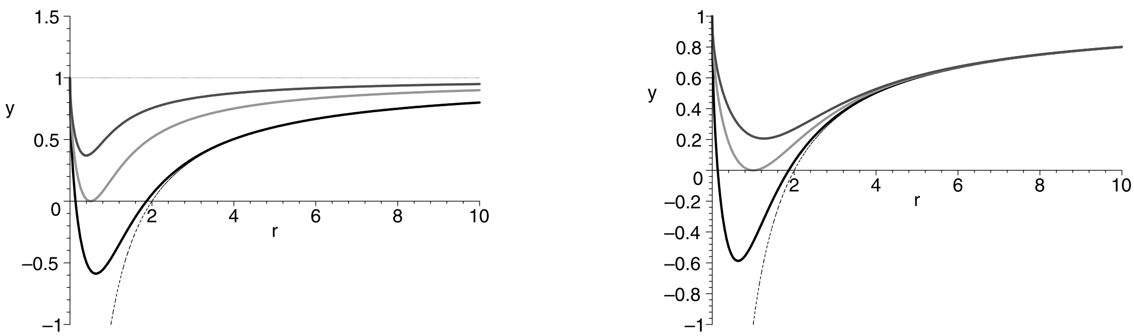

Fig. 1. Plots of $f(r)$ in Hořava gravity for varying M with a fixed ω (left) and for varying ω with a fixed M (right). In particular, in the left we consider $M = 0, 0.25, 0.5, 1$ (top to bottom solid curves) with $\omega = 2$, and in the right $\omega = 0.25, 0.5, 2$ (top to bottom solid curves) with $M = 1$, in contrast with Schwarzschild solution for $M = 1$ in GR (dotted curve).

in the conventional Einstein equation so that there is some "repulsive" interaction at short distances. Moreover, even though the metric converges to the Minkowski's flat spacetime at the origin $r = 0$, *i.e.*, $N^2 = f = 1 - 2\sqrt{\omega M r} + \omega r^2 + \mathcal{O}(r^{7/2})$, its derivative is not continuous so that there is a curvature singularity at $r = 0$, which may be captured by the singularity of $R \sim r^{-3/2}$, $R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} \sim r^{-3}$. So, even though the singularity at $r = 0$ is a time-like line (*i.e.*, time-like singularity) and is milder than that of Schwarzschild black hole, $R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} \sim r^{-6}$ (and also Reissner–Nordström's $R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} \sim r^{-4}$) and surrounded by the (two) horizons provided

$$\omega M^2 \geq \frac{1}{2}, \tag{9}$$

this might indicate that the proposed gravity does not completely resolve the singularity problem of GR still, classically.

This circumstance looks inconsistent with the cosmology solution, where the initial singularity does not exist whence there exist the higher-derivative effects, *i.e.*, non-flat universes [8,9]. Moreover, the black hole singularity becomes naked for $\omega M^2 < 1/2$ so that the cosmic censorship might not work in this edge of solution space, even if M is positive definite.

But according to recent Botta-Cantcheff–Grandi–Sturla (BGS)'s construction of wormholes, it seems that there is another possibility for resolving the unsatisfactory circumstance [14]. What they found was that there exits a wormhole solution also, in addition to the naked singularity solution for $\omega M^2 < 1/2$, without introducing additional (exotic) matters at the throat. Their obtained wormhole solution

$$ds^2 = -N_\pm(r)^2 c^2 dt^2 + \frac{dr^2}{f_\pm(r)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{10}$$

with

$$N_\pm^2 = f_\pm = 1 + \omega_\pm r^2 - \sqrt{r[\omega_\pm^2 r^3 + 4\omega_\pm M_\pm]} \tag{11}$$

is made of two coordinate patches, each one covering the range $[r_0, +\infty)$ in one universe and the two patches joining at the wormhole throat r_0, which is defined as the minimum of the radial coordinate r.

In the conventional approach for (traversable) wormholes, there are basically two unsatisfactory features [15,16]. First, we do not know much about the mechanism for a wormhole formation. This is in contrast to the black hole case, where the gravitational collapse of ordinary matters, like stars, can form a black hole, if its mass is enough. Second, the usual steps of 'constructing' wormholes are too artificial as summarized by the following three steps: (1) Prepare two or several universes; (2) Connect the universes by cut and paste of their throats; (3) Put the needed "exotic" matters, which violate the energy conditions, to the throats so that Einstein's equations are satisfied.

