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Abelian Higgs mechanism in the Schrödinger picture

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We have studied symmetry-breaking phenomena in scalar electrodynamics by evaluating the effective potential at one-loop order in the Schrödinger picture. Contributions to the effective potential by the Higgs particle and the transverse and longitudinal components of a photon are compared with other previous works, and they are found to be consistent.

I. INTRODUCTION

The Schrödinger picture in quantum field theory has recently attracted increasing attention. It was first applied to scalar field theories in flat[1] and curved space-time.[2] Fermionic field theory in this picture also has been developed[3] and dynamical-symmetry-breaking phenomena in Gross-Neveu models in $(1+1)$-dimensional and $(2+1)$-dimensional space-time have been studied using this formalism.[4]

The Schrödinger-picture formalism has not been applied to local gauge theories except in some simple cases.[5] In this paper, as a first step to study gauge theories, we take scalar electrodynamics and analyze its symmetry behavior. Massless scalar electrodynamics was consistently analyzed by Coleman and Weinberg[6] who have shown that the scalar meson and photon acquire masses as a result of radiative corrections. This was done at the one-loop-order approximation of an effective potential in the Landau gauge. Jackiw[7] and Dolan and Jackiw[8] studied in detail the gauge dependence of the effective potential and emphasized the use of a unitary Lagrangian. Since the field space of the unitary Lagrangian is not Cartesian but a polar coordinate system, it is necessary to take into account the effect of the metric in the field space. Recently Russell and Toms[9] calculated the effective potential in the unitary gauge with the geometrical approach and explicitly showed the correction terms due to the field space metric.

We evaluated the effective potential in one-loop order in the Schrödinger picture. All these approaches including ours agree in the Higgs-particle and transverse-photon contributions to the effective potential and they are gauge independent. The would-be Goldstone mode turned longitudinal-photon contribution is, however, gauge and calculation-scheme dependent. Coleman and Weinberg[6] argued that the gauge-dependent part is of higher order in the coupling constant ($e^4 \approx \lambda$) and should be corrected by a next-order calculation. If we drop this higher-order term various calculations agree and are gauge independent in the one-loop level.

In Sec. II the Hamiltonian of scalar electrodynamics and its vacuum expectation value are evaluated using a Gaussian trial wave functional. By taking variations of the energy expectation value with respect to the covariance function we obtain equations that determine the wave functional. In Sec. III we compare our effective potential with others, and perform a renormalization in the massless case. Either in the symmetric or broken phase our calculation correctly reflects physical degrees of freedom, and agrees with other results in the valid region of coupling constants ($\lambda \approx e^4$). In the last section we summarize the results and discuss related problems.

II. SCALAR ELECTRODYNAMICS IN THE SCHRÖDINGER PICTURE

We consider scalar electrodynamics described by the Lagrangian density

$$L = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}(\partial_\mu\phi_1 - eA_\mu\phi_2)^2 + \tfrac{1}{2}(\partial_\mu\phi_2 + eA_\mu\phi_1)^2 - \tfrac{1}{2}\mu^2\phi_a\phi_a - \frac{\lambda}{4!}(\phi_a\phi_a)^2 , \tag{2.1}$$

where $\phi=\phi_1, \phi_2$ are two real components of the charged scalar field, and the action is invariant under local O(2) gauge transformation. Noting that A^0 is not a dynamical variable but a kind of a Lagrange multiplier, we find the Hamiltonian density

$$H=\tfrac{1}{2}(\pi_i\pi_i+B_iB_i)+\tfrac{1}{2}P_aP_a+\tfrac{1}{2}(\partial_i\phi_a+eA_i\epsilon_{ab}\phi_b)(\partial_i\phi_a+eA_i\epsilon_{ac}\phi_c)+\tfrac{1}{2}\mu^2\phi_a\phi_a+\frac{\lambda}{4!}(\phi_a\phi_a)^2-A^0(\partial_i\pi_i+e\phi_a\epsilon_{ab}P_b)\ , \tag{2.2}$$

where $\pi_i=\delta L/\delta\dot{A}_i=\partial_i A^0+\partial_t A_i$, $P_a=\partial L/\partial\phi_a=\partial_t\phi_a-eA_0\epsilon_{ab}\phi_b$, and ϵ_{ab} is the usual antisymmetric real matrix. Our index system is such that i,j,k denote spatial indices 1,2,3 and a,b,c denote scalar field components $\hat{1}$ and $\hat{2}$, and the greek indices α,β includes both of them, i.e., $\alpha=\hat{1},\hat{2},1,2,3$.

