Topological aspects of dual superconductors

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We discuss topological aspects of two-gap superconductors with and without Josephson coupling between gaps. We address nontrivial topological aspects of the dual superconductors and its connections to the Meissner effect and flux quantization. The topological knotted string geometry is also discussed in terms of the Hopf invariant, curvature, and torsion of the strings associated with $U(1)\times U(1)$ gauge group.

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I. INTRODUCTION

There have been considerable attempts to understand condensed matter phenomenology in terms of topological configurations inherited from knot structures.[1–6] The geometry of knotted solitons was studied to show that the total linking numbers during the soliton interactions are preserved,[1] and the anomaly structure of the fermions in a knotted soliton background was shown to be related to the inherent chiral properties of the soliton.[4] Moreover, the curvature and torsion of a bosonic string in 3+1 dimensions were investigated[7] to be employed as Hamiltonian variables in a two-dimensional Ginzburg-Landau gauge field theory.[8] Interactions of vortices were also investigated[9,10] in the Ginzburg-Landau theory. In two and three dimensions, the crossover from weak- to strong-coupling superconductivities was studied to figure out their thermodynamics.[11] Quite recently, the SU(2) Yang-Mills theory was studied to investigate a symmetry between electric and magnetic variables[12] and also to discuss the two-band superconductors with interband Josephson couplings.[13–16] On the other hand, many experiments and *ab initio* calculations show two-band superconductivity in MgB_2—for instance, as in Refs. 17 and 18. The photoemission spectroscopy of the superconductor $NbSe_2$ indicates also two-band superconductivity associated with Fermi-surface sheet-dependent superconductivity in this multiband system.[19] Also theoretical studies indicate the possibility of two-gap superconductivity without an intrinsic Josephson effect in liquid metallic hydrogen, deuterium, and hydrogen alloys under extreme pressures.[20–23]

In this paper we will investigate the two-gap superconductors by exploiting the two-flavor Ginzburg-Landau theory, where we study the magnetic flux quantization of two-gap superconductors. We will explicitly evaluate the London penetration depth and the Meissner and Josephson effects to obtain the nontrivial topological aspects of the two-gap superconductors. The knotted geometry will also be discussed in the framework of the bosonic strings.

II. MODEL FOR TWO-GAP SUPERCONDUCTORS

Now, in order to describe the two-band superconductors with interband Josephson coupling,[13–16] we start with the two-flavor Ginzburg-Landau theory whose free energy density is given by

$$F = \frac{1}{2m_1}\left|\left(\frac{\hbar}{i}\nabla + \frac{2e}{c}\vec{A}\right)\Psi_1\right|^2 + \frac{1}{2m_2}\left|\left(\frac{\hbar}{i}\nabla - \frac{2e}{c}\vec{A}\right)\Psi_2\right|^2 + \frac{1}{8\pi}\vec{B}^2 + V + \eta(\Psi_1^*\Psi_2 + \Psi_2^*\Psi_1), \qquad (2.1)$$

where Ψ_1 and Ψ_2 are order parameters for Cooper pairs of two different flavors and V is a potential of the form $V(|\Psi_{1,2}|^2) = -b_\alpha|\Psi_\alpha|^2 + \frac{1}{2}c_\alpha|\Psi_\alpha|^4$ $(\alpha=1,2)$ (Refs. 2 and 14). Here we introduce η which is a characteristic of the interband Josephson coupling strength.[13–16] In the case of $\eta=0$ vanishing Josephson coupling, we can describe the liquid metallic hydrogen which should allow coexistent superconductivity of protonic and electronic Cooper pairs.[20–23] Moreover, the interband Josephson coupling merely changes the energy of the knot associated with the two-band superconductors. The two condensates are then characterized by different effective masses m_α, coherence lengths $\xi_\alpha = \hbar/(2m_\alpha b_\alpha)^{1/2}$, and densities $\langle|\Psi_\alpha|^2\rangle = b_\alpha/c_\alpha$.