However, in the new approach, the problem of artificiality of a wormhole construction is avoided by observing that the solution is smoothly joined at the throat, so that the additional compensating (singular or non-singular) matters are not needed, if the metric and its derivatives are continuous at the throat. But, if we further consider the reflection symmetric two universes, *i.e.*, $f_+(r) = f_-(r)$, $N_+^2(r) = N_-^2(r)$, with $\omega_+ = \omega_- \equiv \omega$, $M_+ = M_- \equiv M$, the only possible way of smooth patching at the throat is[3]

$$\left.\frac{df_\pm}{dr}\right|_{r_0} = 0 \tag{12}$$

and the throat radius is obtained as

$$r_0 = \left(\frac{M}{2\omega}\right)^{1/3}. \tag{13}$$

Here, it is important to note that the throat is located always inside the black hole horizon, *i.e.*, $r_- < r_0 < r_+$ for $\omega M^2 > 1/2$ so that it is unobservable to an outer observer, whereas the wormhole throat emerges for $\omega M^2 < 1/2$, instead of a naked singularity (Fig. 2). For a fixed ω, the throat radius, after emerging from the coincidence with the extremal black hole radius, $r_0 = r_+ = r_- = M$, decreases monotonically as M decreases and finally vanishes for $M = 0$, *i.e.*, Minkowski vacuum (Fig. 2, left). This is the situation that has been assumed in BGS's paper [14]. But, on the other hand, for a fixed M and varying ω, one finds that the throat radius increases again indefinitely as ω decreases. This means that the wormhole size can be quite large when the coupling constant ω, which could flow under renormalization group, becomes smaller at quantum gravity regime, like the Planck size black hole or wormhole (Fig. 2, right).[4] In the actual quantum gravity process, like black hole evaporations, M, due to Hawking radiations, as well as ω, due to renormalization group flow with the change of energy scales, can vary so that we need to consider some combinations of situations of the left and right in Fig. 2, by extending the original interpretation.

[3] In BGS's paper [14], it is claimed that the junction condition (12) is not always necessary for $\lambda = 1$ but more general class of wormhole solutions could be possible. But, this is only for the case of singular, δ-function discontinuities in the equations of motions. Whereas, for the non-singular discontinuities at the throat, which cannot be properly treated in the BGS's analysis, the condition (12) is still essential for our wormhole construction.

[4] In order that this feature can be seen explicitly, it is important to consider the correct mass parameter M [17], rather than another form of an integration constant $\beta = 4\omega GM/c^2$ [9,11].

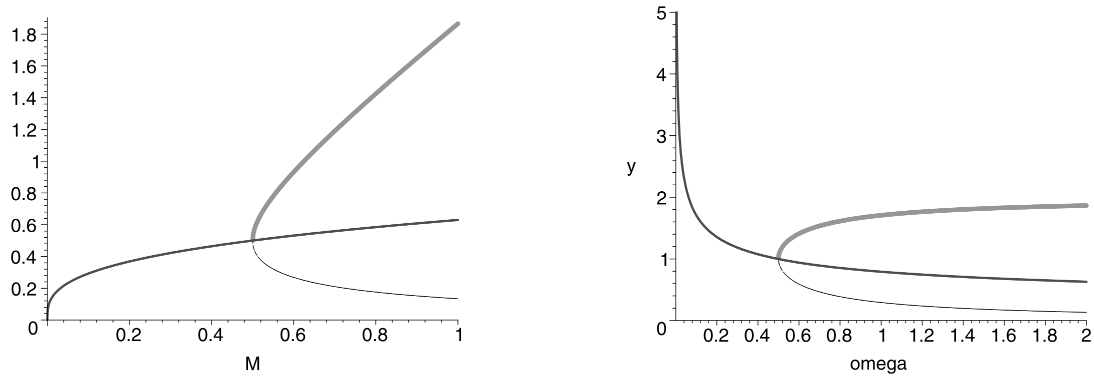

Fig. 2. Black hole horizons (top (r_+) and bottom (r_-) curves) and wormhole throat (middle (r_0) curve) radii for varying M with a fixed $\omega = 2$ (left), and for varying ω with a fixed $M = 1$ (right).

