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Physics Letters B 552 (2003) 111–118

PHYSICS LETTERS B

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Rolling tachyons in string cosmology

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Received 14 October 2002; accepted 22 November 2002

Editor: T. Yanagida

Abstract

We study the role of rolling tachyons in the cosmological model with dilatonic gravity. In the string frame, flat space solutions of both initial-stage and late-time are obtained in closed form. In the Einstein frame, we show that every expanding solution is decelerating.

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Keywords: Rolling tachyon; String cosmology; Dilaton

1. Introduction

Though the situation seems not to be matured yet, *string cosmology* must be an intriguing subject to be tackled at every step of progress in both string theory and observational cosmology [1]. Recently, inspired by string theory D-branes and heterotic M-theory, brane cosmology attracted much attention. Brane cosmology assumes that our universe starts out with branes embedded in the higher-dimensional space-time, either stable or unstable. Instability of D3-brane systems is represented by rolling of tachyon [2] whose action is of Born–Infeld type [3] and has exponentially decaying potential [4]. Since cosmological evolution of accelerating universe was obtained at the early epoch of the rolling tachyon [5,6], the cosmology

involving the rolling tachyon attracts attention in relation with various topics, e.g., inflation, dark matter, and reheating [7], despite stringent difficulty in the simplest versions of this theory [8]. A nice aspect is the absence of open string excitations, which results in the absence of the plane-wave solutions in effective field theory [2,9] so that classical analysis of late-time based on homogeneity and isotropy may lead to a solid prediction.

When we deal with string cosmology, two indispensable fields are gravitons and dilaton among various degrees of freedom. For unstable branes, tachyonic degree of freedom should also be involved. Thus, from the theory side of string cosmology with unstable branes, the system of tachyon, dilaton, and gravitons provides a minimal setting. In this Letter, we consider the cosmology of unstable brane dominated by tachyon and dilaton, based on the effective 4D action. We assume that our brane is either a 3-brane whose transverse dimensions are compactified or a space fill-

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ing brane with compactified extra dimensions. To be a realistic scenario leading to the final radiation, matter, and dark energy dominated universes, other processes should also be clarified, including the birth of (stable or unstable) branes, the compactification of extra dimensions and the dilaton stabilization. Though we need to understand physics behind them and their characteristic time scales to complete the brane world scenario, most of them rely on physics unknown at the moment. Hence our purpose in this Letter is restricted to the study of intermediate stage dynamics when both the dilaton and the tachyon play dominant roles in the evolution of the universe. We do not introduce the stabilization potential for the dilaton, and find the evolution of the scale factor and tachyon energy density while both the dilaton and the tachyon are rolling. Specifically, in the string frame, we obtain flat space solutions at both the early epochs and late-times in closed form. When transformed to the Einstein frame, analysis shows that every expanding universe of the graviton–tachyon–dilaton system should be decelerating irrespective of the specific shape of the tachyon potential.

The Letter is organized as follows. In Section 2, we consider the effective action of a D3-brane system involving graviton, dilaton, and tachyon in the string frame. For the late stage of tachyon-rolling, all possible cosmological solutions are obtained in closed form. In Section 3, we reanalyze the system in the Einstein frame and show that every expanding universe is decelerating. Section 4 is devoted to concluding remarks.

2. Cosmological solutions in the string frame

Let us begin with a cosmological model induced from string theory, which is confined on a D3-brane of tension T_3 and includes graviton $g_{\mu\nu}$, dilaton Φ , and tachyon T . In the string frame, the effective action of the bosonic sector of the D3-brane system is given by [3]

