PHYSICAL REVIEW D VOLUME 46, NUMBER 6 15 SEPTEMBER 1992

Particle creation for time travel through a wormhole

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(Received 10 October 1991)

A time machine can be constructed by the relative motion of one mouth of a wormhole. The model has some remaining problems to be solved. Among the problems, the stability problem arises from the forming of the Cauchy horizon, where rays of early times will accumulate and diverge. This stability problem can be solved at the classical level. For quantum stability, Kim and Thorne recently tried to calculate the vacuum fluctuation of quantized fields by the point-splitting method. It was shown that the vacuum fluctuations produce a renormalized stress-energy tensor that diverges as one approaches the Cauchy horizon, which might be cut off by quantum gravity. However, there is a controversy. Hawking conjectures an observer-independent location for the breakdown in the semiclassical theory. In this paper, we deal with this quantum stability problem using another method: "particle production by an arbitrary gravitational field." When the wormhole forms in the infinite past, the result is finite, while it is divergent near the Cauchy horizon when the wormhole forms at a finite time. If we adopt the Kim-Thorne conjecture, then the divergence might be cut off by quantum gravity; therefore, the total energy cannot prevent the formation of the closed timelike curves when one is within a Planck length.

PACS number(s): 04.20.Cv, 04.60.+n

I. INTRODUCTION

Recently, it was discovered that the generic relative motions of wormhole mouths will produce closed timelike curves (CTC's), a time machine [1,2], as well as generic gravitational redshifts at the wormhole mouths due to the generic gravitational fields [3]. At least this is so if the Cauchy horizon at which the CTC's arise is stable.

Among the problems which remain to be solved for the construction of a time machine, the stability problem arises from the forming of the Cauchy horizon, where rays in the mouth at early times will accumulate; then, the total energy near the Cauchy horizon will diverge. The causality problem was recently considered as a Cauchy problem [4,5] and a billiard ball problem [4,6]. This stability problem can be solved by the zero-measure property of the ray and the diverging lens effect of the wormhole at the classical level; that is, the Cauchy horizon is stable against classical perturbations [1]. But stability against quantum-field perturbations is less certain. Recently, Kim and Thorne have tried to calculate the vacuum fluctuation of quantized fields by the point-splitting method [7]. It was shown that the vacuum fluctuations produce a renormalized stress-energy tensor which diverges as one approaches the Cauchy horizon. It was also recently shown that the divergence of the renormalized stress-energy tensor is a general property of quantum-field theory in a locally static spacetime with a nonpotential gravitational field describing the time machine formation [8]. But the divergence might be cut off by quantum gravity when one is within a Planck length; therefore, vacuum fluctuations cannot prevent the formation of CTC's. [7]. However, Hawking [9] has responded that the criterion for applying quantum gravity is quite different from that used in Ref. [7]. He conjectures that the location at which semiclassical theory breaks down is observer independent, whereas Ref. [7] asserts that the breakdown location depends on the chosen reference frame [8]. Of course, there is a controversy concerning which conjecture is correct for the true quantum theory of gravity.

In this paper we want to analyze this stability problem at the quantum level using another method, "particle production by a gravitational field," to confirm our previous work [7]. For the case of a wormhole mouth moving with a constant velocity and a uniform acceleration, the amount of net energy flux from the wormhole is calculated near the Cauchy horizon following the method of Ford and Parker [10]. The results are divergent as in the previous case, vacuum fluctuations [7]. When the wormhole forms in the infinite past, the result is finite, while it is divergent near the Cauchy horizon when the wormhole forms at a finite time. But if we adopt only the Kim-Thorne conjecture, the divergence can be cut off by quantum gravity; thus, the total energy will be very small and may be ignored. Some stimulated emission problems are considered to obtain the energy fluxes. The results are not so large either, but there are ambiguities for the initial particle distribution. Thus one can say that the particle-production energy through the wormhole from the field ϕ of any state (vacuum or not) at early times is not so large as to prevent the wormhole from forming the Cauchy horizon. The time machine is stable against either classical perturbations or quantum perturbations under the Kim-Thorne conjecture. Therefore, since one can overcome the stability problem in these ways, one can try to construct a time machine if only the other remaining problems can be solved.

