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Large scale suppression of scalar power on a spatial condensation

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We consider a deformed single-field inflation model in terms of three SO(3) symmetric moduli fields. We find that spatially linear solutions for the moduli fields induce a phase transition during the early stage of the inflation and the suppression of scalar power spectrum at large scales. This suppression can be an origin of anomalies for large-scale perturbation modes in the cosmological observation.

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I. INTRODUCTION

Recent measurements of the cosmic microwave background (CMB) by the WMAP and Planck collaborations [1,2] support the inflationary scenario. Most of the inflationary models predict a nearly Gaussian and scale-invariant power spectrum of adiabatic perturbation modes, which can be realized by the single-field slow-roll inflationary model. However, the most recent data released by the Planck collaboration [1] reported statistically significant anomalies at low multipoles, which corresponds a power deficit 5%–10% at a multipole range $l \lesssim 40$ with the 2.5–3σ level. Therefore, the usual single-field inflation model needs some modification to express the large-scale scalar power suppression. This suppression of the scalar power spectrum was then used to explain the observed low quadrupole in the CMB anisotropy.

There are several interesting approaches that address the scalar power suppression and are relevant to the suppression method pursued in this paper. The first one is the mechanism studied by Hazra *et al*. [3], who introduced a steep potential during the first few e-foldings of inflation. It is followed by a fast-roll phase during the large-scale modes cross the horizon, and the resulting scalar power spectrum is suppressed since it is inversely proportional to the inflaton velocity. See also Refs. [4,5]. The second one is related to a nonzero spatial curvature in a single-field inflation model. In this case, one can also induce the suppression of the scalar power spectrum on large scales [6,7]. In the same line of thought, recently White *et al.* revisited the open inflation model [8], which gives rise to a suppression of the scalar power on large scales. Here, the main source of the suppression is also the steepening of the potential due to the barrier that separates the true and false vacua.

In this paper, we consider a modification of the canonical single-field inflation model, which induces large negative running of n_s and results in a suppression of the scalar power spectrum on large scales.[1] As a specific model, we consider a deformation of a single-field inflation model by adding kinetic terms for a number of scalar moduli fields,

$$-\frac{1}{2}\int d^4x\sqrt{-g}\sum_{m=1}^{\bar{N}}\partial_\mu\sigma^m\partial^\mu\sigma^m. \tag{1.1}$$

We also consider a background solution with spatially linear configurations, $\sigma^a \sim x^a \ (a = 1, 2, 3)$ and $\sigma^i = 0$ $(i = 4, \cdots, \bar{N})$. Then, the usual cosmological evolution for the single field under the Friedmann-Robertson-Walker (FRW) metric with the background solution for σ^a guarantees the homogeneity and isotropy of the cosmological principle [16–19]. In the perturbation level, fluctuations for σ^i $(i = 4, \cdots, \bar{N})$ are decoupled and have no influence on cosmological observables [19]. For this reason, we consider the $\bar{N} = 3$ case for simplicity. This model corresponds to the case with $f(\varphi) = 1$ in the work [19]. On the other hand, without the usual single inflaton field contribution, inflation is also possible when one uses a higher-order combination of $X = \partial_\mu\sigma^a\partial^\mu\sigma^a$ $(a = 1, 2, 3)$ with a spatially linear configuration of σ^a. This inflation

[1]The large negative running of n_s can be introduced in the context of reconciling the results of the Planck and BICEP2 collaborations [9], though it has been pointed out that uncertainty from the foreground effect can dominate the excess [10–12] observed by the BICEP2 collaborations. See also for the suppression of the scalar power spectra on large scales Refs. [3,8,13–15], after the BICEP2 observation.

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model is known as the solid inflation [18]. See also Refs. [20].

In our model, the background evolution is the same as that of the single-field model with the curvature term of the open universe. That is, the solution $\sigma^a \sim x^a$ induces the curvature term of the open universe in the Friedmann equation, though we start from the flat FRW metric.[2] The curvature term is proportional to the inverse square of the scale factor, and so the effect of the *spatial condensation* appears during the very early stage of the inflation and disappears quickly as the scale factor grows up. Since we start from the phase in which the curvature term is much more dominant than the potential term of the single field, there appears a phase transition from the curvature term dominant phase to the potential term dominant phase. Because of the phase transition in the early stage of the background evolution, there appears the suppression of the scalar power spectrum. This situation has some resemblance to that of inflation models referred as "whipped inflation" [3,14] and "open inflation" [8,15,21], in which there exist phase transitions from the fast-roll phase to the slow-roll phase of the single scalar field model. These phase transitions during the early stage of inflation induce the suppression of the scalar power spectrum on large scales, though the detailed suppression mechanisms are different from that in our model.

The organization of this paper is as follows. In the next section, we explain the properties of the background evolution under the spatial condensation. In Sec. III, we investigate the effects of the spatial condensation in the linear perturbation level. We find the suppression of the scalar power spectrum and large value of the running of the scalar spectral index on large scales. We conclude in Sec. IV.