Since the functional Schrödinger picture is not manifestly covariant, it is natural to choose the Coulomb gauge

$$\nabla\cdot A=0\ . \tag{2.3}$$

In this gauge the Hamiltonian density can be written as

$$H=\tfrac{1}{2}(\pi_i^t\pi_i^t+B_iB_i)+\tfrac{1}{2}P_aP_a+\tfrac{1}{2}(\partial_i\phi_a+eA_i\epsilon_{ab}\phi_b)(\partial_i\phi_a+eA_i\epsilon_{ac}\phi_c)+\tfrac{1}{2}\mu^2\phi_a\phi_a+\frac{\lambda}{4!}(\phi_a\phi_a)^2+\frac{e^2}{8\pi}\int_{d^3y}\rho(x)\frac{1}{|x-y|}\rho(y)\ , \tag{2.4}$$

where $\rho(x)=\phi_a\epsilon_{ab}P_b$, π_i^t denotes the transverse components of π_i, and x,y denote only the three-dimensional space vector throughout this paper. In order to implement the transverse property we may use the representation

$$\pi_i^t=\left[i\frac{\delta}{\delta A_i}\right]^t=i\left[\delta_{ij}-\frac{\partial_i\partial_j}{\nabla^2}\right]\frac{\delta}{\delta A_j}\ , \tag{2.5}$$

which satisfies the commutation relations

$$[\pi_i(x),A_j(x')]=i\left[\delta_{ij}-\frac{\partial_i\partial_j}{\nabla^2}\right]\delta(x-x')\ .$$

But in the following we will rather follow a different calculation scheme; i.e., we will implement the transversality constraint on the wave functional and the functional integration measure. These two methods completely agree in the final results.

In order to evaluate the effective action at the one-loop level it is sufficient to take a ground-state wave functional in Gaussian form:

$$\psi(\phi,A)=N\exp[-\tfrac{1}{4}(\Phi-V)G(\Phi-V)]\ , \tag{2.6}$$

where $\Phi_\alpha\equiv(\phi_a,A_i)$, $V=(\hat{\phi}_a,0,0,0)$,

$$G_{\alpha\beta}=\begin{bmatrix}G_{ab} & G_{aj}\\ G_{ib} & G_{ij}\end{bmatrix},\quad a,b=\hat{1},\hat{2},\ i,j=1,2,3\ ,\quad \alpha,\beta=\hat{1},\hat{2},1,2,3\ , \tag{2.7}$$

and N is a normalization factor. Here, we use an abbreviation

$$(\Phi-V)G(\Phi-V)\equiv\int\int dx\,dy\sum_{\alpha,\beta}[\Phi_\alpha(x)-V_\alpha(x)]G_{\alpha\beta}(x,y)[\Phi_\beta(y)-V_\beta(y)]\ .$$

The fixed function $V_\alpha(x)$ is introduced to incorporate possible gauge symmetry breaking; i.e., if $\hat{\phi}_a\neq 0$ the ground-state wave functional is not invariant under O(2) and the symmetry is spontaneously broken. Since spatial rotational symmetry is not expected to be broken $\hat{\phi}_i=0$ is inserted from the beginning. We can impose the Coulomb-gauge condition on the wave functional by restricting G such that

$$\nabla_k^x G_{k\alpha}(x,y)=0,\quad k=1,2,3,\quad \alpha=\hat{1},\hat{2},1,2,3\ , \tag{2.8}$$

which implies that only transverse components of A contribute in the wave functional. We also restrict the measure of the functional integral by inserting $\delta(\nabla\cdot A)$. With these restrictions we can replace π^t and A^t in the Hamiltonian density by π and A, respectively.

Given the trial wave functional our problem is to minimize the energy expectation value

$$E=\langle H\rangle=\int d^3x\int d^3A\,d^2\phi\,\delta(\nabla\cdot A)\psi^*H\psi\ . \tag{2.9}$$