In Sec. II a time machine model based on the relative motion of wormhole mouths [1], the stability problem, the vacuum fluctuation of quantized fields, and the quantum solution to the stability problem [7] are briefly reviewed. Particle creation by the gravitational field of a wormhole is considered and the energy fluxes are calculated in Sec. III. The conclusions and discussion are in Sec. IV.

II. TIME MACHINE AND STABILITY

As the time machine model, we consider the spacetime in which two $\mathcal{S}^3 \times \mathcal{R}$ cylinders are eliminated from the Minkowski space and in which the positions of the cylinders are identified at the same proper times. Since the length of the throat of the wormhole (the internal distance between the two mouths of the wormhole, not the external distance) is very short as the structure of the wormhole, it is assumed to be zero for an ideal time machine model by identifying two mouths, i.e., by treating the two mouths the same. Then simultaneous events constitute closed curves which will be gradually changed from closed spacelike curves into closed null curves because of the time dilation from the motion of one mouth. After the closed null curves, the so-called Cauchy horizon $\mathcal{H}$, CTC's, the so-called time machine, will exist.

Among the current main issues concerning time machine construction, the causality problem was recently considered as a Cauchy problem [4,5] and as a billiard ball problem [4,6]. The conclusion was that CTC's (time machines) are benign in the sense that they may not be as nasty as people have assumed; i.e., going back to the past and killing oneself is impossible [4].

To understand how the instability problem arises and is solved, consider the ray from $\mathcal{J}^-$ to the left mouth of the wormhole. The ray from the left mouth goes into the right mouth through external spacetime; then, it comes out from the left mouth again through the throat at the same moment that it goes into the right mouth because of the identification of the two mouths. Next, it travels to the right mouth and comes out from the left mouth again. By such processes the rays accumulate at $\mathcal{H}$, and the energy can be divergent, which will prevent CTC's from forming. However, at the classical level, the energy of the ray near $\mathcal{H}$, after passing an infinite number of times through the wormhole, can be summed up as

$$\rho \simeq \sum_{j=1}^{\infty} \left[\frac{b}{2D}\right]^{2j} \left[\frac{1}{f}\right]^{j} \rho_0 \,, \tag{1}$$

where $f=\sqrt{(1-v)/(1+v)}<1$ is the Doppler-effect factor and $b/2D$ is the diverging lens effect for each ray which passes the wormhole. Here b is the radius of the mouth and D is the external distance between the two mouths just before the onset of the relative motion. We can always make the sum converge by appropriately adjusting b, D, and v so that $b/2D \ll 1$ even though, $1/f>1$.

To investigate the quantum stability near $\mathcal{H}$, we shall now evaluate the order of magnitude of the divergence of the vacuum polarization strength. For detailed calculations and exact expressions, see Ref. [7]. The geodetic intervals between x' and x are $\sigma_{\pm N} \sim D\,\Delta t$ and the Van Vleck–Morette determinant is

$$\Delta_{\pm N}^{1/2} \sim \left[\frac{b}{D}\right]^{N \text{ or } (N-1)} , \tag{2}$$

where N is chosen if x is far from both mouths and $N-1$ if x is near either mouth. Here the time Δt to $\mathcal{H}$ depends on the chosen observer. Then the orders of magnitude of the regularized Hadamard function $G^{(1)}_{\text{reg}}$, which is defined by eliminating its flat-spacetime, vacuum-state value from the Hadamard function [11], and of the regularized stress-energy tensor $T^{\mu\nu}$ obtained from $G^{(1)}_{\text{reg}}$ are

$$G^{(1)}_{\text{reg}} \sim \frac{\Delta_{\pm N}^{1/2}}{\sigma_{\pm N}} \sim \left[\frac{b}{D}\right]^{N \text{ or } (N-1)} \frac{1}{D\,\Delta t} \,, \tag{3}$$

$$T^{\mu\nu} \sim \left[\frac{b}{D}\right]^{N \text{ or } (N-1)} \frac{1}{D(\Delta t)^3} \,. \tag{4}$$

Apparently, this stress-energy tensor is divergent for $x \to \mathcal{H}$, that is, $\Delta t \to 0$. Since the Van Vleck–Morette determinant plays the role of the amplitude of the ray, we hoped that it might give rise to the diverging lens effect as in the classical case. However, even though the Hadamard function is regularized by eliminating its flat-spacetime, vacuum expectation value, the geodetic interval tends to zero when the ray approaches the $\mathcal{H}$ having nullness.