II. BACKGROUND EVOLUTION ON A SPATIAL CONDENSATION

We start from the action for the single-field inflation model with an additional triad of moduli scalar fields,

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm P}^2}{2}R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi) - \frac{1}{2}\partial_\mu\sigma^a\partial^\mu\sigma^a\right], \tag{2.1}$$

where $a = 1, 2, 3$ and $M_{\rm P}$ denotes the Planck mass, $M_{\rm P} \equiv (8\pi G)^{-1/2}$. The SO(3)-symmetric fields σ^a have no potential. Then, equations of motion of the scalar fields σ^a are read as

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\sigma^a) = 0. \tag{2.2}$$

Under the background FRW metric, $ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)$ with the scale factor $a(t)$, the spatially linear configuration

$$\sigma^a = M_{\rm P}^2\alpha x^a \tag{2.3}$$

satisfies the equations (2.2). Here, the constant gradient α is an arbitrary dimensionless parameter. In order to be consistent with the cosmological principle of homogeneity and isotropy, we assume that the field φ depends on time only. Then, the remaining equations of motion of $g_{\mu\nu}$ and φ in (2.1) are given by

$$H^2 = \frac{1}{3M_{\rm P}^2}\left(\frac{1}{2}\dot\varphi^2 + \frac{3M_{\rm P}^4\alpha^2}{2a^2} + V\right),$$
$$\dot H = -\frac{1}{2M_{\rm P}^2}\left(\dot\varphi^2 + \frac{M_{\rm P}^4\alpha^2}{a^2}\right),$$
$$\ddot\varphi + 3H\dot\varphi + V_\varphi = 0, \tag{2.4}$$

where $H \equiv \dot a/a$ and $V_\varphi \equiv dV/d\varphi$. As was discussed in Ref. [19], the α terms in (2.4) correspond to the curvature terms by identifying the curvature constant K as $K = -M_{\rm P}^2\alpha^2/2$. Since the curvature constant is negative in this case, the equations representing the background evolution in (2.4) are the same with those of the open universe in the single-field inflation model. Although the single-field model in the open universe is the same with our model in the background level, they are different in the perturbation level due to the contribution of fluctuation modes of σ^a. In our case, 3 degrees of freedom (one scalar mode and two vector modes) originating from the triad of scalar fields appear in the perturbation level, while there is no additional perturbation degree of freedom in the usual single-field inflation model with a negative curvature constant.

Now, we investigate some characteristic properties of the background evolution of our model. The effect of the α terms in (2.4) is decreasing rapidly during the inflation and has some influence on the early time of the inflation period. Especially, as we see in the first line of (2.4), the Hubble horizon $r_H \equiv 1/H$ starts from a small value when we introduce a large value of the α-dependent term at the initial state, increases during the early stage of the inflation, and approaches the value of the single-field inflation model at late time.

In our model, the suppression of scalar power spectrum on large scales can be achieved by introducing a large value of the α term at the early time of inflation. For that purpose, we consider the case that the α term in the first equation of (2.4) is much larger than the potential term of the inflaton field, i.e.,

[2] We call the remnant of the solution $\sigma^a \sim x^a$ in the background evolution spatial condensation.

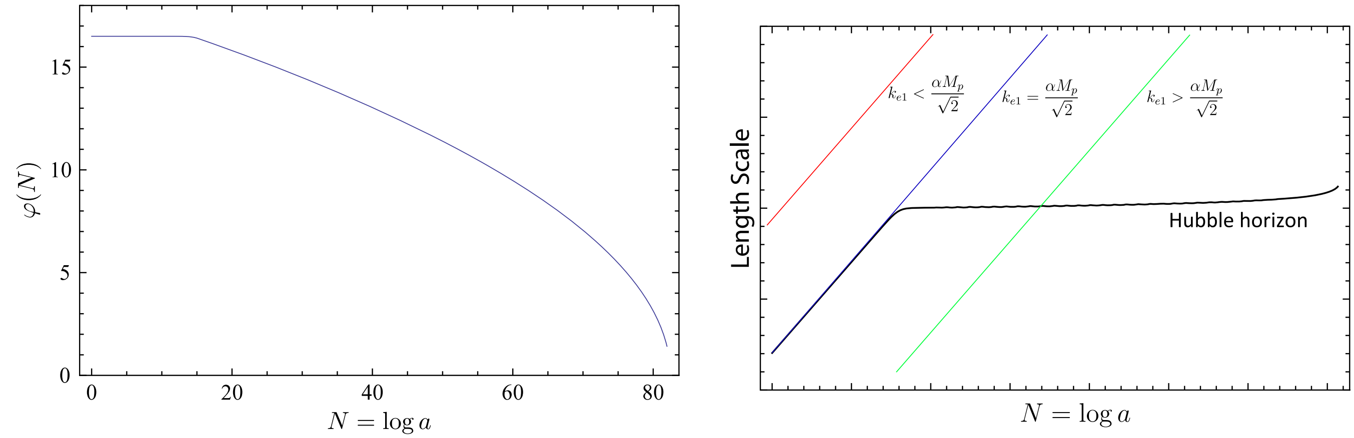

FIG. 1 (color online). The graphs of the inflaton field $\varphi(N)$ (left) and Hubble radius $1/H(N)$ (right). We set the pivot scale $k_0 = 0.05\ \mathrm{Mpc}^{-1}$ and $N_* = 60$. We choose the initial condition as $a_i = 1$, $\varphi_i = 16.5 M_\mathrm{P}$, $\dot{\varphi}_i = 0$, $m = 5.85 \times 10^{-6} M_\mathrm{P}$, and $\alpha = 10^2$.

$$\frac{3M_\mathrm{P}^4\alpha^2}{2a^2} \gg V(\varphi). \tag{2.5}$$

Obtaining an analytic solution for the equations in (2.4) is a formidable task for the potentials of the large-field inflation models. So we rely on a semianalytic way to figure out the behavior of the background evolution governed by the equations in (2.4), based on the numerical method. By employing the simplest scalar potential $V(\varphi) = \frac{1}{2}m^2\varphi^2$, for concreteness, we find that the scalar field φ remains almost constant until the e-folding number $N = 10 \sim 20$. See Fig. 1. Then, there appears a stage that the value of α term is comparable to that of the potential, i.e.,

$$\frac{3M_\mathrm{P}^4\alpha^2}{2a^2} \simeq V(\varphi). \tag{2.6}$$

After the universe passes through this stage, the scalar field starts to decrease and follows the behavior of the canonical slow-roll inflation. The behaviors of the background scalar field and the Hubble horizon with respect to the e-folding number N are plotted in Fig. 1.