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} e^{-2\Phi} (R + 4\nabla_\mu \Phi \nabla^\mu \Phi) - T_3 \int d^4x e^{-\Phi} V(T) \sqrt{-\det(g_{\mu\nu} + \partial_\mu T \partial_\nu T)}, \quad (2.1)$$

where we turned off an Abelian gauge field on the D3-brane and antisymmetric tensor fields of second rank both on the brane and in the bulk, and set the vanishing cosmological constant. Though there seems no consensus and no exact computation of tachyon potential $V(T)$ for all T 's, which measures change of the tension, except for its maximum ($\max(V(T)) = 1$) and minimum ($\min(V(T)) = 0$), its specific form can be chosen, for example, as an exponentially-decreasing potential

$$V(T) = \begin{cases} \exp(-\frac{T^2}{8\ln 2}) & \text{for small } T \text{ but } T \geq 0, \\ \exp(-\frac{T}{\sqrt{2}}) & \text{for large } T, \end{cases} \quad (2.2)$$

which connects two asymptotic expressions smoothly and is consistent with the results of superstring theory [2,4].

For cosmological solutions in the string frame, we try a spatially-homogeneous but time-dependent solution

$$ds^2 = -dt^2 + a^2(t) d\Omega_k^2, \quad \Phi = \Phi(t), \quad T = T(t), \quad (2.3)$$

where $d\Omega_k^2$ corresponds, at least locally, to the metric of S^3 , E^3 , or H^3 according to the value of $k = 1, 0, -1$, respectively. From the action (2.1), we obtain the following equations:

$$3\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) - 2\left(3\frac{\dot{a}}{a}\dot{\Phi} - \dot{\Phi}^2\right) = \kappa^2 e^\Phi \rho_T, \quad (2.4)$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} - 2\left(\ddot{\Phi} + 2\frac{\dot{a}}{a}\dot{\Phi} - \dot{\Phi}^2\right) = -\kappa^2 e^\Phi p_T, \quad (2.5)$$

$$4\left(\ddot{\Phi} + 3\frac{\dot{a}}{a}\dot{\Phi} - \dot{\Phi}^2\right) - 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) = \kappa^2 e^\Phi p_T, \quad (2.6)$$

$$\frac{\ddot{T}}{1 - \dot{T}^2} + \left(3\frac{\dot{a}}{a} - \dot{\Phi}\right)\dot{T} + \frac{1}{V} \frac{dV}{dT} = 0, \quad (2.7)$$

where tachyon energy density ρ_T and pressure p_T defined by $T_v^{T\mu} \equiv \text{diag}(-\rho_T, p_T, p_T, p_T)$ are

$$\rho_T = T_3 \frac{V(T)}{\sqrt{1 - \dot{T}^2}} \quad \text{and} \quad p_T = -T_3 V(T) \sqrt{1 - \dot{T}^2}. \quad (2.8)$$

The tachyon equation (2.7) is equivalent to the following conservation equation

$$\dot{\rho}_T + (3H - \dot{\Phi})\dot{T}^2 \rho_T = 0, \quad (2.9)$$

where $H = \dot{a}/a$ is the Hubble parameter. In the absence of detailed knowledge of $V(T)$, we will examine characters of the solutions based on the simplicity of tachyon equation of state

$$p_T = w_T \rho_T, \quad w_T = \dot{T}^2 - 1. \quad (2.10)$$

By defining the shifted dilaton $\phi = 2\Phi - 3 \ln a$, we rewrite Eqs. (2.4)–(2.7) as

$$\dot{\phi}^2 - 3H^2 + 6\frac{k}{a^2} = 2\kappa^2 e^{\phi/2} a^{3/2} \rho_T, \quad (2.11)$$

$$2(\dot{H} - H\dot{\phi}) + 4\frac{k}{a^2} = \kappa^2 e^{\phi/2} a^{3/2} p_T, \quad (2.12)$$

$$\dot{\phi}^2 - 2\ddot{\phi} + 3H^2 + 6\frac{k}{a^2} = -\kappa^2 e^{\phi/2} a^{3/2} p_T, \quad (2.13)$$

$$\frac{\ddot{T}}{1 - \dot{T}^2} + \frac{1}{2}(3H - \dot{\phi})\dot{T} = -\frac{1}{V} \frac{dV}{dT}. \quad (2.14)$$