Now the physical effects of the *divergent* $T^{\mu\nu}$ will be considered, where

$$T^{\mu\nu} \sim \left[\frac{b}{D}\right]^{N \text{ or } (N-1)} \left[\frac{l_P}{D}\right] \left[\frac{m_P}{(\Delta t)^3}\right] , \tag{5}$$

in cgs units (still $c=1$). The metric perturbation due to the curvature fluctuation produced by the scalar-field stress energy is on the order of

$$\frac{\delta L}{L} \simeq \delta g^{\text{VP}}_{\mu\nu} \sim \left[\frac{b}{D}\right]^{N \text{ or } (N-1)} \left[\frac{l_P}{D}\right] \left[\frac{l_P}{\Delta t}\right] . \tag{6}$$

On any time scale Δt and in any classical time, quantum gravity produces fluctuations of the metric with magnitude

$$\delta g^{\text{QG}}_{\mu\nu} \geq \frac{l_P}{\Delta t} \,. \tag{7}$$

This shows that $\delta g^{\text{QG}}_{\mu\nu} > \delta g^{\text{VP}}_{\mu\nu}$ for $\Delta t \sim l_P$. The quantum-gravity fluctuations dominate over the scalar field's vacuum fluctuations throughout the region considered. Thus an observer will not notice at all the tidal effects of the *divergent* vacuum polarization.

Hawking [9], in response to this, has conjectured that the spacetime near $\mathcal{H}$ remains classical until $D\Delta t \sim l_P^2$ and, correspondingly, until $\delta g^{\text{VP}}_{\mu\nu} \sim 1$ on the grounds that the location at which semiclassical theory breaks down should be observer independent. And he has argued that, as a result, the vacuum polarization divergence will prevent the formation of CTC's. The above discussion based on Ref. [7] (Kim-Thorne conjecture) asserts that since this Δt is frame dependent, the breakdown occurring at $\Delta t \sim l_P$ can be true only in some preferred reference frame. There is a preferred reference frame in which the argument of Ref. [7] gives Hawking's location. This is a frame in which the observer is moving so rapidly that he or she sees the D Lorentz contracted to l_P. The difference between the Hawking conjecture and that of Ref. [7] is so huge (a factor 10^{35} where the semiclassical theory breaks down when $D \sim 1$ m) that there might be some hope of seeing which (if either) is correct for various candidate quantum theories of gravity.

III. PARTICLE PRODUCTION

A. Particle production by an arbitrary gravitational field

This method [10] is applied to the particle-production problem in any spherically symmetric, asymptotically flat spacetime which has a one-to-one mapping between $\mathcal{J}^-$ and $\mathcal{J}^+$, being considered only *in* site. Using the geometrical approximation, we can determine the energy flux through the gravitational field. An incoming null ray $V=t+r=\text{const}$, originating on $\mathcal{J}^-$, propagates through the geometry becoming an outgoing null ray from $U=t-r=\text{const}$ and arriving on $\mathcal{J}^+$ at a value $U=F(V)$. Conversely, one can trace a null ray from U on $\mathcal{J}^+$ to $V=G(U)$ on $\mathcal{J}^-$, where the function G is the inverse of F.

A solution of the two-dimensional massless scalar-field wave equation $\Box\phi=0$ (the minimally coupled case) can be

$$\phi=\int_0^\infty d\omega(a_\omega F_\omega+a_\omega^\dagger F_\omega^*) , \tag{8}$$

where

$$F_\omega \sim \frac{1}{\sqrt{4\pi\omega}}\frac{1}{r}(e^{-i\omega v}+e^{-i\omega G(U)}) . \tag{9}$$

The first term of Eq. (9) is the incoming wave at early times, and the second term is the outgoing wave at late times in accordance with propagation by geometrical optics. Any positive-frequency solution of the scalar wave equation which is incoming on $\mathcal{J}^-$ can be written as a wave packet from the F_ω. The annihilation operator a is defined as $a_\omega|0\rangle=0$ for all ω. The state $|0\rangle$ defined in the *in* state is the state containing no field particles at early times, while $a|0\rangle_{\text{out}}\neq 0$ in the *out* state, which means particle creation.