As we see in Fig. 1, there is a sharp transition point near

$$N_\mathrm{eq} \simeq \ln\sqrt{\frac{3\alpha^2 M_\mathrm{P}^4}{2V(\varphi(N_\mathrm{eq}))}}, \tag{2.7}$$

where $N_\mathrm{eq} \equiv \log a_\mathrm{eq}$ represents the e-folding number when the α term is the same with the potential of the scalar field. After the transition point, the background evolution rapidly follows the behavior of the usual slow-roll inflation by rolling down the potential slop. For this reason, we can approximate the background equations in (2.4) under the assumption of the slow rolling of the scalar field as

$$\text{early time:}\ 3H^2 \simeq \frac{3\alpha^2 M_\mathrm{P}^2}{2a^2} + \frac{\Lambda}{M_\mathrm{P}^2}, \tag{2.8}$$

$$\text{late time:}\ 3H^2 \simeq \frac{3\alpha^2 M_\mathrm{P}^2}{2a^2} + \frac{V(\varphi)}{M_\mathrm{P}^2}, \tag{2.9}$$

where $\Lambda \equiv V(\varphi_i)$ with an initial value of the scalar field φ_i. From the relation (2.7), we also have the relation

$$N_\mathrm{eq} \simeq \ln\sqrt{\frac{3\alpha^2 M_\mathrm{P}^4}{2\Lambda}}. \tag{2.10}$$

In the early time, the background equation (2.8) has a solution [22,23]

$$a(t) \simeq a_\mathrm{eq}\sinh\left(\sqrt{\frac{\Lambda}{3M_\mathrm{P}^2}}(t+t_0)\right), \tag{2.11}$$

where t_0 is the initial time with the scale factor a_0. On the other hand, in the late time for a given value of α, the scale factor a is already very large, and then the α term in (2.9) becomes much smaller than the potential term. Based on the numerical result during the late time in Fig. 1, we see that the scalar field starts to roll down the potential slope matching the behavior of the usual slow-roll approximation. Since the behavior of the background evolution for the case $\frac{3\alpha^2 M_\mathrm{P}^2}{2a^2} \ll \frac{V}{M_\mathrm{P}^2}$ was already investigated in Ref. [19], we omit the detailed background behaviors in this paper.

III. SUPPRESSION OF LARGE SCALE SCALAR POWER SPECTRUM

A. Generality

We consider the linear scalar perturbation of the FRW metric,

$$\begin{aligned} ds^2 = &-(1+2A)dt^2 + 2a\partial_i B dt dx^i \\ &+ a^2[(1-2\psi)\delta_{ij} + 2\partial_i\partial_j E]dx^i dx^j, \end{aligned} \tag{3.1}$$

where A, B, ψ, and E are four scalar modes. In the linear perturbation, there is also a contribution from fluctuations of scalar fields,

$$\varphi(t,x) = \varphi(t) + \delta\varphi(t,x),$$
$$\sigma^a(t,x) = \sigma^a(x) + \delta\sigma^a(t,x). \tag{3.2}$$

The three perturbation modes $\delta\sigma^a$ are decomposed into one scalar and two vector modes,

$$\delta\sigma^a = \delta\sigma^a_\parallel + \delta\sigma^a_\perp. \tag{3.3}$$

In this work, we focus on the scalar mode $\delta\sigma^a_\parallel$ in the perturbation level.[3] Since the kinetic term of σ^a in (2.1) is a well-defined quadratic form, the kinetic term of the three perturbation modes $\delta\sigma^a$'s is also well defined and does not violate the energy condition. Here, we express the longitudinal mode $\delta\sigma^a_\parallel$ in terms of a scalar mode u with a normalization [16,19],

$$\delta\sigma^a_\parallel = \frac{1}{k}\partial_a u, \tag{3.5}$$

where k is the comoving wave number. We introduce the $1/k$ factor to define the canonical kinetic term for the scalar mode u in the form of the perturbed Lagrangian.

Employing the spatially flat gauge ($\psi = 0$ & $E = 0$), the perturbed scalar equations are reduced to

$$\ddot{Q}_\varphi + 3H\dot{Q}_\varphi + \left(\frac{k^2}{a^2} + \frac{\dot{\varphi}V_\varphi}{M_\mathrm{P}^2 H} + V_{\varphi\varphi}\right)Q_\varphi + 2\left(\frac{\dot{H}\dot{\varphi}}{H} - \ddot{\varphi}\right)A = 0,$$
$$\ddot{Q}_u + 3H\dot{Q}_u + \left(\frac{k^2}{a^2} + \frac{2\alpha^2 M_\mathrm{P}^2}{a^2}\right)Q_u = 0, \tag{3.6}$$

where $Q_\varphi \equiv \delta\varphi - \frac{\dot{\varphi}}{H}\psi$ and $Q_u = u - \alpha k M_\mathrm{P}^2 E$ are gauge-invariant quantities [19]. Scalar modes A and B satisfy the constraints

$$3AH^2 - \frac{k^2 BH}{a} = \frac{1}{2M_\mathrm{P}^2}(A\dot{\varphi}^2 - \dot{\varphi}\dot{Q}_\varphi - V_\varphi Q_\varphi) + \frac{\alpha k}{2a^2}Q_u,$$
$$2AH = \frac{\dot{\varphi}Q_\varphi}{M_\mathrm{P}^2} + \frac{\alpha}{k}Q_u - \frac{\alpha^2 M_\mathrm{P}^2 B}{a}. \tag{3.7}$$