Note that $\sqrt{-g}$, or a^3 is not a scalar quantity even in flat spatial geometry, the shifted dilaton ϕ is not a scalar field in 3 + 1 dimensions. The conservation equation (2.9) becomes

$$\dot{\rho}_T + \frac{1}{2}(3H - \dot{\phi})\dot{T}^2 \rho_T = 0. \quad (2.15)$$

Now Eqs. (2.11)–(2.13) and Eq. (2.10) are summarized by the following two equations:

$$2\ddot{\phi} - \dot{\phi}^2 + 2H\dot{\phi} - 3H^2 - 2\dot{H} = -10\frac{k}{a^2}, \quad (2.16)$$

$$w_T \dot{\phi}^2 + 4H\dot{\phi} - 3w_T H^2 - 4\dot{H} = -(8 + 6w_T)\frac{k}{a^2}. \quad (2.17)$$

Let us consider only the flat metric ($k = 0$) in the rest part of the Letter. If we express the dilaton ϕ as a function of the scale factor $a(t)$, $\phi(t) = \phi(a(t))$, we can introduce a new variable ψ such as

$$\psi \equiv a\phi' = \frac{\dot{\phi}}{H}, \quad (2.18)$$

where the prime denotes the differentiation with respect to a , and the second equality shows that ψ is the ratio between $\dot{\phi}$ and H . Then Eqs. (2.16) and (2.17)

are combined into a single first-order differential equation for ψ :

$$4a\psi' + (\psi^2 - 3)(w_T\psi + 2 - w_T) = 0. \quad (2.19)$$

From now on we look for the solutions of Eq. (2.19). Above all one may easily find a constant solution $\psi = \mp\sqrt{3}$ which is consistent with Eqs. (2.11)–(2.13) only when $\rho_T = 0$:

$$a(t) = a_0(1 \mp \sqrt{3} H_0 t)^{\mp 1/\sqrt{3}}, \quad (2.20)$$

$$H(t) = \frac{H_0}{1 \mp \sqrt{3} H_0 t},$$

$$\Phi(t) = \Phi_0 - \frac{1 \pm \sqrt{3}}{2} \ln(1 \mp \sqrt{3} H_0 t), \quad (2.21)$$

where $H_0 = H(t = 0)$, $a_0 = a(t = 0)$, and $\Phi_0 = \Phi(t = 0)$ throughout this section. However, exactly-vanishing tachyon density $\rho_T = 0$ from Eq. (2.11) restricts strictly the validity range of this particular solution to that of vanishing tachyon potential, $V(T) = 0$, which leads to $T = \infty$. The tachyon equation (2.14) forces $\dot{T} = 0$ and $\dot{T} = 1$ so that the tachyon decouples ($w_T = p_T = 0$). Therefore, the obtained solution (2.20), (2.21) corresponds to that of string cosmology of the graviton and the dilaton before stabilization but without the tachyon.

Since it is difficult to solve Eq. (2.19) with dynamical w_T , let us assume that w_T is time-independent (or equivalently $a(t)$ -independent). We can think of the cases where the constant w_T can be a good approximation. From the tachyon potential (2.2), the first case is onset of tachyon rolling around the maximum point and the second case is late-time rolling at large T region. In fact we can demonstrate that those two cases are the only possibility as far as no singularity evolves.

When w_T is a nonzero constant, Eq. (2.19) allows a particular solution

$$\psi = \frac{w_T - 2}{w_T} \equiv \beta. \quad (2.22)$$

This provides a consistent solution of Eqs. (2.11)–(2.13)

$$a(t) = a_0 \left(1 + \frac{w_T^2 + 2}{2w_T} H_0 t \right)^{2w_T/(w_T^2 + 2)},$$

$$H(t) = H_0 \left(1 + \frac{w_T^2 + 2}{2w_T} H_0 t \right)^{-1}, \quad (2.23)$$

$$\Phi(t) = \Phi_0 + \frac{2(2w_T - 1)}{w_T^2 + 2} \ln\left(1 + \frac{w_T^2 + 2}{2w_T} H_0 t\right). \tag{2.24}$$