The total power radiated at late times across a sphere of radius r at late times with the stress-energy tensor $T_{\mu\nu}$ by the field ϕ which satisfies the field equation is

$$P=\frac{1}{24\pi}\left[\frac{3}{2}\left[\frac{G''}{G'}\right]^2-\frac{G'''}{G'}\right] , \tag{10}$$

since the average energy flux of particles radiated to $\mathcal{J}^+$ is given formally by $\langle 0|T^r{}_t|0\rangle$. One can also write Eq. (10) in terms of the function F and obtain the formula for the conformally coupled case.

Therefore the main issue for particle production by a gravitational field is to find the explicit form of the function $G(U)$ or $F(V)$; then, we can calculate the energy flux. The restriction for applying this method is that $|G(U)|$ must be small for large r at late times so that the outgoing wave packet arrives in the asymptotic region at late times.

B. Wormhole cases

1. Constant-velocity case

Consider the Minkowski spacetime from which two cylinders have been eliminated and identify the positions which have the same proper times, since the wormhole's throat is assumed to be an extremely short one. Let the event of the point where the left mouth meets the line between the centers of the two mouths be $\mathcal{P}=(0,0,0,0)$ and that of the point where the right mouth meets the line be $\mathcal{Q}=(0,D,0,0)$ when the wormhole forms. By moving the right mouth with constant velocity v, the world lines of $\mathcal{P}$ and $\mathcal{Q}$ are $(t,0,0,0)$ and $(\gamma t, D-\gamma vt,0,0)$, respectively, at later times $t>0$ (see Fig. 1).

This model should be made such that the ray in this wormhole can travel through the throat as if it were confined. Thus we must consider the head-on traversal case in which the positions of the mouths where the rays come in and go out are $x\neq 0$ and $y=z=0$ as in the two-dimensional model. If it is not so, rays which have entered one mouth can escape from the wormhole sometime after passing through the throat a finite number of times. The latter is not applicable to our problem since the rays must remain in the wormhole until they approach $\mathcal{H}$; then, they come out to $\mathcal{J}^+$. Just in this case, we can deal

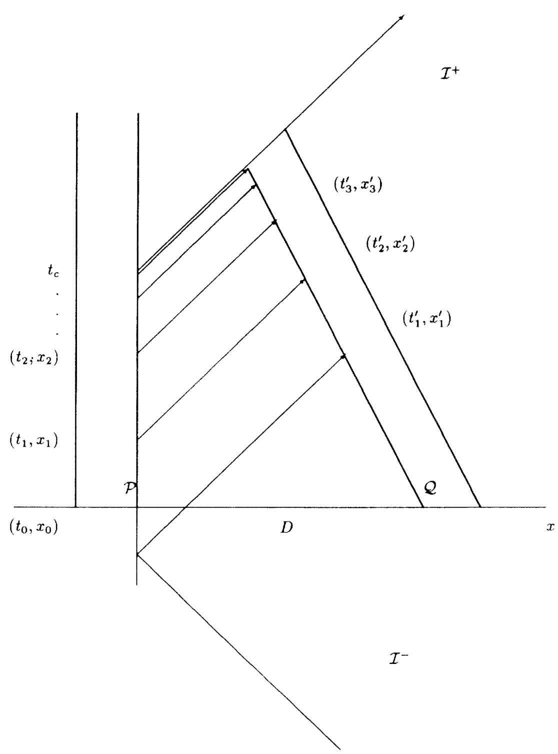

FIG. 1. Particle production when the right mouth moves with constant velocity v. The events (t_j, x_i) and $t_j', x_j')$ are identical. The right mouth disappears just before forming the Cauchy horizon at t_c.

with the particle-production problem near $\mathcal{H}$. Thus, based on the above arguments, a two-dimensional model is appropriate for particle production and Eq. (10) will be more useful than others.

As a model for particle production from the wormhole, one can consider the ray from $\mathcal{J}^-$ which reflects at the origin at the time t_0 before wormhole formation and goes into the right mouth. That is to say, the retarded time V should be in the range $-D \le V < 0$. The reason is that the wormhole forms at $t=0$ $(-D \le V)$ and that there is no wormhole, but only a reflection at the origin before that time $(V<0)$. The equality comes from the assumption that at wormhole formation the reflection does not exist, but the identification does.