By taking only to one-loop order ($\hbar$ order) we obtain

$$E=\int d^3x\lim_{y\to x}\left[\tfrac{1}{4}\mathrm{tr}(G-\tfrac{1}{2}G\Lambda G)-\tfrac{1}{2}\nabla^2\Lambda_{\alpha\alpha}(x,y)+\tfrac{1}{2}\nabla_k\nabla_l\Lambda_{kl}(x,y)+\tfrac{1}{2}e^2\hat{\phi}^2\Lambda_{kk}(x,y)+e\hat{\phi}_a\epsilon_{ab}\nabla_k\Lambda_{kb}(x,y)+\left[\tfrac{1}{2}\mu^2+\frac{2}{4!}\lambda\hat{\phi}^2\right]\Lambda_{aa}(x,y)+\frac{\lambda}{3!}\hat{\phi}_a\hat{\phi}_b\Lambda_{ab}(x,y)\right]+\tfrac{1}{2}\mu^2\hat{\phi}^2+\frac{\lambda}{4!}(\hat{\phi}^2)^2+\frac{1}{32\pi}\int\int\int\int d^3x\,d^3y\,d^3z\,d^3\omega\,\hat{\phi}_a\epsilon_{ab}G_{b\mu}(x,z)\Lambda_{\mu\nu}(z,y)G_{\nu c}(\omega,y)\epsilon_{cd}\hat{\phi}_d\frac{e^2}{|x-y|}\ . \tag{2.10}$$

Here $\hat{\phi}_a$ is a constant, $\hat{\phi}^2=\hat{\phi}_a\hat{\phi}_a$, ∇_k operates on the x variable only, and the limit $y\to x$ is taken after ∇ operations. We have used the relations

$$\delta(\nabla\cdot A)=\lim_{\alpha\to 0}\exp\left[\frac{-1}{2\alpha}\int d^3x(\nabla\cdot A)^2\right] \tag{2.11}$$

and

$$\Lambda^{-1}_{\mu\nu} \equiv G_{\mu\nu} - \frac{1}{\alpha}\partial_k \partial_l \delta_{\mu k}\delta_{\nu l} \ . \tag{2.12}$$

Using the functional derivative operators

$$\pi_k = i\frac{\delta}{\delta A_k}, \quad P_a = \frac{1}{i}\frac{\delta}{\delta \phi_a} \tag{2.13}$$

we get terms containing G, and other terms including only Λ come from the functional integration of $\psi^* \psi$. We neglected Λ^2 terms because they are of $\hbar^2$ order.

Inserting (2.12) into (2.10) and taking Fourier transformations we obtain the effective potential

$$V = \tfrac{1}{2}\mu^2 \hat{\phi}^2 + \frac{\lambda}{4!}\hat{\phi}^4 + \int_p \left[\mathrm{tr}\tfrac{1}{8}\Lambda^{-1}(p) - \frac{1}{2\alpha^2}p^2 p_k \Lambda_{kl} p_l + \frac{1}{8}\frac{e^2}{p^2}\hat{\phi}_a \epsilon_{ab}\Lambda^{-1}_{bc}\epsilon_{cd}\hat{\phi}_d + M_{\alpha\beta}(p)\Lambda_{\beta\alpha}(p) \right] , \tag{2.14}$$

where $\int_p = \int d^3p/(2\pi)^3$ and

$$M_{\alpha\beta}(p) \equiv \tfrac{1}{2}p^2 \delta_{\alpha\beta} - \tfrac{1}{2}p_\alpha p_\beta + e(\hat{\phi}\epsilon)_\alpha (ip)_\beta + \left[\frac{\mu^2}{2} + \frac{2}{4!}\lambda \hat{\phi}^2 \right] \Pi_\phi + \frac{e^2 \hat{\phi}^2}{2}\Pi_A + \frac{\lambda}{3!}\hat{\phi}_\alpha \hat{\phi}_\beta \ . \tag{2.15}$$

Here we have introduced an extended index system,

$$p_\alpha = (0,0,p_1,p_2,p_3) \quad \text{and} \quad \hat{\phi}_\alpha = (\hat{\phi}_{\hat{1}},\hat{\phi}_{\hat{2}},0,0,0) \tag{2.16}$$

and the projection operators $\Pi_\phi = \mathrm{diag}(1,1,0,0,0)$, and $\Pi_A = \mathrm{diag}\ (0,0,1,1,1)$.