From Eq. (2.11) and Eq. (2.15), the tachyon energy density ρ_T is given by

$$\rho_T(t) = \frac{2 - 2w_T - w_T^2}{w_T^2 \kappa^2 e^{\Phi_0}} H_0^2 \times \left(1 + \frac{w_T^2 + 2}{2w_T} H_0 t\right)^{-2(1+w_T)^2/(w_T^2+2)}. \tag{2.25}$$

Since the obtained solution is a constant solution of ψ , it has only three initially-undetermined constants. Specifically, the solution should satisfy $\dot{\Phi} = [(2w_T - 1)/w_T]H$ so that the initial conditions also satisfy a relation $\dot{\Phi}_0 = [(2w_T - 1)/w_T]H_0$. Once we assume general solutions of $a(t)$ -dependent ψ with keeping constant nonzero w_T , they should be classified by four independent parameters $(a_0, H_0, \Phi_0, \dot{\Phi}_0)$ instead of three in Eq. (2.24).

According to the aforementioned condition for valid w_T values, the obtained solution in Eq. (2.24) may be physically relevant as the onset solution of $w_T = -1$ ($\psi = 3$). In this case, $\rho_T(t)$ is reduced to a constant $\rho_T(t) = 3e^{-\Phi_0} H_0^2 / \kappa^2$. Comparing this with the definition of ρ_T in Eq. (2.8), the initial Hubble parameter H_0 is related to the dilaton as $H_0 = \pm \kappa e^{\Phi_0/2} \sqrt{T_3/3}$. Then, with $T(t) = 0$, the tachyon equation of motion is automatically satisfied and hence Eq. (2.24) becomes an exact solution of the whole set of equations of motion (2.4)–(2.7). Since the tachyon field remains as constant at the maximum of the potential, this solution describes the expanding or shrinking solution depending on the initial Hubble parameter, with a constant vacuum energy corresponding to brane tension due to tachyon sitting at the unstable equilibrium point.¹

In order to study the behavior of the tachyon rolling down from the top of the potential, now we slightly perturb this solution, i.e., look for a solution with nonzero but small T dependence. So we treat

¹ The interpretation as expanding or shrinking solution needs to be more careful, since we are working in the string frame. Actually the behaviors are reversed in the Einstein frame as we will see in Section 3.

T as a small expansion parameter and work up to the first-order in T . Since the unperturbed solution satisfies $3H = \dot{\Phi}$, the tachyon equation of motion (2.7) becomes, to the first-order in T ,

$$\ddot{T} = -\frac{1}{V} \frac{dV}{dT}. \tag{2.26}$$

This can easily be integrated to

$$\frac{1}{2} \dot{T}^2 = -\ln V + \text{const} = \frac{T^2}{8 \ln 2} + \text{const}, \tag{2.27}$$

where we used the form of the potential near the origin (2.2). Given the initial condition that $T = T_0$ and $\dot{T} = 0$ at $t = 0$, we can solve Eq. (2.27) and obtain

$$T(t) = T_0 \cosh \alpha t, \tag{2.28}$$

where $\alpha = 1/2\sqrt{\ln 2}$. Therefore tachyon starts to roll down the potential as a hyperbolic cosine function. Taking derivative, we find $\dot{T} = \alpha T_0 \sinh \alpha t$. The range for which \dot{T} remains small is then $t \lesssim t_r \equiv 2\sqrt{\ln 2} \sinh^{-1}(2\sqrt{\ln 2}/T_0)$, during which the approximation $w_T \simeq -1$ is good. Unless the initial value T_0 is fine-tuned, the tachyon follows the onset solution (2.24), (2.25) for $t \lesssim t_r$ and enters into rolling mode.

For more general solutions, the first-order differential equation (2.19) can be integrated to

$$a = C \left[\frac{\psi^2 - 3}{(\psi - \beta)^2} \left(\frac{\psi - \sqrt{3}}{\psi + \sqrt{3}} \right)^{\beta/\sqrt{3}} \right]^{(\beta-1)/(\beta^2-3)}, \tag{2.29}$$

where C is an integration constant. Note that this algebraic equation does not provide a closed form of ψ in terms of the scale factor $a(t)$ except for a few cases, e.g., $w_T = 0, -1/(\sqrt{3} - 1/2), -2/(3\sqrt{3} - 1)$.