Using these constraints, one can express the modes A and B in terms of Q_φ and Q_u. In this multifield perturbation system, the comoving curvature perturbation is written as [19]

$$\mathcal{R} = H\left[\frac{\dot{\varphi}Q_\varphi - \alpha M_\mathrm{P}^2(\frac{\alpha M_\mathrm{P}^2 B}{a} - \frac{\dot{Q}_u}{k})}{\dot{\varphi}^2 + \frac{\alpha^2 M_\mathrm{P}^4}{a^2}}\right]. \tag{3.8}$$

Differently from the single-field inflation model, there is also a nonvanishing isocurvature perturbation [19]. However, here we only concentrate on the adiabatic curvature perturbation.

B. Suppression of the scalar power spectrum

As we discussed in Sec. 2, there are two inflation phases in the background evolution, the α-term dominant phase and the scalar potential dominant phase. Because of the phase transition during the inflation, the computation of the power spectrum is different from that of the usual single scalar-field model. To calculate the power spectrum and related observational quantities, such as the scalar spectral index n_s and the running of the spectral index α_s, we use the method developed in Ref. [4], in which the authors calculated the power spectrum of the single scalar-field model with the potential having a step transition. Because of the shape of the scalar potential, there are two inflationary phases, the fast-roll phase and slow-roll phase. The origin of the phase transition in Ref. [4] is different from ours, but there is a robust similarity between these two cases in the sense that there is a transition during the inflation and the background evolution approaches the usual slow-roll inflation phase of the single-field model at late time. For this reason, we follow the method developed in Ref. [4] to compute the power spectrum and related perturbation quantities. Because of the phase transition in the background level, there is also a phase transition to the perturbed equations in (3.6). We try to solve the perturbed equations for the α-term dominant phase and the scalar potential dominant phase separately and apply the matching condition at the transition point.

1. Early time

In the early time, having the limit $\frac{3M_\mathrm{P}^4\alpha^2}{2a^2} \gg V$, we obtain A, B from (3.7) in the leading order of the limit as

[3] In the linear perturbation level, the two vector modes $\delta^a_\perp$ have no contribution to the scalar mode [19]. To see this fact explicitly, one can read the $(0,i)$ component of the perturbed Einstein equation $\delta G^0_i = M_\mathrm{P}^{-2}\delta T^0_i$ as

$$-2\partial_i(HA + \dot{\psi}) = M_\mathrm{P}^{-2}\delta T^0_i = M_\mathrm{P}^{-2}\left(-\partial_i(\dot{\varphi}\delta\varphi) - \alpha\delta\dot{\sigma}^i + \frac{\alpha^2}{a}\partial_i B\right). \tag{3.4}$$

Taking the curl of both sides of (3.4), we obtain $\epsilon_{ijk}\partial^j\delta\sigma^k = 0$. That is, only the longitudinal scalar mode can satisfy this relation.

$$A \approx \frac{\frac{\alpha}{k} M_{\rm P} Q_u}{2 + 3\frac{\alpha^2 M_{\rm P}^2}{k^2}}, \qquad B \approx -\frac{\sqrt{\frac{4V_0}{3\alpha^2 M_{\rm P}^4}} Q_u}{z M_{\rm P}^2 (2 + 3\frac{\alpha^2 M_{\rm P}^2}{k^2})}. \qquad (3.9)$$

Using the relation (3.9) and the fact that φ is almost a constant during the α-term dominant phase, we conclude that the A-dependent term in (3.6) is negligible. One can also neglect V_φ- and $V_{\varphi\varphi}$-dependent terms in (3.6) since $V(\varphi)$ is almost constant in the early time phase. Introducing the Sasaki–Mukhanov variables,

$$\mathcal{V} \equiv a Q_\varphi, \qquad \mathcal{U} \equiv a Q_u, \qquad (3.10)$$

and the conformal time coordinate $\tau = \int dt/a$, we obtain the decoupled differential equations for $\mathcal{V}$ and $\mathcal{U}$ as

$$\mathcal{V}_e'' + \left(k_{e1}^2 - \alpha^2 M_{\rm P}^2 \mathrm{csch}^2\left(\frac{\alpha(-\tau)}{\sqrt{2}}\right)\right)\mathcal{V}_e = 0,$$
$$\mathcal{U}_e'' + \left(k_{e2}^2 - \alpha^2 M_{\rm P}^2 \mathrm{csch}^2\left(\frac{\alpha(-\tau)}{\sqrt{2}}\right)\right)\mathcal{U}_e = 0, \qquad (3.11)$$

where the prime represents the differentiation with respect to the conformal time, the subscript e denotes the early time phase, and

$$k_{e1} \equiv \sqrt{k^2 - \frac{\alpha^2 M_{\rm P}^2}{2}}, \qquad k_{e2} \equiv \sqrt{k^2 + \frac{3\alpha^2 M_{\rm P}^2}{2}}. \qquad (3.12)$$