Next, the ray comes out from the left mouth through the throat because of the identification, and it goes into the right mouth again. This process repeats again and again until the ray approaches $\mathcal{H}$. When it is near $\mathcal{H}$, the right mouth suddenly disappears in order for the ray to escape from the wormhole to $\mathcal{J}^+$ [12]. There, particle production will be detected. Thus the ray escapes to $\mathcal{J}^+$ for the time $U<t_c$, where t_c is the formation time of $\mathcal{H}$. Even if the right mouth can exist for $U>t_c$, after the formation of $\mathcal{H}$ the ray can be confined in the wormhole because of its time travel to the past or future. For $U>t_c$ there is no ray to $\mathcal{J}^+$. However, escape is possible for $U>t_c$ in the four-dimensional case, which does not admit head-on traversals with arbitrary incident angles in general. But we will not deal with these cases because the rays can time travel to the past or future for $U>t_c$, and this will not be strongly relevant to the stability of $\mathcal{H}$.

With the identification of the two proper times of the two mouths, then

$$\begin{aligned} t_i' &= \gamma t_i \ , \\ x_i' &= D - \gamma v t_i \ , \end{aligned} \tag{11}$$

where (t_i, x_i) is the position at which the ray meets the left mouth on the world line of the left mouth and (t_i', x_i') is the corresponding position for the right mouth. The $\gamma = (1-v^2/c^2)^{-1/2}$ is the well-known factor of special relativity. Of course, $x_i=0$ in our case because the reflection occurs at $\mathcal{P}=(t,0,0,0)$. The Cauchy horizon $\mathcal{H}$ is determined by the equation $-t_c+\gamma t_c = D-\gamma v t_c$; thus, $t_c = Df/(1-f)$. The relation of t_n with t_{n-1} is

$$t_{n-1} = \gamma(1+v)t_n - D \ , \tag{12}$$

and this relation shows

$$t = f^n t_0 + \frac{(1-f^n)fD}{1-f} \ . \tag{13}$$

We can check this expression for t_n as $n \to \infty$ such that it becomes t_c:

$$\lim_{n\to\infty} t_n = D\frac{f}{1-f} = t_c \ . \tag{14}$$

Since $x_i=0$, $U_n=t_n$, and $V=t_0$, the relation of U with V is

$$U_n - t_c = f^n(V-t_c) \tag{15}$$

or

$$V = G(U) = t_c - \frac{t_c - U}{f^n} \ . \tag{16}$$

One can find the restriction on n in terms of U from the relation of Eq. (16) so that the value of n is the smallest integer such that

$$1 - \frac{U}{t_c} > f^n \ . \tag{17}$$

If the wormhole forms in the *infinite past* and the ray $U=U_0$ has no real quanta at very early times, then the wormhole has no beginning. There is *no point* at which U reached the start of the wormhole. It means that $n=$const (independent of U) and that

$$G(U) = \frac{1}{f^n}U + t_c\left[1 - \frac{1}{f^n}\right] \ , \tag{18}$$

which is a linear function of U. Therefore $G' = 1/f^n =$const and $G''=G'''=\cdots=0$, which gives *no particle production* in this case.

If, however, the wormhole forms at a *finite time* (by topology change) rather than in the infinite past, then n is not a constant any more, but a function of U. Since $-D \le V = t_0 < 0$,

$$(n-1) < \frac{\ln(1-U/t_c)}{\ln f} < n \ . \tag{19}$$

Thus

$$\begin{aligned} &-D \le U < 0, \quad G(U) = U \ , \\ &0 \le U < fD, \quad G(U) = t_c\left[1 - \frac{1}{f}\right] + \frac{U}{f} \ , \\ &fD \le U < fD + f^2 D, \quad G(U) = t_c\left[1 - \frac{1}{f^2}\right] + \frac{U}{f^2} \ , \\ &\qquad \vdots \qquad\qquad\qquad \vdots \end{aligned} \tag{20}$$

The function $G(U)$ is a discrete function that has jumps at

$$X_n = D(f + f^2 + \cdots + f^m) = fD\frac{1-f^m}{1-f} \ ,$$

which means that $G', G'', \ldots$ do not exist at X_m (see Fig. 2). One cannot adopt the formula of Eq. (11) under this

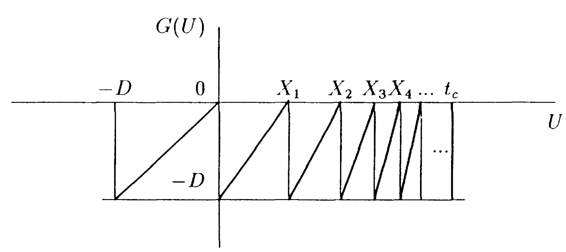

FIG. 2. Function $G(U)$ when the wormhole forms at finite time. There are jumps at $U=X_m$.