Fortunately, for the late-time case of vanishing w_T , we can obtain the solution in closed form

$$\psi = \sqrt{3} \frac{C_+ \left(\frac{a}{a_0}\right)^{\sqrt{3}/2} + C_- \left(\frac{a}{a_0}\right)^{-\sqrt{3}/2}}{C_+ \left(\frac{a}{a_0}\right)^{\sqrt{3}/2} - C_- \left(\frac{a}{a_0}\right)^{-\sqrt{3}/2}}. \tag{2.30}$$

Then the scale factor a and the dilaton Φ are explicitly expressed as functions of time t by solving Eqs. (2.12), (2.13):

$$a(t) = a_0 \left(\frac{C_- t + 2}{C_+ t + 2} \right)^{1/\sqrt{3}}, \tag{2.31}$$

$$H(t) = \frac{4H_0}{(C_- t + 2)(C_+ t + 2)},$$

$$\Phi(t) = \Phi_0 + \ln \left[2 \frac{(C_- t + 2)^{(\sqrt{3}-1)/2}}{(C_+ t + 2)^{(\sqrt{3}+1)/2}} \right], \quad (2.32)$$

where $C_{\pm} = (3 \mp \sqrt{3})H_0 - 2\dot{\Phi}_0$. We also read the tachyon density ρ_T from Eq. (2.11)

$$\rho_T = C_+ C_- e^{-\Phi_0} \frac{(C_+ t + 2)^{(\sqrt{3}-1)/2}}{(C_- t + 2)^{(\sqrt{3}+1)/2}}. \quad (2.33)$$

Note that C_{\pm} should have the same sign from the positivity of the tachyon density (2.33). Let us first consider that both C_+ and C_- are positive. When $C_- > C_+$ or equivalently $H_0 > 0$, the scale factor a is growing but saturates to a finite value such as $a(\infty) = a_0(C_-/C_+)^{1/\sqrt{3}}$ in the string frame. When $C_- < C_+$, it decreases. When $C_- = C_+$, $H_0 = 0$ so that the scale factor is a constant, $a(t) = a_0$. For all of the cases, the dilaton Φ approaches negative infinity. Note that $w_T = 0$ means late-time, the tachyon density decreases like $\rho_T \sim 1/t$ as $t \rightarrow \infty$. Consistency check by using Eq. (2.15) or equivalently by Eq. (2.14) provides us the expected result, $\dot{T} \rightarrow 1$. If both C_+ and C_- are negative, there appears a singularity at finite time irrespective of relative magnitude of C_+ and C_- .

3. Analysis in the Einstein frame

In the previous section, it was possible to obtain the cosmological solutions analytically in a few simple but physically meaningful limiting cases. To study the physical implications of what we found, however, we need to work in the Einstein frame. In this section, we will convert the cosmological solutions obtained in the string frame to those in the Einstein frame and discuss the physical behaviors. In the Einstein frame, the metric has the form²

$$ds^2 = e^{2\Phi} (-dt^2 + a^2(t) d\Omega_k^2), \quad (3.34)$$

and hence the time t and the scale factor a are related to those in the string frame as

$$a_s = ae^{\Phi}, \quad dt_s = e^{\Phi} dt. \quad (3.35)$$

² In this section all the quantities are in the Einstein frame except the variables with subscript s which denote the quantities in the string frame.