We then introduce fields ρ and z_α defined as

$$\Psi_\alpha = (2m_\alpha)^{1/2}\rho z_\alpha, \qquad (2.2)$$

where the modulus field ρ is given by the condensate densities and masses, $\rho^2 = (1/2m_1)|\Psi_1|^2 + (1/2m_2)|\Psi_2|^2$, and the CP^1 complex fields z_α are chosen to satisfy the geometrical constraint

$$z_\alpha^* z_\alpha = |z_1|^2 + |z_2|^2 = 1. \qquad (2.3)$$

In the two-gap superconductors, the gauge-invariant supercurrent is given by[2]

$$\vec{J} = -\frac{e}{2m_1}\left[\Psi_1^*\left(\frac{\hbar}{i}\nabla + \frac{2e}{c}\vec{A}\right)\Psi_1 - \Psi_1\left(\frac{\hbar}{i}\nabla - \frac{2e}{c}\vec{A}\right)\Psi_1^*\right] + \frac{e}{2m_2}\left[\Psi_2^*\left(\frac{\hbar}{i}\nabla - \frac{2e}{c}\vec{A}\right)\Psi_2 - \Psi_2\left(\frac{\hbar}{i}\nabla + \frac{2e}{c}\vec{A}\right)\Psi_2^*\right], \tag{2.4}$$

which can be rewritten in terms of the fields ρ and z_α as follows:

$$\vec{J} = -\hbar e\rho^2\left(\vec{C} + \frac{4e}{\hbar c}\vec{A}\right), \tag{2.5}$$

where

$$\vec{C} = i(\nabla z^\dagger z - z^\dagger\nabla z) = i(z_1\nabla z_1^* - z_1^*\nabla z_1 - z_2\nabla z_2^* + z_2^*\nabla z_2), \tag{2.6}$$

with $z = (z_1, z_2^*)$.

Since the CP^1 model is equivalent to the O(3) nonlinear sigma model[24] (NLSM) at the canonical level, one can introduce the dynamical physical fields n_a $(a=1,2,3)$ which are mappings from the space-time manifold (or the direct product of a compact two-dimensional Riemann surface M^2 and time dimension R^1) to the two-sphere S^2—namely, $n_a : \mathsf{M}^2 \otimes R^1 \to S^2$. On the other hand, the dynamical physical fields of the CP^1 model are z_α which map the spacetime manifold $\mathsf{M}^2 \otimes R^1$ into S^3—namely, $z_\alpha : \mathsf{M}^2 \otimes R^1 \to S^3$. Since S^3 is homeomorphic to the SU(2) group manifold and the CP^1 model is invariant under a local U(1) gauge symmetry,

$$z \to e^{i\xi/2} z, \tag{2.7}$$

for arbitrary space-time-dependent ξ (Ref. 25), the physical configuration space of the CP^1 model is that of the gauge orbits which form the coset $S^3/S^1 = S^2 = CP^1$. In order to associate the physical fields of the CP^1 model with those of the O(3) NLSM, we exploit the projection from S^3 to S^2—namely, the Hopf bundle[25,26]

$$n_a = z^\dagger \sigma_a z, \tag{2.8}$$

with the Pauli matrices σ_a and the n_a fields satisfying the geometrical constraint $n_a n_a = 1$—to yield the free energy

$$F = \hbar^2(\nabla\rho)^2 + \frac{1}{4}\hbar^2\rho^2(\nabla n_a)^2 + \frac{1}{4e^2\rho^2}\vec{J}^2 + \frac{1}{8\pi}\vec{B}^2 + V + K\rho^2 n_1,$$

where $K = 2\eta(m_1 m_2)^{1/2}$. Introducing gauge-invariant vector fields $\vec{S}$ in terms of the supercurrent $\vec{J}$ in Eq. (2.4), $\vec{S} = (1/\hbar e\rho^2)\vec{J}$, one can arrive at the free energy density of the form

$$F = \hbar^2(\nabla\rho)^2 + \frac{1}{4}\hbar^2\rho^2[(\nabla n_a)^2 + \vec{S}^2] + \frac{\hbar^2 c^2}{128\pi e^2} \times \left(\nabla\times\vec{S} + \frac{1}{2}\epsilon_{abc} n_a \nabla n_b \times \nabla n_c\right)^2 + V + K\rho^2 n_1.$$

III. MEISSNER EFFECTS

Now, we discuss the Meissner effect in the two-flavor topological NLSM, where the magnetic field $\vec{B}$ is expressed in terms of the fields ρ, n_a, and $\vec{S}$:

$$\vec{B} = \nabla\times\vec{A} = -\frac{\hbar c}{4e}\left(\nabla\times\vec{S} + \frac{1}{2}\epsilon_{abc} n_a \nabla n_b \times \nabla n_c\right). \tag{3.1}$$