In order to minimize the effective potential with respect to the variation of Λ we need to take care of the constraint (2.8). So we modify the potential using Lagrange multipliers C^μ as

$$V' = V + \int_p p_k \left[\Lambda^{-1}_{k\mu} - \frac{1}{\alpha}p_k p_\mu \right] C^\mu \ . \tag{2.17}$$

By varying with respect to Λ we obtain equations

$$\begin{aligned} \frac{1}{2}\left[\frac{\delta V'}{\delta \Lambda_{\alpha\beta}} + \frac{\delta V'}{\delta \Lambda_{\beta\alpha}} \right] = 0 \\ = \frac{-1}{8}\Lambda^{-2}_{\alpha\beta} \\ - \frac{1}{8}\frac{e^2}{p^2}\hat{\phi}_a \epsilon_{ab}\Lambda^{-1}_{b\alpha}\Lambda^{-1}_{\beta c}\epsilon_{cd}\hat{\phi}_d \\ - \frac{p^2}{2\alpha^2}p_k p_l \delta_{k\alpha}\delta_{l\beta} + M_{\alpha\beta} \\ - \tfrac{1}{2}p_k(\Lambda^{-1}_{k\alpha}\Lambda^{-1}_{\beta\mu} + \Lambda^{-1}_{k\beta}\Lambda^{-1}_{\alpha\mu})C^\mu \ . \end{aligned} \tag{2.18}$$

We should solve this equation with the constraint

$$p_k \left[\Lambda^{-1}_{k\mu} - \frac{1}{\alpha}p_k p_\mu \right] = 0 \ . \tag{2.19}$$

Noting that V is a function of only $\hat{\phi}^2 = \hat{\phi}_1^2 + \hat{\phi}_2^2$ and $p^2 = p_1^2 + p_2^2 + p_3^2$ we can greatly simplify the computations by putting $\hat{\phi}_a = (\hat{\phi},0)$, $p_k = (p,0,0)$. Then we have a diagonalized form of the Λ matrix. First we find that $p[\Lambda^{-1}_{3\mu} - (1/\alpha)p^2 p_\mu] = 0$ which means

$$\Lambda^{-1}_{3\mu} = 0 \quad \text{for } \mu = \hat{1},\hat{2},1,2 \ , \tag{2.20a}$$

and

$$\Lambda^{-1}_{33} = \frac{1}{\alpha}p^2 \ . \tag{2.20b}$$

Since Λ^{-1} is symmetric its last row and column vanish except the diagonal element. So we need to determine only the 4×4 part of Λ^{-1}. Now from (2.18) the Lagrange multiplier part vanishes if neither α nor β is 3. The equation is simply

$$\tfrac{1}{8}\Lambda^{-2}_{\alpha\beta} + \frac{1}{8}\frac{e^2\hat{\phi}^2}{p^2}\Lambda^{-1}_{2\alpha}\Lambda^{-1}_{\beta 2} - M_{\alpha\beta} = 0 \quad \text{if } \alpha,\beta = \hat{1},\hat{2},1,2 \ , \tag{2.21}$$

where $M_{\alpha\beta}$ is a diagonal matrix (omitting α or $\beta = 3$):

$$M = \tfrac{1}{2}\mathrm{diag}(p^2 + m_H^2, p^2 + m_G^2, p^2 + e^2\hat{\phi}^2, p^2 + e^2\hat{\phi}^2) \ , \tag{2.22}$$

where

$$m_H^2 = \mu^2 + \frac{\lambda}{2}\hat{\phi}^2, \quad m_G^2 = \mu^2 + \frac{\lambda}{6}\hat{\phi}^2 \ . \tag{2.23}$$

The solution of Eq. (2.21) is

$$\Lambda^{-1}_{\alpha\beta} = 2\mathrm{diag}\left[(p^2 + m_H^2)^{1/2}, \left[(p^2 + m_G^2) \Big/ \left[1 + \frac{e^2\hat{\phi}^2}{p^2} \right] \right]^{1/2}, (p^2 + e^2\hat{\phi}^2)^{1/2}, (p^2 + e^2\hat{\phi}^2)^{1/2}, \frac{1}{2\alpha}p^2 \right] , \tag{2.24}$$

where the last element is determined by the constraint and it is the only element explicitly dependent on the gauge parameter.

By inserting solution (2.24) into the effective potential (2.14) we obtain

$$V=\frac{1}{2}\mu^2\hat{\phi}^2+\frac{\lambda}{4!}\hat{\phi}^4+\frac{\hbar}{2}\int_p\left\{(p^2+m_H^2)^{1/2}+2(p^2+e^2\hat{\phi}^2)^{1/2}+\left[(p^2+m_G^2)\left[1+\frac{e^2\hat{\phi}^2}{p^2}\right]\right]^{1/2}+\frac{\alpha e^2\hat{\phi}^2}{p^2}-\frac{p^2}{2\alpha}\right\}, \quad (2.25)$$

where we write $\hbar$ explicitly for later use. The constant term $[(1/2\alpha)p^2]$ can be dropped and finally the limit $\alpha\to 0$ should be taken.