Then the equations of motion for the flat case ($k = 0$) in the string frame (2.4)–(2.7) are converted to

$$H^2 = \frac{1}{3}\dot{\Phi}^2 + \frac{1}{3}\kappa^2 e^{3\Phi} \rho_T, \quad (3.36)$$

$$\dot{H} = -\dot{\Phi}^2 - \frac{1}{2}\kappa^2 e^{\Phi} \rho_T \dot{T}^2, \quad (3.37)$$

$$\ddot{\Phi} + 3H\dot{\Phi} = -\frac{1}{2}\kappa^2 e^{3\Phi} (\rho_T - 2p_T), \quad (3.38)$$

$$\frac{\ddot{T}}{1 - e^{-2\Phi} \dot{T}^2} + 3H\dot{T} + \dot{\Phi} \dot{T} \frac{1 - 2e^{-2\Phi} \dot{T}^2}{1 - e^{-2\Phi} \dot{T}^2} + e^{2\Phi} \frac{1}{V} \frac{dV}{dT} = 0. \quad (3.39)$$

Tachyon energy density ρ_T and pressure p_T in the Einstein frame are obtained by the replacement $\dot{T}_s = e^{-\Phi} \dot{T}$ in Eq. (2.8) according to Eq. (3.35), and thereby w_T is given as $w_T \equiv p_T/\rho_T = e^{-2\Phi} \dot{T}^2 - 1$. Demanding constant w_T is nothing but asking a strong proportionality condition between the dilaton and tachyon, $\dot{T} \propto e^{\Phi}$. Note that the pressure p_T as shown in Eq. (2.8) is always negative irrespective of both specific form of the tachyon potential ($V(T) \geq 0$) and the value of the kinetic term ($e^{-2\Phi} \dot{T}^2 \leq 1$), and the value of w_T interpolates smoothly between -1 and 0 .

First we observe that the right-hand side of Eq. (3.36) is always positive, which means that the Hubble parameter $H(t)$ is either positive definite or negative definite for all t and it cannot change the sign in the Einstein frame. Obviously it is a natural consequence of the weak energy condition. Let us first consider the case of positive Hubble parameter, $H(t) > 0$. Eq. (3.37) shows \dot{H} consists of two terms both of which are negative definite for all t . Since $H > 0$ by assumption, the only consistent behavior of H in this case is that \dot{H} vanishes as $t \rightarrow \infty$, which, in turn implies that $\dot{\Phi}$ and $e^{\Phi} \rho_T \dot{T}^2$ go to zero, separately. It also means that H should be a regular function for all t . In order to find the large t behavior of $H(t)$, one has to study $e^{-\Phi} \dot{T}$ in large t limit which appears in the definition of w_T in the Einstein frame. Knowing that the functions are regular, it is not difficult to show that the only possible behavior is $e^{-\Phi} \dot{T} \rightarrow 1$ as $t \rightarrow \infty$ after some straightforward analysis of Eqs. (3.36)–(3.38). Combining it with the fact that $\dot{\Phi}$ and $e^{\Phi} \rho_T \dot{T}^2$ vanish, we can immediately conclude from Eq. (3.36) that $H(t)$ should go to zero in large t limit.

The asymptotic behavior of fields in case of the positive Hubble parameter can be found from the solution (2.32) since w_T is essentially zero for large t as we just have seen above. The only thing to do is to transform the expressions in the string frame to those in the Einstein frame, using the relation (3.35). Therefore, for large t , we find

$$\begin{aligned} a(t_s) &= a_s(t_s)e^{-\Phi(t_s)} \\ &\simeq \frac{1}{2}a_{s0}e^{-\Phi_0}(C_+t_s + 2)^{(\sqrt{3}+1)/(2\sqrt{3})} \\ &\quad \times (C_-t_s + 2)^{(\sqrt{3}-1)/(2\sqrt{3})}, \\ t &= \int dt_s e^{-\Phi} \\ &\simeq 2e^{-\Phi_0} \int dt_s \frac{(C_+t_s + 2)^{(\sqrt{3}+1)/2}}{(C_-t_s + 2)^{(\sqrt{3}-1)/2}}. \end{aligned} \quad (3.40)$$

One can also identify the initial Hubble parameter H_0 in terms of C_{\pm} as

$$H_0 = \frac{1}{4}e^{\Phi_0} \left[\left(1 - \frac{1}{\sqrt{3}}\right)C_- + \left(1 + \frac{1}{\sqrt{3}}\right)C_+ \right]. \quad (3.41)$$