Combining Eqs. (2.5) and (3.1) and the identity $\nabla\times\vec{C} = \frac{1}{2}\epsilon_{abc} n_a \nabla n_b \times \nabla n_c$, we obtain the two-gap equation in terms of the ρ and n_a fields,

$$\nabla\times\vec{J} = -\frac{4e^2}{c}\rho^2\vec{B} + \frac{2}{\rho}\nabla\rho\times\vec{J} - \frac{\hbar e}{2}\rho^2\epsilon_{abc} n_a \nabla n_b \times \nabla n_c, \tag{3.2}$$

which can also be rewritten in terms of the vector fields $\vec{S}$: $\nabla\times\vec{S} = (-4e/\hbar c)\vec{B} - \frac{1}{2}\epsilon_{abc} n_a \nabla n_b \times \nabla n_c$. Note that in the two-gap equation (3.2) there exists topological contribution proportional to $\epsilon_{abc} n_a \nabla n_b \times \nabla n_c$ which originates from interactions of Cooper pairs of two different flavors.

Next, we consider the Meissner effect[27] and the corresponding London penetration depth in the two-gap superconductor where the Maxwell equation reads $\nabla\times\vec{B} = (4\pi/c)\vec{J}$. Here the rate of time variation is assumed to be so slow that the displacement current can be ignored. Combining the above Maxwell equation with the two-gap equation (3.2), we arrive at the two-gap equations for $\vec{J}$ and $\vec{B}$:

$$\begin{aligned}\nabla^2\vec{J} &= \left(\frac{16\pi e^2}{c^2} + \frac{2}{\rho}\nabla^2\rho - \frac{2}{\rho^2}(\nabla\rho)^2\right)\vec{J} + \frac{8e^2}{c}\rho\nabla\rho\times\vec{B} \\ &\quad + \frac{2}{\rho^2}(\nabla\rho\cdot\vec{J})\nabla\rho + \frac{2}{\rho}[(\nabla\rho\cdot\nabla)\vec{J} - (\vec{J}\cdot\nabla)\nabla\rho] \\ &\quad + \frac{\hbar e}{2}\rho^2\nabla\times(\epsilon_{abc} n_a \nabla n_b \times \nabla n_c) \\ &\quad + \hbar e\rho\nabla\rho\times(\epsilon_{abc} n_a \nabla n_b \times \nabla n_c), \\ \nabla^2\vec{B} &= \frac{16\pi e^2}{c^2}\rho^2\vec{B} - \frac{8\pi}{c\rho}\nabla\rho\times\vec{J} + \frac{2\pi\hbar e}{c}\rho^2\epsilon_{abc} n_a \nabla n_b \\ &\quad \times \nabla n_c.\end{aligned} \tag{3.3}$$

Note that the spatial variation of the order parameter magnitude $\nabla\rho$ couples the $\vec{J}$ and $\vec{B}$ field equations. From Eq. (3.3), we can investigate the two-gap Meissner effect at low temperature $T < T_c$ as below.

At low temperature $T < T_c$ where the order parameter magnitude ρ varies only very slightly over the superconductor, we obtain $\nabla\times\vec{J} = -(4e^2/c)\rho^2\vec{B} - (\hbar e/2)\rho^2\epsilon_{abc} n_a \nabla n_b \times \nabla n_c$, so that we can arrive at the decoupled equations for the $\vec{J}$ and $\vec{B}$:

$$\nabla^2\vec{J} = \frac{16\pi e^2}{c^2}\rho^2\vec{J} + \frac{\hbar e}{2}\rho^2\nabla\times(\epsilon_{abc} n_a \nabla n_b \times \nabla n_c),$$

$$\nabla^2\vec{B}=\frac{16\pi e^2}{c^2}\rho^2\vec{B}+\frac{2\pi\hbar e}{c}\rho^2\epsilon_{abc}n_a\nabla n_b\times\nabla n_c. \quad (3.4)$$