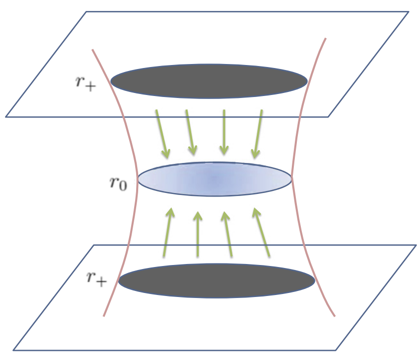

Fig. 3. A black hole pair which are connected by a wormhole throat r_0 inside the black hole horizons r_+. The arrows represent the time-like geodesics inside the horizons.

So, the wormhole solution is obtained, without the problem of artificiality, when the black hole horizon disappears for $\omega M^2 < 1/2$. Whereas, the solution for $\omega M^2 > 1/2$ with the throat inside the horizon is *not* the "traversable" wormhole since the throat will be still inside the black hole horizon of the "mirror" black hole in another universe. This is similar to the Einstein–Rosen bridge [18] but the difference is that the throat may not coincide with the black hole horizon generally in the present case. Moreover, in the present case the throat is located at the fixed "time" r_0 so that any time-like trajectories should meet the throat if exits (Fig. 3). This means that the black hole solution (10), (11) with the "time-like" throat for $\omega M^2 > 1/2$ should be considered as a physically distinct object from the black hole solution (6), (7) having the (hidden) singularity at $r = 0$. And in order to avoid a rather strange situation that the wormhole throat "suddenly" emerges from the extremal black hole which has a singularity at $r = 0$ still, it would be *natural* to consider the time-like throat in (10), (11) for $\omega M^2 > 1/2$ as the "would-be" wormhole throat. In order to be distinguished from the usual black hole solution (6), (7), we may call the solution (10), (11) as the "Black Wormhole" solution.

Now, we have a completely regular vacuum solution, without the curvature singularities, which interpolates between the black hole state for $\omega M^2 > 1/2$ and wormhole state for $\omega M^2 < 1/2$. This seems to support Wheeler's foam picture of the quantum spacetime in quantum gravity but as a real static, not as a virtual time-dependent, wormhole. And in our quantum gravity theory, where the concept of horizon emerges at low energies, escaping the horizons is not impossible for high energy particles with Lorentz-violating dispersion relations. But for low energy point of view, without probing the interior structure of real black holes, the observable consequences of the black wormhole solution would be expressed in the form of a "Generalized Cosmic Censorship",[5] suggesting that "the naked singularity does not appear still by forming wormholes even after the horizons disappear in quantum gravity" though, before evaporation, there would be no naked singularity by the existence of horizons as usual.

Another important new implication of the black wormhole solution to low energy observers is that there would be transformations between black holes and wormholes,[6] which is known to be impossible in GR, due to the no-go theorem for topology change [20]. In other words, once a (primordial) wormhole is formed in the quantum gravity regime, due to quantum fluctuation in the early universe, it may evolve into a black hole state by the combinations of Fig. 2 (right) from renormalization group flows to GR and Fig. 2 (left) from accretion of matters. It would be interesting to see whether this could be a mechanism for the primordial black holes and supermassive black holes, which are believed to be formed very early in the universe and distinguished from the stellar-mass black holes which are (or believed to be) generated from collapsing stars.

On the other hand, according to the black wormhole solution, once a black hole is formed in our quantum gravity, it always has the would-be wormhole inside the horizon but this inside wormhole is exposed to outer observers when the horizon disappears after the complete evaporation. But we know that the inside wormhole is absent in the GR limit, as can be seen in Fig. 2 (right). Then, where does the inside wormhole come from in quantum gravity regime? This is the question about the physical mechanism of a wormhole formation, which has been lacking. It seems that the only possible answer to this question could be found in the Hawking radiation process, which involves virtual pairs of particles near the event horizon, one of the pair enters into the black hole while the other escapes: The escaped particle is observed as a real particle with a *positive* energy with respect to an observer at infinity and then, the absorbed particle must have a *negative* energy in order that the energy is conserved [4]. This implies that the negative energy particles that fall into black hole could be an "ex-

[5] In [14], this notion was considered as a "complementary" between the two mechanisms to censure the singularities.

[6] Hayward has suggested a similar black hole–wormhole transformation on the grounds of "trapping" horizons which may describe black holes and wormholes unifiedly [19]. But in his framework, assuming the *bifurcating* black hole horizons is essential and it is not clear how to extend his framework to our case of non-bifurcating horizons of extremal state where wormhole throat and extremal black hole horizon coincides. Moreover, he did not suggest that interior structure of a black hole might be changed after the transformation from a wormhole so that the singularity might not occur in the black hole state, also.