Note that C_{\pm} have the same sign as the Hubble parameter H . Now, with $C_{\pm} > 0$, one can easily confirm from Eq. (3.40) that all the functions indeed behave regularly. In $t_s \rightarrow \infty$ limit, $a \sim t_s$ and $t \sim t_s^2$ so that the asymptotic behavior of the scale factor becomes $a \sim t^{1/2}$. This power law expansion in flat space is contrasted with the result of Einstein gravity without the dilaton Φ , where ultimately the scale factor ceases to increase, $\lim_{t \rightarrow \infty} a(t) \rightarrow \text{const.}$ The behavior of tachyon density ρ_T can be read from Eq. (2.33) with t replaced by t_s , which shows that $\rho_T \sim t^{-1/2}$. Since w_T also goes to zero, the fluid of condensed tachyon becomes pressureless. Differently from ordinary scalar matter where matter domination of pressureless gas is achieved for the minimum kinetic energy ($\dot{T} \rightarrow 0$), it is attained for the maximum value of time dependence ($e^{-\Phi}\dot{T} \rightarrow 1$ as $T \rightarrow \infty$) for the tachyon potential given in Eq. (2.2).

When the Hubble parameter H is negative, the situation is a bit more complicated. Since $\dot{H} < 0$ always, H becomes more and more negative and there is a possibility that eventually H diverges to negative infinity at some finite time. Indeed, it turns out that all solutions in this case develop a singularity at some

finite time at which $H \rightarrow -\infty$ and $a \rightarrow 0$. These big crunch solutions may not describe viable universes in the sense of observed cosmological data. Depending on initial conditions, the dilaton Φ diverges to either ∞ or $-\infty$ and \dot{T} goes to either ∞ or zero with the factor $e^{-\Phi}\dot{T}$ remaining finite. It is rather tedious and not much illuminating to show this explicitly, so here we will just content ourselves to present a simple argument to understand the behavior. Since the tachyon field T rolls down from the maximum of the potential to the minimum at infinite T , it is physically clear that $\dot{T}_s = e^{-\Phi}\dot{T}$ would eventually go to one unless there is a singularity at some finite time. Suppose that there appeared no singularity until some long time had passed so that $e^{-\Phi}\dot{T}$ approached to one sufficiently closely. Then Eq. (3.40) should be a good approximate solution in this case. However, we know that both C_{\pm} are negative when $H < 0$ and Eq. (3.40) is clearly singular in this case. We have also verified the singular behavior for various initial conditions using numerical analysis.

As mentioned in the previous section, the tachyon T is decoupled when $e^{-\Phi}\dot{T} = 1$ and $T = \infty$. In this decoupling limit, characters of the Einstein equations (3.36) and (3.37) that $H^2 > 0$ and $\dot{H} < 0$ do not change so that all the previous arguments can be applied. Well-known cosmological solution of the dilaton gravity before stabilization of the dilaton is

$$\begin{aligned} a(t) &= a_0(1 + 3H_0t)^{1/3}, \\ H(t) &= \frac{H_0}{1 + 3H_0t}, \end{aligned} \quad (3.42)$$

$$\Phi(t) = \Phi_0 \pm \frac{1}{\sqrt{3}} \ln(1 + 3H_0t), \quad (3.43)$$

where the (\pm) sign in Eq. (3.43) is due to the reflection symmetry ($\Phi \leftrightarrow -\Phi$) in Eqs. (3.36)–(3.38). This solution can also be obtained throughout a transformation (3.35) from Eqs. (2.20), (2.21). For $H_0 < 0$, it is a big crunch solution ($a \rightarrow 0$) which encounters singularity ($H \rightarrow \infty$, $\Phi \rightarrow \mp\infty$) as $t \rightarrow 1/3|H_0|$. For $H_0 > 0$, it is an expanding but decelerating solution. Since $a \sim t^{1/3}$, the power of expansion rate is increased from $1/3$ to $1/2$ by the tachyonic effect as expected.