state of the function $G(U)$. To treat the smoothed function instead of this pathological function, one can modify the function $G(U)$ with the proper constant similar to the topologist's sine curve. It is tuned as

$$G(U) \simeq -\frac{D}{2}\cos\frac{2\pi\ln(1-U/t_c)}{\ln f} - \frac{D}{2} , \tag{21}$$

which is an oscillatory function between $-D$ and 0, and $G(X_m)=0$. The two most important things relating to the particle production for the function $G(U)$ are the contracting period of the repeating oblique line, which depends on the value of $\ln(1-U/t_c)/\ln f$, and the inclination of $G(U)$. As one model for approximating $G(U)$, we can assume a sinusoidal function with a contracting period such as Eq. (21). Though Eq. (21) is somewhat far from $G(U)$ near the origin, it approaches very near to $G(U)$ when $U \to t_c$. Of course, the exact $G(U)$ can be represented as the Fourier series of a sawtooth wave, but the basis function of the series is also of the type of Eq. (21), i.e., a sinusoidal function whose argument is $2\pi\ln(1-U/t_c)/\ln f$.

Using this modified function, the net flux (two-dimensional case) is

$$\begin{aligned} P(U) &= \frac{1}{24\pi}\frac{1}{(t_c-U)^2}\left[\frac{3}{2} - 2(1-\zeta)^2 - \zeta\coth(U) + \frac{3}{2}\zeta^2\cot^2 h(U)\right] \\ &\sim \frac{1}{(t_c-U)^2} , \end{aligned} \tag{22}$$

where $\zeta = -2\pi/\ln f$ and $h(U) = -\zeta\ln(1-U/t_c)$. The fourth term in the square brackets is greater than the third term for large $\zeta\coth(U)$, though ζ can be smaller than unity. Then the flux may be set as a positive one. The net flux is divergent when $U \to t_c$, and it looks as if an attempt to form the Cauchy horizon will lead to strong back reaction. The particle production may preclude a civilization from forming a wormhole at a finite time and moving one mouth to produce a Cauchy horizon.

However, the order of magnitude of the net energy flux is similar to the case of vacuum fluctuation [6]:

$$P \sim \frac{1}{(\Delta t)^2} = \frac{l_P m_P}{(\Delta t)^2} , \tag{23}$$

where $\Delta t = U - t_c$ is frame dependent. Thus, using the Kim-Thorne criterion under which the semiclassical theory breaks down, the net energy out of the mouth is

$$E \sim \int_{-\infty}^{t_c - l_P} dU \frac{l_P m_P}{(U-t_c)^2} \sim m_P , \tag{24}$$

which is very small, and the net energy out of the mouth can be *cut off* by quantum gravity on the same basis as vacuum fluctuations (see Sec. II). Hence the net energy out of the mouth has no significant effect on the wormhole.

2. Uniformly accelerated case

Using the same procedures as for the constant-velocity case, except the world line $\mathcal{Q}$ (see Fig. 3), the net flux from a wormhole, one of whose mouths moves with a uniform acceleration g (hyperbolic motion) [2], can be calculated using Eq. (10). With the coincidence of the two positions which have the same proper times,

$$\begin{aligned} t_j' &= \frac{1}{g}\sinh g t_j , \\ x_j' &= D - \frac{1}{g}(\cosh g t_j - 1) . \end{aligned} \tag{25}$$

The null curve at $t = t_c$, where the Cauchy horizon forms, shows

$$\frac{1}{g}\sinh g t_c - t_c = D - \frac{1}{g}(\cosh g t_c - 1) . \tag{26}$$

We can determine the time t_c such that

$$\exp(g t_c) - g t_c = gD + 1 . \tag{27}$$

The relation of t_n with t_{n-1} is

$$g t_n = \ln[g t_{n-1} + (1+gD)] . \tag{28}$$

Since Eq. (28) is very complicated, it is not easy to solve the equation and to obtain a rigorous expression for $t_n = G(U)$. Hence we now consider the extremely low acceleration case $g \ll 1$. Then, by neglecting higher-order