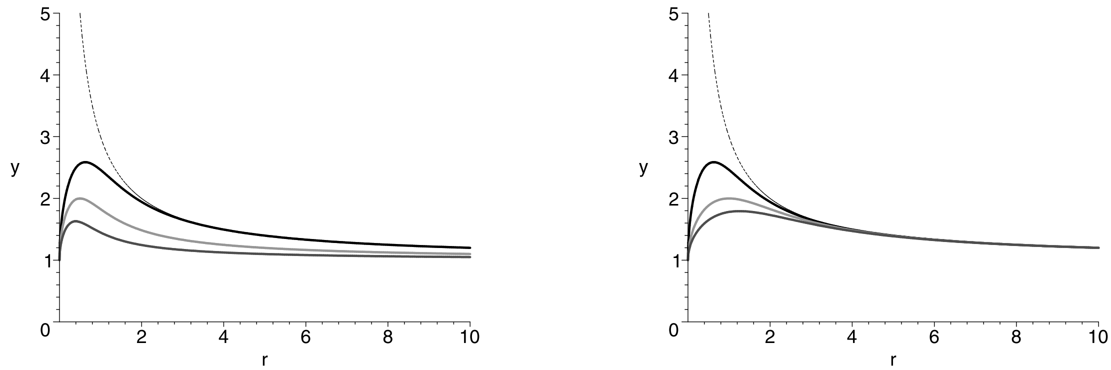

Fig. 4. Plots of the *negative mass* solution of $f(r)$ in Hořava gravity for varying M with a fixed ω (left) and for varying ω with a fixed M (right). In particular, in the left we consider $M = -0.25, -0.5, -1$ (bottom to top solid curves) with $\omega = -2$, and in the right $\omega = -0.25, -0.5, -2$ (bottom to top solid curves) with $M = -1$, in contrast with the negative mass Schwarzschild solution for $M = -1$ in GR (dotted curve).

otic matter" source for a wormhole formation inside the horizon. But one can easily discover that this would be quite implausible in GR since negative masses (or energies) repel (*i.e.*, produce the outward accelerations of), as positive masses attract (*i.e.*, produce the inward accelerations of), all other bodies regardless of their (positive or negative) masses from the *equivalence principle*. We have never observed the negative mass object yet and it could produce some strange phenomena, known as "the runaway motion" of a pair of positive and negative mass particles, but the repulsive nature of gravity for negative masses is another remarkable consequence of GR [21]. In the language of the Newtonian approximation, where the gravitational potential of a spherically symmetric body is given by $\varphi(r) = (f(r) - 1)/2$, that property is indicated by the fact that df/dr, which is related to the (radial) acceleration $a = -d\varphi/dr = -(1/2)(df/dr)$, is always positive for the potential of a positive mass $M > 0$, whereas df/dr is always negative for the potential of a negative mass $M < 0$. This property implies that negative mass could not form a (stable) structure, *naturally*. This could explain why it would not be possible to form a wormhole by collapsing of exotic matters, in particular, negative energy matters, including the negative energy particles that fall into black hole in GR, in contrast to the case of a black hole formation. Actually, previously the negative energy particle has been thought to result in just reducing the black hole mass by some compensation process with positive energy sources, which may exist inside the horizon, for the positive black hole mass.

However, the situation is quite different in our quantum gravity context, where the gravity becomes weaker at short distance and changes its (attractive or repulsive) nature after passing the surface of $df/dr = 0$, which we call the "zero-gravity surface" for convenience. In other words, for the positive black hole mass $M > 0$, the gravity becomes repulsive inside the zero-gravity surface (Fig. 1, solid curves). This now implies that even the positive mass could not form a structure *inside the zero-gravity surface*, naturally, in contrast to the outside the zero-gravity surface where the its gravity is still attractive so that a stable structure can be formed, as in GR.