In the free-field limit ($\lambda=e=0$) the effective potential is simply

$$V_{\text{free}}=\frac{1}{2}\mu^2\hat{\phi}^2+\frac{\hbar}{2}\int_p\{2\sqrt{p^2+\mu^2}+2\sqrt{p^2}\} \quad (2.26)$$

which correctly reveals the degrees of freedom: two scalars and two transverse photons. In the decoupled system ($\lambda>0$, $e=0$) the photon sector is free and the scalar meson sector has a global SO(2) symmetry. The effective potential of the scalar sector in this case is

$$V_{\text{scalar}}=\frac{1}{2}\mu^2\hat{\phi}^2+\frac{\lambda}{4!}\hat{\phi}^4+\frac{\hbar}{2}\int_p\left[\left[p^2+\mu^2+\frac{\lambda}{2}\hat{\phi}^2\right]^{1/2}+\left[p^2+\mu^2+\frac{\lambda}{6}\hat{\phi}^2\right]^{1/2}\right], \quad (2.27)$$

which agrees well with the results of other works revealing the Higgs mode and the Goldstone mode when the spontaneous symmetry-breaking condition $[\mu^2+(\lambda/6)\hat{\phi}^2=0]$ is met. Thus the Schrödinger picture shows correctly the relevant degrees of freedom either in the symmetric phase ($\mu^2>0$, $\hat{\phi}=0$) or the broken case $[\mu^2+(\lambda/6)\hat{\phi}^2=0]$.

III. GAUGE DEPENDENCE AND COMPARISON WITH OTHER WORKS

Since the seminal work of Coleman and Weinberg[6] on massless scalar electrodynamics there have been several attempts to treat the subject using different techniques to clarify the gauge dependence of the effective potential. We will compare our result with the results of other works. Our effective potential can be written as

$$V=\frac{1}{2}\mu^2\hat{\phi}^2+\frac{\lambda\hat{\phi}^4}{4!}+\frac{\hbar}{2}\int_p\left\{(p^2+m_H^2)^{1/2}+2(p^2+e^2\hat{\phi}^2)^{1/2}+\left[(p^2+m_G^2)\left[1+\frac{e^2\hat{\phi}^2}{p^2}\right]\right]^{1/2}\right\}, \quad (3.1)$$

where $m_H^2=\mu^2+(\lambda/2)\hat{\phi}^2, m_G=\mu^2+(\lambda/6)\hat{\phi}^2$. We note that the three terms in the momentum integral reflect corresponding physical degrees of freedom. In the symmetric case ($\mu^2>0$) the effective potential is a minimum at $\hat{\phi}^2=0$, and there are two scalar mesons ($2\sqrt{p^2+\mu^2}$ terms) and two transverse photons ($2\sqrt{p^2}$). In the broken phase ($\mu^2<0$) there are a Higgs mode $[(p^2+m_H)^{1/2}]$, two transverse photon modes ($2\sqrt{p^2+e^2\hat{\phi}^2}$), and the Goldstone-photon mixed mode

$$\left[(p^2+m_G^2)\left[1+\frac{e^2\hat{\phi}^2}{p^2}\right]\right]^{1/2}.$$

Since $\mu^2+(\lambda/6)\hat{\phi}^2=0$ up to $\hbar^0$ order we can put $m_G^2=0$ in the last mode, and then we have the correct masses for the longitudinal photon mode ($\sqrt{p^2+e^2\hat{\phi}^2}$) and the Higgs particle ($m_H^2=-2\mu^2$).

Coleman and Weinberg kept manifest covariance by choosing the Lorentz gauge but they had one spurious degree of freedom in their effective potential of the massless case ($\mu^2=0$), that is,

$$V_{\text{CW}}=\frac{\lambda}{4!}\hat{\phi}^4+\frac{\hbar}{2}\int\frac{d^4p}{(2\pi)^4}\left[\ln\left[p^2+\frac{\lambda}{2}\hat{\phi}^2\right]+\ln\left[p^2+\frac{\lambda}{6}\hat{\phi}^2\right]+3\ln(p^2+e^2\hat{\phi}^2)\right] \quad (3.2a)$$

$$=\frac{\lambda}{4!}\hat{\phi}^4+\frac{\hbar}{2}\int_p\left[\left[p^2+\frac{\lambda}{2}\hat{\phi}^2\right]^{1/2}+\left[p^2+\frac{\lambda}{6}\hat{\phi}^2\right]^{1/2}+3\left[p^2+e^2\hat{\phi}^2\right]^{1/2}\right], \quad (3.2b)$$