Here note that we have the topological contribution with $\epsilon_{abc}n_a\nabla n_b\times\nabla n_c$. The equation for $\vec{B}$ in Eq. (3.4) then yields the two-gap London penetration depth

$$\Lambda=\left(\frac{m_1c^2}{4\pi e^2n_{1s}}\right)^{1/2}\left(1+\frac{m_1n_{2s}}{m_2n_{1s}}\right)^{-1/2}, \quad (3.5)$$

where the superfluid densities $n_{\alpha s}$ are given by $n_{\alpha s}=2|\Psi_\alpha|^2$ (Ref. 28). Here, we have derived the quantity Λ in Eq. (3.5) in London limit when $|\Psi_\alpha|=\text{const}$ and thus $\epsilon_{abc}n_a\nabla n_b\times\nabla n_c=0$. Note that the two-gap surface supercurrents screen out the applied field to yield the two-gap Meissner effect. Moreover, the two-gap London penetration depth in Eq. (3.5) is reduced to the single-gap London penetration depth (3.7) below in the one-flavor limit with $n_{2s}=0$.

Next, we consider the nontopological one-flavor limit with $n_{2s}=0$ and $\nabla\times\vec{C}=0$. In this limit, Eqs. (3.2) and (3.3) are reduced to the form

$$\nabla\times\vec{J}=-\frac{e^2n_{1s}}{m_1c}\vec{B}+\frac{1}{n_{1s}}\nabla n_{1s}\times\vec{J},$$

$$\nabla^2\vec{J}=\left(\frac{4\pi e^2}{m_1c^2}n_{1s}+\frac{1}{n_{1s}}\nabla^2n_{1s}-\frac{1}{n_{1s}^2}(\nabla n_{1s})^2\right)\vec{J}+\frac{e^2}{m_1c}\nabla n_{1s}\times\vec{B}$$
$$+\frac{1}{2n_{1s}^2}[(\nabla n_{1s}\cdot\vec{J})\nabla n_{1s}+\nabla n_{1s}(\vec{J}\cdot\nabla)n_{1s}]$$
$$+\frac{1}{n_{1s}}[(\nabla n_{1s}\cdot\nabla)\vec{J}-(\vec{J}\cdot\nabla)\nabla n_{1s}],$$

$$\nabla^2\vec{B}=\frac{4\pi e^2}{m_1c^2}n_{1s}\vec{B}-\frac{4\pi}{cn_{1s}}\nabla n_{1s}\times\vec{J}. \quad (3.6)$$

Note that in the more restricted low temperature limit $T<T_c$, we have the well-known single-gap equations $\nabla\times\vec{J}=-(e^2n_{1s}/m_1c)\vec{B}$, $\nabla^2\vec{J}=(4\pi e^2/m_1c^2)n_{1s}\vec{J}$, and $\nabla^2\vec{B}=(4\pi e^2/m_1c^2)n_{1s}\vec{B}$, which yield the single-gap London penetration depth[29]

$$\Lambda=\left(\frac{m_1c^2}{4\pi e^2n_{1s}}\right)^{1/2}=41.9\left(\frac{r_s}{a_0}\right)^{3/2}\left(\frac{n_e}{n_{1s}}\right)^{1/2}\text{ Å}, \quad (3.7)$$

where $r_s=(3/4\pi n_e)^{1/3}$, a_0 is the Bohr radius, and n_e is the total electron density given by $n_e=n_{1n}+n_{1s}$ with the normal (superfluid) electron density n_{1n} (n_{1s}).

Exploiting the relation in Eq. (3.7), we can rewrite the two-gap London penetration depth (3.5) as

$$\Lambda=41.9\left(\frac{r_s}{a_0}\right)^{3/2}\left(\frac{n_e}{n_{1s}}\right)^{1/2}\left(1+\frac{m_1n_{2s}}{m_2n_{1s}}\right)^{-1/2}\text{ Å}. \quad (3.8)$$

Note that, in the two-gap London penetration depth (3.8), with respect to the single-gap case we have more degrees of freedom associated with the physical parameters m_2 and n_{2s} to adjust theoretical predictions to experimental data for the London penetration depth.