Using general solutions for $\mathcal{V}_e$ and $\mathcal{U}_e$, we obtain normalized solutions [22],

$$\mathcal{V}_e(\tau) = \frac{M_{\rm P}^{\frac{3}{2}}}{\sqrt{2k_{e1}}(-\frac{\alpha M_{\rm P}}{\sqrt{2}} + i k_{e1})} \times \left(-\frac{\alpha M_{\rm P}}{\sqrt{2}} \coth\left(\frac{\alpha(-\tau)}{\sqrt{2}}\right) + i k_{e1}\right) e^{-i k_{e1}\tau},$$
$$\mathcal{U}_e(\tau) = \frac{M_{\rm P}^{\frac{3}{2}}}{\sqrt{2k_{e2}}(-\frac{\alpha M_{\rm P}}{\sqrt{2}} + i k_{e2})} \times \left(-\frac{\alpha M_n}{\sqrt{2}} \coth\left(\frac{\alpha(-\tau)}{\sqrt{2}}\right) + i k_{e2}\right) e^{-i k_{e2}\tau}. \qquad (3.13)$$

Here, we adjusted the integration constants for the solutions $\mathcal{V}_e$ and $\mathcal{U}_e$ to get the Bunch–Davies vacua in the $\tau \to -\infty$ limit,

$$\mathcal{V}_e(\tau) = \frac{M_{\rm P}^{\frac{3}{2}}}{\sqrt{2k_{e1}}} e^{-i k_{e1}\tau}, \quad \mathcal{U}_e(\tau) = \frac{M_{\rm P}^{\frac{3}{2}}}{\sqrt{2k_{e2}}} e^{-i k_{e2}\tau}. \qquad (3.14)$$

As we see in (3.14), effective wave numbers for oscillation modes $\mathcal{V}_e$ and $\mathcal{U}_e$ at early time are k_{e1} and k_{e2}, which are deformed from the wave number k due to the nonvanishing value of α. Then, we find that there is a minimum value of the comoving wave number k. The modes below the minimum value always stay in the superhorizon scale and never cross the horizon, so those modes do not contribute to current observable quantities. Now, we try to obtain the minimum value of the wave number. As we will see later, the leading contribution to the power spectrum comes from the $\mathcal{V}_e$ mode in the limit we are considering. So we focus on the mode $\mathcal{V}_e$. The horizon crossing condition for the mode $\mathcal{V}_e$ at the early stage of inflation is given by

$$k_{e1} = a_* H_*, \qquad (3.15)$$

and the corresponding conformal time τ_* is

$$\tau_* = -\frac{\sqrt{2}}{\alpha M_{\rm P}} \tanh^{-1}\left(\frac{\alpha M_{\rm P}}{\sqrt{2} k_{e1}}\right). \qquad (3.16)$$

From this relation, we notice that at the early stage of inflation the Hubble crossing occurs only when perturbation modes satisfy the condition to give a real value of τ_*,

$$\frac{\alpha M_{\rm P}}{\sqrt{2} k_{e1}} < 1. \qquad (3.17)$$

This condition determines the minimum value of the comoving wave number,

$$k_{\rm min} = \alpha M_{\rm P}. \qquad (3.18)$$

In the current observation for perturbation modes, the minimum comoving wave number is in the range $k_{\rm min} \lesssim 10^{-2}\ \mathrm{Mpc}^{-1}$. Because of this fact, one can roughly estimate the value of α as

$$\alpha \sim \frac{\mathrm{Mpc}^{-1}}{M_{\rm P}} \ll 1. \qquad (3.19)$$

2. *Late time*

On the other hand, in the late time satisfying the condition $\frac{3\alpha^2 M_{\rm P}^4}{2a^2} \ll V(\varphi)$, one can express the scalar modes A and B in terms of the gauge-invariant variables, Q_φ and Q_u from the constraint (3.7),

$$A \simeq \frac{\dot\varphi}{2HM_{\rm P}^2} Q_\varphi + \frac{\alpha}{2kH} Q_u,$$
$$B \simeq \frac{\alpha}{2k}\left(\frac{3a}{k^2} - \frac{1}{aH}\right) Q_u + \frac{a\dot\varphi}{2Hk^2 M_{\rm P}^2} \dot Q_\varphi. \qquad (3.20)$$

Using (3.20), one can easily see that the coefficient of the A-dependent term in the first line of (3.6) belongs to higher order for slow-roll parameters. For this reason, the differential equations for Q_φ and Q_u are decoupled in the leading contribution of slow-roll parameters. Then, we obtain differential equations for $\mathcal{V}_l$ and $\mathcal{U}_l$ in the conformal time coordinate as

$$\mathcal{V}_l'' + \left(k_{l1}^2 - \frac{\mu_1^2 - \frac{1}{4}}{\tau^2}\right)\mathcal{V}_l = 0,$$
$$\mathcal{U}_l'' + \left(k_{l2}^2 - \frac{\mu_2^2 - \frac{1}{4}}{\tau^2}\right)\mathcal{U}_l = 0, \tag{3.21}$$

where the subscript l denotes the late time phase and

$$k_{l1}^2 \equiv k^2 - \frac{\alpha^2 M_P^2}{6}, \qquad k_{l2}^2 \equiv k^2 + \frac{11\alpha^2 M_P^2}{6},$$
$$\mu_1 \simeq \frac{3}{2} + 3\epsilon - \eta, \qquad \mu_2 \simeq \frac{3}{2} + \epsilon. \tag{3.22}$$