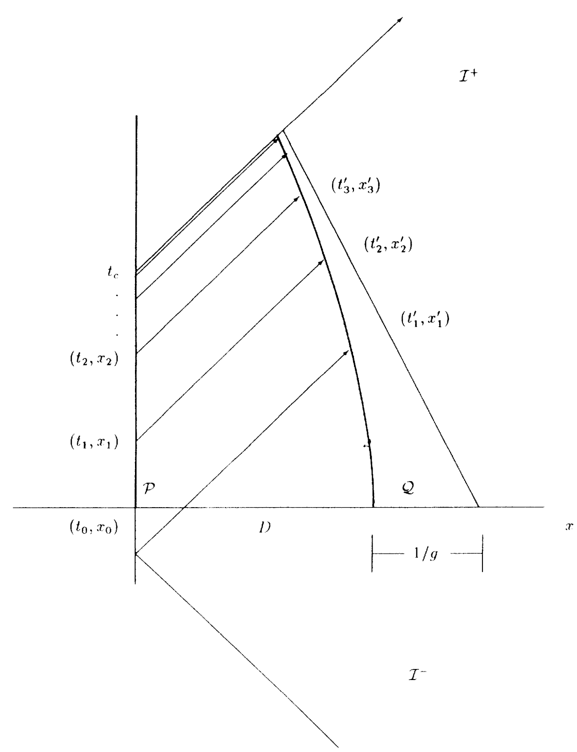

FIG. 3. Particle production when the right mouth moves with uniform acceleration g. The thick lines are the world lines of the two events $\mathcal{P}$ and $\mathcal{Q}$. The right oblique line is the asymptote of $\mathcal{Q}$.

small terms,

$$G(U)\simeq t_c+(U-t_c)(gU+gt_c+2)\prod_{i=1}^{n-1}\frac{gU_i+gt_c+2}{2^n}\ , \tag{29}$$

$$t_c\simeq\left[\frac{2D}{g}\right]^{1/2} . \tag{30}$$

When the wormhole forms at infinite past, n and U_i can be considered as constants independent of U. Thus

$$G(U)\simeq t_c+(U-t_c)(gU+gt_c+2)\Delta\ , \tag{31}$$

where Δ is a positive constant. Hence the net flux is

$$P(U)=\frac{1}{16\pi}\left[\frac{2}{(U+t_c+2/g)+(U-t_c)}\right]^2 \tag{32}$$

$$=\frac{1}{16\pi}\frac{1}{(U+1/g)^2}\ . \tag{33}$$

Compared with the constant-velocity case of Eq. (22), the term $U+t_c+2/g$ in the denominator of Eq. (32) is an additional term due to the acceleration g. This term will prevent the net flux from being divergent for $U\to t_c$. If $g\to 0$ (the constant-velocity case), then $P(U)\to 0$; i.e., no particle production occurs. Of course, this result is consistent with the case of a wormhole formed at infinitely early times and moving with constant velocity [$G(U)$ is a linear function].

When U approaches t_c (Cauchy horizon), the net flux of Eq. (33) becomes finite, and

$$\lim_{U\to t_c}P(U)=\frac{1}{16\pi}\left[\frac{g}{1+\sqrt{2gD}}\right]^2 . \tag{34}$$

Therefore there is a positive *finite flux* which means no instability in the formation of the Cauchy horizon for this uniformly accelerated motion. This result is as stable and as convergent as the former case of wormhole formation in the infinite past with constant velocity. However, because wormhole formation by topology change at a finite time yields much more complicated results for the case of accelerated motion than for the case of constant velocity, we will not treat the accelerated-motion case any further.

3. *Stimulated emission*

So far, spontaneous emission has been treated only in the vacuum *in* state which contains no particle. Thus , if the *in* state is not a vacuum, i.e., contains particles, there may be stimulated emission added to the spontaneous emission.