IV. FLUX QUANTIZATION AND JOSEPHSON EFFECTS

Now, we consider the magnetic flux quantization of the two-gap superconductors to discuss interspecies Cooper pair tunneling—namely, the Josephson effects.[30] We consider a two-gap superconductor in the shape of a cylinderlike ring where there exists a cavity inside the inner radius. In order to evaluate the magnetic flux inside the two-gap superconductor, we embed within the interior of the superconducting material a contour encircling the cavity. Since at low temperature $T<T_c$ appreciable supercurrents can flow only near the surface of the superconductor and the order parameter magnitude ρ vary only very slightly over the two-gap superconductor, integration of the supercurrent $\vec{J}$ in Eq. (2.5) over a contour vanishes to arrive at the magnetic flux $\Phi=\oint A$ carried by vortex of the superconductor. On the other hand, to explicitly evaluate the phase effects of the two-gap superconductor, we parametrize the z_α fields as follows:

$$z_1=|z_1|e^{i\phi_1}=e^{i\phi_1}\cos\frac{\theta}{2},\quad z_2=|z_2|e^{i\phi_2}=e^{i\phi_2}\sin\frac{\theta}{2} \quad (4.1)$$

to satisfy the constraint (2.3). After some algebra, we obtain

$$\vec{C}=2(|z_1|^2\nabla\phi_1-|z_2|^2\nabla\phi_2). \quad (4.2)$$

Here note that even though there exists a $\nabla\theta$ dependence of $z_\alpha\nabla z_\alpha^*-z_\alpha^*\nabla z_\alpha$ $(\alpha=1,2)$ in each flavor channel, these contributions to $\vec{C}$ cancel each other to yield vanishing overall effects. Since the order parameters Ψ_α are single valued in each flavor channel, their corresponding phases should vary 2π times integers p_α when the ring is encircled, to yield $\oint\nabla\phi_\alpha\cdot d\vec{l}=2\pi p_\alpha$ so that we can obtain

$$\oint C=4\pi(|z_1|^2p_1-|z_2|^2p_2). \quad (4.3)$$

Exploiting Eq. (4.3), we arrive at

$$|\Phi|=(|z_1|^2p_1-|z_2|^2p_2)\Phi_0,$$

which is also written in terms of the n_a fields to yield the fractional magnetic flux quantized with the vortex of the two-gap superconductors:

$$|\Phi|=\frac{1}{2}(p_1-p_2+(p_1+p_2)n_3)\Phi_0, \quad (4.4)$$

with the fluxoid $\Phi_0=hc/2e=2.0679\times10^{-7}$ G cm^2. Here note that the interband Josephson coupling does not change the flux quantization since its role converts circularly symmetric vortex to a two-dimensional sine-Gordon vortex.[14,15] To investigate the physical meaning of the magnetic flux (4.4) for the two-gap superconductor, we consider a particular case of $p_1=p_2=1$. In this case, we can find the magnetic flux carried by the vortex in terms of the angle θ:

$$|\Phi|=n_3\Phi_0=\Phi_0\cos\theta,$$

which shows that such a vortex can possess an arbitrary fraction of a magnetic flux quantum since $|\Phi|$ depends on the parameter $\cos\theta$ measuring the relative densities of the two condensates in the superconductor as shown in Eq. (4.1). Moreover, in the case of $p_1=-p_2$, the magnetic flux (4.4) is reduced to the well-known single-gap magnetic flux quantization, $|\Phi|=p_1\Phi_0$, where we can readily find $\theta=0$ to yield $|z_1|=1$ and $|z_2|=0$. Note that, exploiting the above identity (4.2), $\nabla\times\vec{J}$ in Eq. (3.6) can be also rewritten in terms of the phase ϕ_1 as $\nabla\times\vec{J}=-(e^2 n_{1s}/m_1 c)\vec{B}-(\hbar e/2m_1)\nabla n_{1s}\times\nabla\phi_1 -(e^2/m_1 c)\nabla n_{1s}\times\vec{A}$, where we have the explicit phase-dependent term.

V. KNOTTED STRING GEOMETRY

Now, we consider bosonic string knot geometry associated with the two-gap superconductors. It is shown to be an equivalence between the two-flavor Ginzburg-Landau theory and a version of the O(3) NLSM introduced in Ref. 31. Moreover, the model in Ref. 31 describes topological excitations in the form of stable, finite-length knotted closed vortices[32] to lead to an effective string theory.[33] This equivalence can thus imply that the two-gap superconductors similarly support topologically nontrivial, knotted solitons.