Here, the slow-roll parameters, ϵ and η, are defined as

$$\epsilon = \frac{\dot{\varphi}^2}{2M_P^2 H^2}, \qquad \eta = \frac{V_{\varphi\varphi}}{3H^2}. \tag{3.23}$$

General solutions of $\mathcal{V}_l$ and $\mathcal{U}_l$ modes for differential equations in (3.21) are given by

$$\mathcal{V}_l(\tau) = M_P^{\frac{3}{2}}\sqrt{-\tau}(C_1 H_{\mu_1}^{(1)}(-k_{l1}\tau) + C_2 H_{\mu_1}^{(2)}(-k_{l1}\tau)),$$
$$\mathcal{U}_l(\tau) = M_P^{\frac{3}{2}}\sqrt{-\tau}(D_1 H_{\mu_1}^{(1)}(-k_{l2}\tau) + D_2 H_{\mu_2}^{(2)}(-k_{l2}\tau)), \tag{3.24}$$

where $H_\mu^{(i)}(x)$ $(i = 1, 2)$ are the first and second kinds of the Hankel functions and $C_{1,2}$, $D_{1,2}$ are integration constants.

3. Matching condition

As we discussed in the previous section, there are two phases in our model, and we obtained perturbation modes for each phase separately. Then, all perturbed modes should satisfy matching conditions at the transition point $\tau_{\rm eq}$ in conformal time,

$$\mathcal{V}_e(\tau)|_{\tau=\tau_{\rm eq}} = \mathcal{V}_l(\tau)_{\tau=\tau_{\rm eq}}, \qquad \mathcal{V}_e'(\tau)_{\tau=\tau_{\rm eq}} = \mathcal{V}_l'(\tau)_{\tau=\tau_{\rm eq}},$$
$$\mathcal{U}_e(\tau)_{\tau=\tau_{\rm eq}} = \mathcal{U}_l(\tau)_{\tau=\tau_{\rm eq}}, \qquad \mathcal{U}_e'(\tau)_{\tau=\tau_{\rm eq}} = \mathcal{U}_l'(\tau)_{\tau=\tau_{\rm eq}}. \tag{3.25}$$

Here, we notice that the perturbed modes $\mathcal{V}$ and $\mathcal{U}$ satisfy the same type of differential equations with different parameters. So in what follows, we only consider the matching condition for the mode $\mathcal{V}$. Then, the results can be extended to the case of the mode $\mathcal{U}$ as well. From the matching condition in (3.25), we obtain the corresponding integration constants,

$$C_1 - C_2 = \frac{e^{-i\beta\tau_{\rm eq}}\sqrt{\pi}}{\sqrt{2}k\sqrt{-k_{e1}\tau_{\rm eq}}}\Bigg[(k_{e1} + i\alpha M_P)k_{l1}\tau_{\rm eq}J_{\mu_1-1} + \left((k_{e1} + i\alpha M_P)\left(\mu - \frac{1}{2}\right) + (i\alpha^2 M_P^2 + 2\alpha k_{e1}M_P - 2ik_{e1}^2)\tau_{\rm eq}\right)J_{\mu_1}\Bigg],$$
$$C_1 + C_2 = \frac{-ie^{-i\beta\tau_{\rm eq}}\sqrt{\pi}}{\sqrt{2}k\sqrt{-k_{e1}\tau_{\rm eq}}}\Bigg[(k_{e1} + i\alpha M_P)k_{l1}\tau_{\rm eq}Y_{\mu_1-1} + \left((k_{e1} + i\alpha M_P)\left(\mu - \frac{1}{2}\right) + (i\alpha^2 M_P^2 + 2\alpha k_{e1}M_P - 2ik_{e1}^2)\tau_{\rm eq}\right)Y_{\mu_1}\Bigg], \tag{3.26}$$

where we used the relations between the Hankel functions and Bessel functions, $H_\mu^{(1,2)}(x) \equiv J_\mu(x) \pm iY_\mu(x)$, and defined the quantities at the transition point as

$$J_\mu = J_\mu(-k_{l1}\tau_{\rm eq}), \qquad Y_\mu = Y_\mu(-k_{l1}\tau_{\rm eq}),$$
$$\tau_{\rm eq} = -\frac{\sqrt{2}\coth^{-1}(\sqrt{2})}{\alpha M_P}. \tag{3.27}$$

4. Power spectrum

Now, we try to obtain the power spectrum for the curvature perturbation $\mathcal{R}$ in (3.8) and related observational quantities. For a single scalar model, one usually reads the power spectrum at the horizon crossing point since it is guaranteed in the absence of the transition point that curvature perturbations of perturbed modes are frozen after the horizon crossing. In our case with two inflationary phases, however, reading the power spectrum at the horizon crossing point can cause some possible errors for large-scale modes that are deformed due to the presence of the nonvanishing α term. That is, one cannot guarantee the freezing of the curvature perturbation after the horizon crossing for large-scale modes. For this reason, we read the power spectrum at the limit $\tau \to 0$ for all values in the region $k > \alpha M_P$.