In order to see how this problem could be resolved dramatically by the negative masses in our quantum gravity context, we now turn to consider the *negative mass* solution in Hořava gravity, which has never been studied in the literature. In order to find a negative mass solution, one might first try to consider $M < 0$ for the solution (7) but one can easily find that this could not be the right solution: The solution is *not defined* below a certain radius, where the quantity inside the square root become zero for $\omega > 0$ or shows a different asymptotic ($r \to \infty$) behavior, *i.e.,* not asymptotically flat, $N^2 = f = 1 + 2\omega r^2 + 2M/r + \mathcal{O}(r^{-4})$, for $\omega < 0$. However, by noting that the solution of the metric ansatz (6) is unique up to the ($\pm$) sign in front of the square root in (7) generally, we find that the negative mass solution as

$$N^2 = f = 1 + \omega r^2 \text{“} + \text{”} \sqrt{r[\omega^2 r^3 + 4\omega M]} \tag{14}$$

for the different sign of the square root term, compared to the solution (7). This solution recovers the Schwarzschild solution in the IR limit for $\omega < 0$, as (7) does for $M > 0$ and $\omega > 0$ (Fig. 4); for $\omega > 0$, the solution (14) neither recovers the Schwarzschild solution nor the metric is defined for the whole space region. [But in this case, in order that GR is recovered, we need to consider $\mu^2 < 0$ as in asymptotically de Sitter space [9] and we will discuss about this again later.]

One can prove that the new solution (14) does not have horizons, as can be easily observed in Fig. 4 also but it has a curvature singularity at $r = 0$, where the metric is finite but discontinuous, with the same degrees of divergences as the solution (7). So, this solution itself represents a naked singularity as the negative mass Schwarzschild solution does in GR. Actually, this may be one of main obstacle for considering the negative mass solution as a viable solution in GR and this is not much improved for the solution (14) alone, in our Hořava gravity context.

But, as in the positive mass case, we have another (exact) solution of a wormhole, for the reflection symmetric two universes, with the same formula (13) for the throat r_0. In other words, in Hořava gravity, there exists a regular, *i.e., singularity free*, negative mass solution so that it could be viable but only in the form of a wormhole geometry. But here we will not consider about the negative mass wormhole solution further since its formation mechanism is unclear at present.

Rather, we consider the negative mass solution (14) in order to see how does the negative masses, like the negative energy particles that fall into black hole could interact and form a structure, like the (black) wormhole with a positive mass. Actually, the solution (14) indicates that the gravity of a negative mass is repulsive at large distance but becomes weaker, *i.e.*, less-repulsive, at short distance and moreover becomes attractive inside the zero-gravity surface of $df/dr = 0$. This implies that the negative masses could form a structure naturally, at short distance inside the zero-gravity surface, in contrast to the case of positive masses which could not form a structure inside their *own* zero-gravity surface as we have explained already. This is the remarkable consequence of Hořava gravity which could justify our picture that *wormholes are created and sustained by the continuous inflow of negative energy particles via the Hawking radiation process*, which is impossible in GR. This is the main claim of this paper and in a more compact form, this can be expressed as: *The quantum black hole could be a wormhole factory!*

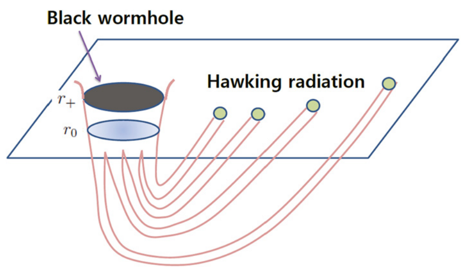

Fig. 5. The microscopic would-be wormholes created by negative energy quanta which have entered the black hole horizon with its Hawking radiated, positive energy, partner quanta outside the horizon. This describes the quantum black hole as a factory of microscopic wormholes which would merge to a macroscopic black wormhole solution, in conformity with "*ER* = *EPR*" proposal.

Interestingly, this may provide a physical origin of the Einstein–Rosen bridge in the recent "*ER* = *EPR*" proposal for resolving the issue of entanglement in a black hole spacetime [22] claiming that "the black hole and its Hawking radiation are entangled via Einstein–Rosen bridges" (Fig. 5). Following their proposal, one could understand the would-be wormholes inside the black hole horizon as the results of microscopic wormholes created by negative energy quanta which have entered the black hole horizon but entangled with its Hawking radiated, positive energy, partner quanta, which now lives in a single universe, widely separated from its mother black hole. This may be consistent with our reflection symmetric configuration, but it is not clear how to extend this picture to more general configurations which cannot be interpreted as a configuration in a single universe. Moreover, it is not clear either how the throats of Einstein–Rosen's bridges evolve into our would-be wormhole in detail.