So far we discussed generic properties and asymptotic behaviors of solutions in the Einstein frame. Now we consider the behavior at the onset. The solution

(2.24) obtained by assuming constant w_T is transformed to the Einstein frame as

$$a(t) = a_0 \left[1 + \frac{(w_T - 2)^2}{2(1 - w_T)} H_0 t \right]^{2(1-w_T)/(w_T-2)^2},$$

$$e^{\Phi(t)} = e^{\Phi_0} \left[1 + \frac{(w_T - 2)^2}{2(1 - w_T)} H_0 t \right]^{2(2w_T-1)/(w_T-2)^2},$$
(3.44)

where the initial Hubble parameter H_0 is related to that in the string frame by $H_0 = e^{\Phi_0} H_{0s} (1 - w_T) / w_T$. Note that H_0 and H_{0s} have opposite signs since $w_T < 0$. Therefore the expanding (shrinking) solution in the string frame corresponds to the shrinking (expanding) solution in the Einstein frame. For the onset solution with $w_T = -1$, the tachyon energy density ρ_T is a constant as before, $\rho_T(t) = 3e^{-3\Phi_0} H_0^2 / 4\kappa^2$. Then the initial Hubble parameter is given by $H_0 = \pm 2\kappa e^{3\Phi_0/2} \sqrt{T_3/3}$, which describes the exact solution that tachyon remains at the origin as explained in Section 2. Under a small perturbation tachyon starts rolling down according to Eq. (2.28) with t replaced by t_s . The rest of the discussion on the rolling behavior is the same as in the string frame and the details will not be repeated here.

In conclusion the cosmological solution can be classified into two categories depending on the value of the Hubble parameter $H(t)$ in the Einstein frame. When the initial Hubble parameter H_0 is positive, the solution is regular and the universe is expanding but decelerating as $a(t) \sim \sqrt{t}$ while $e^{\Phi(t)}$ vanishes. When H_0 is negative, there appears a singularity at some finite time t at which the universe shrinks to zero.

4. Concluding remarks

In this Letter we have discussed cosmological solutions of the effective theory of rolling tachyon coupled to gravitons and dilaton. In the study of homogeneous and isotropic universes, we found initial-stage and late-time solutions in closed form in addition to the known solution of the gravitons and dilaton in the decoupling limit of the tachyon. Those obtained solutions included expanding universes, big crunch solutions, and even the static universe in the string frame. The dilatonic part of those also indicated that the dila-

ton could not be stabilized in the system of our interest without the dilaton potential.

We have also provided a description of cosmological solutions in the Einstein frame. Einstein equations were summarized by the positivity of square of the Hubble parameter and the negativity of derivative of the Hubble parameter irrespective of specific shape of the rolling tachyon potential. This implies that once the universe starts expanding, then it continues expanding eternally but decelerating.

We conclude with the list of intriguing questions for further study. One would like to generalize this to that with the Abelian gauge field on the brane [10] and ask whether or not radiation dominated era is possible. As was the original string cosmology, stabilization mechanism of dilaton should be understood and its positive effect to our unsatisfactory decelerating universe solutions is carefully investigated [11]. Various topics for the rolling tachyons asked in the Einstein gravity should be addressed again in the context of dilaton gravity such as existence of inflationary era, possibility as a source of quintessence, reheating without oscillating tachyon modes, cosmological perturbation and structure formation, and so on.

Acknowledgements

We would like to thank Nakwoo Kim and Jung Tay Yee for discussions on D-brane systems. H.B. Kim and Y. Kim would like to acknowledge the hospitality of Korea Institute for Advanced Study where a part of this work has been done. This work was supported by Korea Research Foundation Grant KRF-2002-070-C00025 (C.K.) and the Swiss Science Foundation, grant 21-58947.99 (H.B.K.), and is the result of research activities (Astrophysical Research Center for the Structure and Evolution of the Cosmos (ARC-SEC) and the Basic Research Program, R01-2000-000-00021-0) supported by Korea Science & Engineering Foundation (Y.K.).

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