In order to investigate the stringy features of the two-flavor Ginzburg-Landau theory, we recall that in the Hopf bundle (2.8), n_a remains invariant under the U(1) gauge transformation (2.7). Exploiting the parametrization (4.1), n_a can be rewritten in terms of the angles θ and $\beta=\phi_1+\phi_2$:

$$\vec{n}=(\cos\beta\sin\theta,-\sin\beta\sin\theta,\cos\theta). \tag{5.1}$$

Note that n_a is independent of the angle $\alpha=\phi_1-\phi_2$ so that α can be considered as a coordinate generalization of parameter s of the string coordinates $\vec{x}(s)\in\mathbb{R}^3$, which describe the knot structure involved in our two-gap superconductor. In fact, the knot theory in the two-gap superconductor can be constructed in terms of a bundle of two strings. Moreover, the U(1) gauge transformation (2.7) is related with the angle α in such a way that

$$\alpha\to\alpha+\xi, \tag{5.2}$$

to yield reparametrization invariance $s\to\tilde{s}(s)$.

In order to evaluate the Hopf invariant associated with the knot structure of the two-gap superconductor, we substitute Eq. (4.1) into Eq. (2.6) to obtain

$$C=\cos\theta d\beta+d\alpha, \tag{5.3}$$

which is also attainable from Eq. (4.2). Note that C in Eq. (5.3) transforms under Eq. (2.7) as

$$C\to\cos\theta d\beta+d(\alpha+\xi), \tag{5.4}$$

so that C can be identified as the U(1) gauge field and its exterior derivative produces the pull-back of the area two-form on the two-sphere S^2,

$$H=dC=\frac{1}{2}\vec{n}\cdot d\vec{n}\wedge d\vec{n}=\sin\theta d\beta\wedge d\theta,$$

and the corresponding dual one-form $G_i=\frac{1}{2}\epsilon_{ijk}H_{jk}$, which can be rewritten in terms of the angles θ and β:

$$G=\frac{1}{2}\sin\theta d\beta\wedge d\theta.$$

The Hopf invariant Q_H is then given by

$$Q_H=\frac{1}{8\pi^2}\int H\wedge C=\frac{1}{8\pi^2}\int\sin\theta d\alpha\wedge d\beta\wedge d\theta.$$

Note that if there exists a nonvanishing Hopf invariant, the bundle of two strings forms a knot so that the flat connection $d\alpha$ cannot be removed through the gauge transformation (5.4).

Next, to figure out the knot structure more geometrically we employ a right-handed orthonormal basis defined by a triplet $(\vec{n},\vec{e}_1,\vec{e}_2)$ where $\vec{n}$ is given by Eq. (5.1) and

$$\vec{e}_1=(\cos\beta\cos\theta,-\sin\beta\cos\theta,-\sin\theta),\ \vec{e}_2=(\sin\beta,\cos\beta,0).$$

Using this orthonormal basis, we define with $\vec{e}_\pm=\vec{e}_2\pm i\vec{e}_1$ a curvature and a torsion:

$$\kappa_i^\pm=\frac{1}{2}e^{\pm\alpha}\vec{e}_\pm\cdot\partial_i\vec{n}=\frac{1}{2}e^{\pm\alpha}(-\sin\theta\partial_i\beta\pm i\partial_i\theta),$$

$$\tau_i=\frac{i}{2}\vec{e}_-\cdot(\partial_i+i\partial_i\alpha)\vec{e}_+=\cos\theta\partial_i\beta-\partial_i\alpha.$$

Here one can readily check that the curvature $\kappa_i^\pm$ and the torsion τ_i are invariant under the $U(1)\times U(1)$ gauge transformations defined by Eqs. (2.7) and (5.2) and also they are not independent to yield flatness relations between them:

$$d\tau+2i\kappa^+\wedge\kappa^-=0,\ d\kappa^\pm\pm i\tau\wedge\kappa^\pm=0.$$

Here we emphasize that the knotted stringy structures of the two-gap superconductors are constructed only in terms of the CP^1 complex fields z_α in the order parameters Ψ_α in Eq. (2.2), since the modulus field ρ associated with the condensate densities does not play a central role in the geometrical arguments involved in the topological knots of the system.

VI. CONCLUSIONS

We have studied the current equations in two-gap superconductor to yield the nontrivial topological aspects and discussed its relationship to Meissner effects. We have also discussed the knotted string geometry in terms of the Hopf invariant, curvature and torsion of the strings associated with $U(1)\times U(1)$ gauge group.

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