In conclusion, we have considered a vacuum and static *black wormhole* solution which is regular, *i.e.*, singularity-free, and interpolates between the black hole state for $\omega M^2 > 1/2$ and wormhole state for $\omega M^2 < 1/2$ through the coincidence state of an extremal black hole and a (kind of) Einstein–Rosen bridge for $\omega M^2 = 1/2$. From this, we have suggested the transformation between the black hole and wormhole states, and its resulting generalized cosmic censorship. And furthermore, we have argued that the would-be wormholes inside the black hole horizon could be understood as the results of microscopic wormholes created by "negative" energy quanta which have entered the black hole horizon in Hawking radiation processes so that *the quantum black hole could be a wormhole factory.*

But then, "Can the transformation really occur dynamically?" In order to answer to this question, we first consider the transformation from a black hole state to a wormhole state. Of course, there is some obstacle in GR context since the black hole should pass[7] the extremal black hole to become a wormhole state but the extremal black hole has vanishing Hawking radiation and temperature so that it is believed as the stable ground state in the black hole states. However recently, certain classical instability, known as *Aretakis instability*, for extremal black holes have been found in GR so that the usual belief may not be quite correct [25]. A heuristic argument about this instability suggests that the inner horizon instability of near-extremal black holes [26] might cause an instability of the coincided inner and outer horizons [27]. If this is the case, we may expect a similar instability in our extremal black holes also, due to the (expected) inner horizon instability.[8]

Now then, let us consider the transformation from a wormhole state to a black hole state. In this case, there does not seem to exist any classical obstacle against this transformation, which resembles the collapse of stars made of ordinary matter to make a black hole, where the entropy would increase in the process. Actually, in the conventional wormhole context with exotic matters in GR, the collapse of the Morris–Thorne wormhole into a black hole was observed from non-linear instability under ordinary matter perturbations [28]. We expect the similar non-linear instability, though linearly stable, exits for our wormhole state too so that it transforms to the extremal black hole state, a seed of large black holes.

Finally, two further remarks are in order.

First, even though we have obtained the solution in a particular quantum gravity model which is power-counting renormalizable without ghost problems, the features of the small scale structure seems to be quite generic if *the vanishing Hawking temperature for a Planck mass black hole*, which implying the existence of the wormhole throat r_0 satisfying $df/dr|_{r_0} = 0$, is considered as a verifiable prediction that any theory of quantum gravity makes. For example, so-called "renormalization group improved black hole spacetimes" would have the similar wormhole structure and our discussion may not be limited to Hořava gravity [5]. But we can easily check that it is not applicable to Kerr nor Reissner–Nordström black holes in GR: In these cases, there are more metric functions or additional matter fields but there is no solution for the throat where all the metric functions or fields join smoothly.[9]

Second, we have found that, in order to describe the negative mass solution (14), we need to consider the coupling constant of $\mu^2 < 0$ so that one can recover the well-defined GR parameters $c^2, G > 0$ in the IR limit. This does not affect the couplings of the second-derivative terms in the action (2) (note that we are considering the case of $\Lambda_W = 0$) but only affects other UV couplings [11], which differ from those for the positive mass solution (7). This indicates that the (weak) *equivalence principle* may be violated by UV effects generally, depending on the sign of particle's mass. It would be a challenging problem that the runaway motions of a pair of positive and negative mass particles could be avoided from the inequivalent motions of the positive and negative mass particles in UV.

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[7] Recently, there have been quite active researches and debates about whether a non-extremal black hole can "jump over" the extremality, by capturing a particle with appropriate parameters [23]. In particular, a quantum jump or tunneling seems to be a quite promising mechanism for breaking extremal black holes in our quantum gravity set-up [24].

[8] The inner horizon instability [26] may be related to the *negative* temperature for the inner horizon, *i.e.*, $T_- \equiv (\hbar/4\pi)(df/dr)_{r_-} < 0$ [29], which seems to be a quite generic property of the inner horizons, including those of the black holes in Hořava gravity [9].

[9] Similarly, it looks like that the existence of the throat for more general black holes with charges or rotations in the Hořava gravity would be very difficult, unless some accidental coincidences occur. This seems to implies that the wormhole throat in the spherically symmetric configurations could be easily destroyed by adding other hairs. Conversely, the wormhole throat could be formed only after losing all the hairs except the mass.

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