Using the relation (3.20) in the limit $\frac{3\alpha^2 M_P^4}{2a^2} \ll V(\varphi)$, we obtain leading contributions for the curvature perturbation [19],

$$\mathcal{R} \simeq \frac{1}{2 + \frac{\alpha^2 M_P^2}{a^2 H^2 \epsilon}}\left(-\frac{\sqrt{2}}{M_P\sqrt{\epsilon}}Q_\varphi + \frac{\alpha}{kH\epsilon}\dot{Q}_u\right). \tag{3.28}$$

As we see in (3.18), the comoving wave number has a minimum value, and so we notice that all comoving wave numbers relevant to the observation are in the range $\frac{\alpha M_P}{k} < 1$. Because of this fact, from now on, we neglect the contribution of $\dot{Q}_u$ in (3.28) by keeping the leading contribution of $\frac{\alpha M_{\rm P}}{k}$ since the $\dot{Q}_u$ term in (3.28) gives an $\mathcal{O}(\frac{\alpha M_{\rm P}}{k})^4$ contribution to the resulting power spectrum [19]. Then, the power spectrum of the curvature perturbation is given by

$$\mathcal{P}_{\mathcal{R}}(k) \equiv \frac{k^3}{2\pi^2}\langle \mathcal{R}\mathcal{R}^*\rangle_* \simeq \left(1+\frac{\alpha^2 M_{\rm P}^2}{2\epsilon_* k^2}\right)^{-2} \frac{H_*^2}{2\epsilon_* M_{\rm P}^2}\lim_{\tau\to 0} k_{1l}\langle \mathcal{V}_l(\tau)\mathcal{V}_l(\tau)^*\rangle, \tag{3.29}$$

where the subscripted asterisk indicates the value at the horizon crossing point $k_{l1} = aH$ and we take the late time limit $\tau \to 0$ to read the power spectrum. Plugging the first line of (3.24) into (3.29), we obtain

$$\mathcal{P}_{\mathcal{R}} \simeq \mathcal{P}_{\mathcal{R}}^{(0)} \frac{|C_1 - C_2|^2}{(1+\frac{\alpha^2 M_{\rm P}^2}{2\epsilon k^2})^2}, \tag{3.30}$$

where $\mathcal{P}_{\mathcal{R}}^{(0)}$ denotes the power spectrum of the canonical single inflation model,

$$\mathcal{P}_{\mathcal{R}}^{(0)} = \frac{H_*^2}{8\pi^2 M_{\rm P}^2}\frac{1}{\epsilon}(1+(2-2C)\eta + (6C-8)\epsilon). \tag{3.31}$$

Here, $C = 2 - \ln 2 - \gamma$ with the Euler–Mascheroni constant $\gamma \approx 0.5772$ and

$$\begin{aligned}|C_1 - C_2|^2 = -\frac{\pi\alpha M_{\rm P}}{X k_{e1}}\Bigg[&\beta^2\left(1+\frac{\alpha^2 M_{\rm P}^2}{2k^2}\right)\frac{k_{l1}^2}{\alpha^2 M_{\rm P}^2} J_{\mu_1 - 1}^2 \\ &+ 2\beta\left\{\left(1+\frac{\alpha^2 M_{\rm P}^2}{2k^2}\right)\left(\mu - \frac{1}{2}\right) + \frac{\beta\alpha^2 M_{\rm P}^2}{k^2}\right\}\frac{k_{l1}}{\alpha M_{\rm P}} J_{\mu_1} J_{\mu_1 - 1} \\ &+ \left\{\left(1+\frac{\alpha^2 M_{\rm P}^2}{2k^2}\right)\left(\mu-\frac{1}{2}\right)^2 + \frac{\beta\alpha^2 M_{\rm P}^2}{k^2}\left(\mu - \frac{1}{2}\right)\right. \\ &\left. + \beta^2\left(1+\frac{3\alpha^2 M_{\rm P}^2}{4k^2}\right)\right\} J_{\mu_1}^2\Bigg],\end{aligned} \tag{3.32}$$

where $\beta \equiv M_{\rm P}\alpha\tau_{\rm eq} = -\sqrt{2}\coth^{-1}(\sqrt{2})$.

As discussed previously, we notice that during the early time phase modes satisfying the condition $k > \alpha M_{\rm P}$ can only cross the Hubble horizon. In other words, modes in the range $k < \alpha M_{\rm P}$ stay outside the Hubble horizon and never cross the horizon. Therefore, those superhorizon modes are causally disconnected to our Universe and irrelevant to observational quantities. As we see in the plot of the power spectrum in Fig. 2, the power spectrum in our model asymptotically approaches that of the single-field model (dashed blue line) by increasing the comoving wave number k, while it is strongly suppressed by decreasing the value k.

In this paper, we investigate the behavior of power spectrum in terms of the value k. To do that, we divide the values of k into two regions, $k \gg \alpha M_{\rm P}$ and $k \gtrsim \alpha M_{\rm P}$. At first, in the region $k \gg \alpha M_{\rm P}$, from (3.30) we have the following asymptotic form of power spectrum:

$$\mathcal{P}_{\mathcal{R}} \simeq \mathcal{P}_{\mathcal{R}}^{(0)}\left[1 - \frac{\alpha^2 M_{\rm P}^2}{\epsilon k^2} + \frac{\alpha^2 M_{\rm P}^2}{\beta^2 k^2}\sin^2\left(\frac{\beta k}{\alpha M_{\rm P}}\right)\right]. \tag{3.33}$$

The power spectrum is almost scale invariant but modulated with small oscillation. This oscillation behavior of the power spectrum was also reported in a different inflation model with phase transition [3,4]. The corresponding spectral index $n_{\mathcal{R}}$ and the running of the spectral index $\alpha_{\mathcal{R}}$ at the pivot scale k_0 in the leading order of α/k with small slow-roll parameters are given by

$$n_{\mathcal{R}} \simeq n_{\mathcal{R}}^{(0)} + \frac{2\alpha^2 M_{\rm P}^2}{\epsilon_* k_0^2}, \qquad \alpha_{\mathcal{R}} \simeq -\frac{4\alpha^2 M_{\rm P}^2}{\epsilon_* k_0^2}, \tag{3.34}$$

where $n_{\mathcal{R}}^{(0)}$ denotes the spectral index at the pivot scale k_0 for the single-field inflation model. We notice that the