where we use

$$\int\frac{d^4p}{(2\pi)^4}\ln(p^2+a^2)=\int\frac{d^3p}{(2\pi)^3}\sqrt{p^2+a^2}+\text{const},$$

and neglected infinite constants in the effective potential. Jackiw[7] obtained it in a wide class of gauges as

$$V_J=\frac{\lambda}{4!}\hat{\phi}^4+\frac{\hbar}{2}\int\frac{d^4p}{(2\pi)^4}\left[\ln\left[p^2+\frac{\lambda}{2}\hat{\phi}^2\right]+\ln\left[p^2+\frac{\lambda}{6}\hat{\phi}^2+\frac{\alpha\lambda\hat{\phi}^4e^2}{6p^2}\right]+3\ln(p^2+e^2\hat{\phi}^2)\right], \quad (3.3)$$

where α is a gauge parameter and $\alpha=0$ is the Coleman-Weinberg case.

Dolan and Jackiw[8] considered the gauge dependence problem in detail and argued for the unitary Lagrangian where only physical degrees of freedom contribute. Their result is

$$V_{\rm DJ}=\frac{\mu^2\hat{\phi}^2}{2}+\frac{\lambda\hat{\phi}^4}{4!}+\frac{\hbar}{2}\int\frac{d^4p}{(2\pi)^4}[\ln(p^2+m_H^2)+3\ln(p^2+e^2\hat{\phi}^2)]\ . \tag{3.4}$$

The use of the unitary Lagrangian is limited to the broken phase and the mixing between Goldstone and longitudinal photon modes is not traceable. Recently Russell and Toms[9] reconsidered the unitary gauge to the effective action. Their result is

$$V_{\rm RT}=\frac{\mu^2\hat{\phi}^2}{2}+\frac{\lambda\hat{\phi}^2}{4!}+\frac{\hbar}{2}\int\frac{d^4p}{(2\pi)^4}[\ln(p+m_H^2)+2\ln(p^2+e^2\hat{\phi}^2)+\ln(p^2+e^2\hat{\phi}^2+\beta m_G^2p^2)]\ , \tag{3.5}$$

where $\beta=0$ corresponds to the Dolan-Jackiw case. The geometrical approach ($\beta=1$) takes into account the effect of the metric and connection[10] on the field space of the unitary Lagrangian and agrees with Fradkin and Tseytlin[11] and Kunstatter.[11]

Comparing the various results we find that the Higgs mode and the transverse photon mode are common and gauge independent, but the mixing mode of longitudinal photon and would-be Goldstone field is gauge and formalism dependent which is expected from the very definition of the effective potential as Dolan and Jackiw pointed out. This gauge dependence of the longitudinal photon mode persists even after renormalization as we will now show explicitly in the simple massless case.

Since scalar electrodynamics is a renormalizable theory the one-loop approximation should be shown to be finite by introducing suitable counterterms. In the massless case ($\mu^2=0$) the effective potential is

$$\begin{aligned}V=&\frac{\lambda}{4!}\hat{\phi}^4+\frac{B}{2}\hat{\phi}^2+\frac{C}{4!}\hat{\phi}^4+\frac{\hbar(\Lambda\hat{\phi})^2}{16\pi^2}\left[\left[\frac{\lambda}{2}\right]^2+2e^2+\left[\frac{\lambda}{6}+e^2\right]\right]\\&+\frac{\hbar\hat{\phi}^4}{64\pi^2}\left[\left[\frac{\lambda}{2}\right]^2\left[\ln\frac{\left[\frac{\lambda}{2}\hat{\phi}^2\right]}{\Lambda^2}+\frac{5}{2}-2\ln2\right]+2e^4\left[\ln\frac{e^2\hat{\phi}^2}{\Lambda^2}+\frac{5}{2}-2\ln2\right]\right]\\&+\frac{\hbar\hat{\phi}^4}{64\pi^2}\left[\frac{\lambda}{6}-e^2\right]^2\left[\ln\left[\frac{(\sqrt{\lambda/6}+e)^2\hat{\phi}^2}{\Lambda^2}\right]-3\ln2\right]+\frac{\hat{\phi}^4}{128\pi^2}(\sqrt{\lambda/6}-e)\ ,\end{aligned} \tag{3.6}$$

where B and C are infinite constants of counterterms and Λ^2 is the momentum cutoff. Here the parameter λ is the renormalized coupling constant. Now we can determine B and C by requiring