In general, the expectation value of the stress-energy tensor for an *in* state with particles is given as [11,13,14]

$$\langle {}^1n_{k_1},{}^2n_{k_2},\ldots|T_{\mu\nu}|\,{}^1n_{k_1},{}^2n_{k_2},\ldots\rangle$$
$$=\sum_k T_{\mu\nu}[F_k,F_k^*]+2\sum_i {}^i n T_{\mu\nu}[F_{k_i},F_{k_i}^*]\ , \tag{35}$$

where $T_{\mu\nu}[\phi,\phi]$ is defined as

$$T_{\mu\nu}[\phi,\phi]=\phi_{,\mu}\phi_{,\nu}-\tfrac{1}{2}g_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha}$$

and F is defined as Eq. (9). The n-particle state is

$$|{}^1n_{k_1},{}^2n_{k_2},\ldots,{}^jn_{k_j}\rangle$$
$$=({}^1n!{}^2n!\cdots{}^jn!)^{-1/2}(a_{k_1}^\dagger)^{{}^1n}(a_{k_2}^\dagger)^{{}^2n}\cdots(a_{k_j}^\dagger)^{{}^jn}|0\rangle\ . \tag{36}$$

The first term of Eq. (35) is just the spontaneous emission $\langle 0|T_{\mu\nu}|0\rangle$ [cf. Eq. (10)], and the second term represents the stimulated emission and scattering [13,14] due to initial particles.

Since the model in our case is two dimensional (l and m are meaningless), the quantum number which will be substituted for k_i in the massless case is ω, and using the same procedures as for the spontaneous-emission case [10] in Sec. III A, the additional term in Eq. (35) simply becomes

$$\int d\omega\, n(\omega)[F_{\omega,t}F_\omega^{*,r}+F_\omega^{,r}F_{\omega,t}^*]\ . \tag{37}$$

The sum on i in Eq. (35) is changed into the integral on ω in Eq. (37). The only difference from the previous spontaneous-emission case is $n(\omega)$ in the integrand. In Eq. (37), $n(\omega)$ depends on the particle distribution of the initial state. Thus, when $n(\omega)$ is constant for all ω, then the additional term simply becomes $n\langle 0|T_t^r|0\rangle$, n times the spontaneous emission, and seems to have no other special effect on our stability problem. For example, one particle state gives the particle creation of $2\langle 0|T_{\mu\nu}|0\rangle$. For an arbitrary distribution $n(\omega)$, one should calculate the Fourier transform of $\omega n(\omega)$ for a positive-definite ω as

$$\int_0^\infty d\omega\,\omega n(\omega)e^{i\omega\epsilon}=\hat{n}(\epsilon)\ . \tag{38}$$

With this $\hat{n}(\epsilon)$, one can set up an approximate formula for the energy flux P in terms of G and its derivatives in the limit of infinitesimally small ϵ, and

$$P=\frac{1}{4\pi}\lim_{\epsilon\to 0}[G'(U)G'(U+\epsilon)\hat{n}(\alpha)-\hat{n}(\epsilon)]\ , \tag{39}$$

where $\alpha=G(U+\epsilon)-G(U)$.

Stimulated emission can be applied to various particle distributions $n(\omega)$ of the initial state, but it seems that this stimulated-emission term does not affect seriously our result on particle production.

IV. CONCLUSIONS

In this paper the quantum stability of the Cauchy horizon when the wormhole can be described by the time machine model is investigated. The particle creation by the gravitational field of the wormhole is considered and the ability of the energy flux to break the Cauchy horizon is examined when the initial *in* state is a vacuum and when it is a nonvacuum state with particles. Three cases near the Cauchy horizon are examined: the constant-velocity, uniform-acceleration, and stimulated-emission cases. In the case of wormhole formation in the infinite past, the net flux is zero near the Cauchy horizon when the mouth moves with constant velocity, and it is finite when the mouth moves with uniform acceleration. Thus fluxes cannot be divergent as one approaches the Cauchy horizon. The Cauchy horizon is safe under quantum perturbations. In the case of wormhole formation in the finite past by topology change, the particle production can be divergent near the Cauchy horizon, but the amounts of the energy fluxes within a Planck scale of the Cauchy horizon are still small compared with the fluctuations of the quantum theory of gravity and are the same order of magnitude as vacuum fluctuations. Therefore, under the Kim-Thorne conjecture, the Cauchy horizon is also stable at the quantum level as well as at the classical level.

One can try such problems as particle creation for a ray after the Cauchy horizon. One can also extend this problem using other quantum natures which might also give other results for the range in which the quantum theory of gravity is or is not available.

ACKNOWLEDGMENTS

The author would like to thank Kip S. Thorne, Lenoard Parker, Lawrence Ford, Thomas Roman, and Edward J. Button for their ideas and encouragement. This paper was supported in part by the Non Directed Research Fund, Korea Research Foundation, 1990, and in part by the Center for Theoretical Physics of Seoul National University.

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