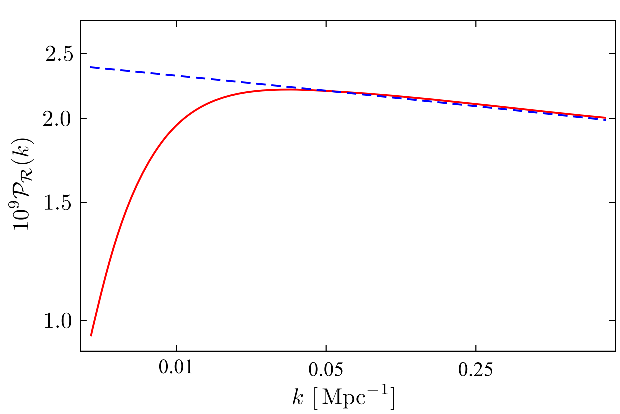

FIG. 2 (color online). The primordial power spectrum of curvature perturbation for the usual single-field model (dashed blue line) and the inflation model on the spatial condensation (solid red line).

spectral index is slightly increasing and the running is negative and slightly decreasing by decreasing the value k due to the spatial condensation. On the other hand, for the region $k \gtrsim \alpha M_{\rm P}$, we obtain the behavior of the power spectrum as

$$\mathcal{P}_{\mathcal{R}} \sim \frac{k^4}{\alpha^4 M_p^4} \mathcal{P}_{\mathcal{R}}^{(0)}. \tag{3.35}$$

This behavior was plotted in Fig. 2 in terms of the red line in the logarithmic scale of the wave number. Then, the corresponding spectral index and its running are given by

$$n_{\mathcal{R}} \simeq 5, \qquad \alpha_{\mathcal{R}} \simeq 0. \tag{3.36}$$

From this behavior of the power spectrum in the spatial condensation, one can clearly notice a strong suppression of the scalar power spectrum on large scales. Since the spectral index is approaching a constant on these large-scale limits, the running of the spectral index becomes almost zero.

We analyzed the behavior of the power spectrum in terms of semianalytic methods for large scales $k \gtrsim \alpha M_{\rm P}$ and small scales $k \gg \alpha M_{\rm P}$ in the previous paragraph. However, as we see in the numerical result shown in Fig. 2, there is a sudden transition of the power spectrum for intermediate scales between $k \gtrsim \alpha M_{\rm P}$ and $k \gg \alpha M_{\rm P}$. Therefore, for this region, the spectral index grows suddenly by decreasing the comoving wave number, i.e., the scalar power spectrum starts to be suppressed strongly, and then one has large negative running of the spectral index in the intermediate region.

IV. CONCLUSION

There are several models to accomplish the suppression. One common property of these models is that there exists a phase transition of the background evolution and it is connected to the slow-roll phase of the single-field inflation model at late time. In this paper, we showed that a deformed single-field inflation model in terms of the spatial condensation has a phase transition that is similar to that of models in Refs. [14,15] and the suppression of the scalar power spectrum on large scales.

We deformed a single-field inflation model in terms of three SO(3) symmetric moduli fields σ^a. On the solution with constant gradient $\sigma^a = \alpha x^a$, the background evolution is equivalent to that of the single inflation model with the curvature term of the open universe. During very early time, the background evolution is governed by the curvature term, but soon after the curvature term is rapidly decreased. Then, at the late time, the evolution is governed by the potential term of the single scalar field and asymptotically approaches that of a single-field inflation model. This means that there exists a phase transition of the background evolution, and so, for an analytic approach, we divided the background evolution into two phases, the α-term dominant phase and the potential term dominant phase.

During the α-term dominant phase, we assumed that the inflation started with a very large value of the α term (curvature term) and then the e-folding could be accumulated very rapidly. Therefore, during the early time phase with a short cosmic time process, the single scalar field remains almost constant. Assuming the scalar potential is a constant, there is an exact solution governing the background evolution. On the other hand, in the late time phase, the α term becomes very small, and the evolution is governed by the potential term and asymptotically approaches that of the single-field inflation. Under the slow-roll assumption, the system is governed by slow-roll parameters and a small contribution of the α term.

Under the above circumstance of the background evolution, we investigated the behavior scalar modes in the linear perturbation level. We considered the perturbation modes in the early and late time phases separately. For perturbed modes in the two phases, we applied the junction condition at the transition point. Then, we obtained the power spectrum, the spectral index, and the running of the spectral index for scalar modes. We found that the power spectrum is apparently suppressed by decreasing the comoving wave number, while it approaches the value of the single-field inflation model for a large value of the comoving wave number. Therefore, one can obtain large negative running of the scalar spectral index on large scales. We also found a oscillation behavior of the power spectrum at late time.

We focused on the suppression of the scalar power spectrum on large scales. However, we also expect that there will be a nontrivial contribution to the isocurvature perturbation since our model has two perturbed scalar modes. Actually, our model introduces a free gradient parameter α to a single-field inflation model in an isotropic and homogeneous way. Therefore, to accommodate observational data, a similar analysis to what we did in this paper can be applied to various inflation models by adjusting the free parameter α.

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