$$\left.\frac{d^2V}{d\hat{\phi}^2}\right|_{\hat{\phi}=0}=0 \tag{3.7}$$

and

$$\left.\frac{d^4V}{d\hat{\phi}^4}\right|_{\hat{\phi}=M}=\lambda\ , \tag{3.8}$$

which gives the renormalized effective potential as

$$V=\frac{\lambda}{4!}\hat{\phi}^4+\frac{\hbar\hat{\phi}^4}{64\pi^2}\left[\left[\frac{\lambda}{2}\right]^2+2e^4+\left[\frac{\lambda}{6}-e^2\right]^2\right]\times\left[\ln\frac{\hat{\phi}^2}{M^2}-\frac{25}{6}\right]\ , \tag{3.9}$$

where M is an unspecified mass scale. For comparison we list the renormalized effective potentials of other works:

$$V_{\rm CW}=\frac{\lambda}{4!}\hat{\phi}^4+\frac{\hbar\hat{\phi}^4}{64\pi^2}\left[\left[\frac{\lambda}{2}\right]^2+2e^4+\left[\frac{\lambda}{6}\right]^2+e^4\right]\times\left[\ln\frac{\hat{\phi}^2}{M^2}-\frac{25}{6}\right]\ , \tag{3.10a}$$

$$V_{\rm DJ}=\frac{\lambda}{4!}\hat{\phi}^4+\frac{\hbar\hat{\phi}^4}{64\pi^2}\left[\left[\frac{\lambda}{2}\right]^2+2e^4+e^4\right]\left[\ln\frac{\hat{\phi}^2}{M^2}-\frac{25}{6}\right]\ , \tag{3.10b}$$

and

$$V_{\rm RT}=\frac{\lambda}{4!}\hat{\phi}^4+\frac{\hbar\hat{\phi}^4}{64\pi^2}\left[\left[\frac{\lambda}{2}\right]^2+2e^4+\left[\frac{\lambda}{6}\right]^2+\frac{2\lambda}{3}e^2+e^4\right]\times\left[\ln\frac{\hat{\phi}^2}{M^2}-\frac{25}{6}\right]\ . \tag{3.10c}$$

The coefficients in front of the logarithm reflect the contributing modes, and, as mentioned earlier, the Higgs parts $[(\lambda/2)^2]$ and two transverse photon parts $(2e^4)$ are in agreement with one another, but the part of the Goldstone-longitudinal-photon mixing mode differs from one approach to another. In regard to this disagreement, we recall the Coleman-Weinberg renormalization-group arguments which state, in brief, that λ is of e^4 order. In this case all these approaches are in agreement within e^4 order as

$$V=\frac{\lambda}{4!}\hat{\phi}^4+\frac{\hbar\hat{\phi}^4}{64\pi^2}3e^4\left[\ln\left[\frac{\hat{\phi}^2}{M^2}\right]-\frac{25}{6}\right] . \quad (3.11)$$

This potential has a minimum at nonzero $\hat{\phi}$; the symmetry is spontaneously broken and the mass ratio of the Higgs scalar and the vector meson is

$$\frac{m^2(S)}{m^2(V)}=\frac{3}{2\pi}\frac{e^2}{4\pi} , \quad (3.12)$$

which was shown by Coleman and Weinberg.

In the massive case ($\mu^2\neq 0$) the effective potential is

$$\begin{aligned}V=&\frac{\mu^2}{2}\hat{\phi}^2+\frac{\lambda}{4!}\hat{\phi}^4+\frac{B}{2}\hat{\phi}^2+\frac{C}{4!}\hat{\phi}^4+\frac{\hbar}{16\pi^2}\Lambda^2\left[\mu^2+\frac{\lambda}{2}\hat{\phi}^2+2e^2\hat{\phi}^2+\left[\mu^2+\frac{\lambda}{6}\hat{\phi}^2\right]+e^2\hat{\phi}^2\right]\\&-\frac{\hbar}{64\pi^2}\ln\Lambda^2\left[\left[\mu^2+\frac{\lambda}{2}\hat{\phi}^2\right]^2+2e^4\hat{\phi}^4+\left[\mu^2+\frac{\lambda}{6}\hat{\phi}^2-e^2\hat{\phi}^2\right]^2\right]\\&+\frac{\hbar}{64\pi^2}\left\{\left[\mu^2+\frac{\lambda}{2}\hat{\phi}^2\right]\ln\left[\mu^2+\frac{\lambda}{2}\hat{\phi}^2\right]+2e^4\hat{\phi}^4\ln(e^2\hat{\phi}^2)+\left[\mu^2+\frac{\lambda}{6}\hat{\phi}^2-e^2\hat{\phi}^2\right]^2\ln\left[\left[\mu^2+\frac{\lambda}{6}\hat{\phi}^2\right]^{1/2}+e\hat{\phi}\right]^2\right\}\\&+\frac{\hbar}{64\pi^2}\left\{\left[\left[\mu^2+\frac{\lambda}{2}\hat{\phi}^2\right]^2+2e^4\hat{\phi}^4\right](\tfrac{5}{2}-2\ln 2)-3\left[\mu^2+\frac{\lambda}{6}\hat{\phi}^2-e^2\hat{\phi}^2\right]^2\ln 2\right\}\\&+\frac{\hbar}{128\pi^2}\left[\left[\mu^2+\frac{\lambda}{6}\hat{\phi}^2\right]^{1/2}-e\hat{\phi}\right]^4 , \end{aligned}\quad (3.13)$$

where μ^2 and λ are renormalized quantities. The renormalization conditions are

$$\left.\frac{d^2V}{d\hat{\phi}^2}\right|_{\hat{\phi}=0}=\mu^2 \quad (3.14)$$

and

$$\left.\frac{d^4V}{d\hat{\phi}}\right|_{\hat{\phi}=M}=\lambda ,$$

which gives the renormalized effective potential

$$V=\frac{\mu^2}{2}\hat{\phi}^2+\frac{\lambda}{4!}\hat{\phi}^4+f(\hat{\phi})-\frac{f^{(2)}(0)}{2}\hat{\phi}^2-\frac{f^{(4)}(M)}{4!}\hat{\phi}^4 , \quad (3.15)$$

where

$$\begin{aligned}f(\hat{\phi})=&\frac{\hbar}{64\pi^2}\left[\left[\mu^2+\frac{\lambda}{2}\hat{\phi}^2\right]^2\ln\left[\mu^2+\frac{\lambda}{2}\hat{\phi}^2\right]+2e^4\hat{\phi}^4\ln\left[e^2\hat{\phi}^2\right]\right]\\&+\frac{\hbar}{64\pi^2}\left\{\left[\mu^2+\frac{\lambda}{6}\hat{\phi}^2-e^2\hat{\phi}^2\right]^2\ln\left[\left[\mu^2+\frac{\lambda}{6}\hat{\phi}^2\right]^{1/2}+e\hat{\phi}\right]^2+\frac{1}{2}\left[\left[\mu^2+\frac{\lambda}{6}\hat{\phi}^2\right]^{1/2}-e\hat{\phi}\right]^4\right\} ,\end{aligned}\quad (3.16)$$

and

$$f^{(2)}(0)\equiv\left.\frac{d^2f}{d\hat{\phi}^2}\right|_{\hat{\phi}=0} , \quad f^{(4)}(M)=\left.\frac{d^4f}{d\hat{\phi}^4}\right|_{\hat{\phi}=M} ,$$

which we do not write explicitly because they are extremely complicated without being particularly illuminating. We note that for positive μ^2 one can take $M=0$, but for negative μ^2 one should choose $M^2>-6\mu^2/\lambda$ because of the branch point singularity appearing in the effective potential. We also note that the effective potential (3.9) computed in the Coulomb gauge is the same as the Feynman-gauge result, and the potential (3.13) for the massive case is manifestly renormalizable.

IV. DISCUSSION

We have obtained the ground-state wave functional and the effective potential of scalar electrodynamics in the one-loop order in the Schrödinger picture. We have studied spontaneous symmetry breaking and the Higgs mechanism and compared our results with the results of other approaches in the same approximation order. All these methods including ours agree in the Higgs and the transverse photon sectors, but they disagree in the longitudinal photon mode which is also gauge dependent. Following the Coleman-Weinberg renormalization-group argument[6] we note that this disagreeing gauge-dependent part is of higher order, and would be corrected by next-order calculations.

The two-loop-order calculation in the Schrödinger picture has not been developed even in $\lambda\phi^4$ scalar theories. Furthermore in electrodynamics (either in scalar or spinor) one photon coupling term in the Hamiltonian does not contribute if the wave functional is of the Gaussian form. So the $\hbar^2$-order evaluation of the effective potential in the Schrödinger picture is difficult at the present time. The Gaussian approximation which is well developed and captures part of the two-loop-order effects might be obtainable, and we will work on this problem. Another interesting work to be done is to extend the present work to non-Abelian gauge theories.

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