SO(14) flavor unification

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An SO(14) grand unification model is proposed to solve the flavor problem. There are four families of quarks and leptons, which are divided into three types by a new quantum number. The bare and renormalized Weinberg angle and the proton-decay mass scale are similar to those of the Georgi-Glashow SU(5) theory. Right-handed neutrinos of electrons and muons can be superheavy.

I. INTRODUCTION

The successes of the Weinberg-Salam SU(2) $\times$U(1) theory of weak and electromagnetic interactions[1] has spurred model-building activities of unifying weak, electromagnetic, and strong interactions by a single gauge group. Among the models proposed so far, the simplest and most successful ones are the Georgi-Glashow SU(5) theory[2] and its extension to the SO(10) scheme.[3] Especially the latter naturally synthesizes the Pati-Salam idea[4] of lepton number as a fourth color with the idea of grand unification first exhibited in SU(5).[2] With all their amazing features the SU(5) and the SO(10) theories have, however, an important shortcoming: They give no explanation for the existence of more than one family of quarks and leptons. This is called the flavor problem. The purpose of this Communication is to offer a new solution of the flavor problem while keeping the good features of the SO(10) [or SU(5)] theory.

II. DEFINITION OF THE MODEL

We begin with the SO(14) Clifford algebra by introducing notations. Γ matrices are defined by tensor products of Pauli and 2×2 identity matrices:

$$\Gamma_1=\sigma_1^{(7)},\quad \Gamma_{14}=\sigma_2\times I^{(6)} \ , \tag{1a}$$

$$\Gamma_{2k}=\sigma_1^{(7-k)}\times\sigma_2\times I^{(k-1)},\quad \Gamma_{2k+1}=\sigma_1^{(7-k)}\times\sigma_3\times I^{(k-1)} \ , \tag{1b}$$

where k runs from 1 to 6. The tensor products may be illustrated by two examples:

$$\sigma_1^{(2)}\equiv\sigma_1\times\sigma_1,\quad \sigma_1\times A=\begin{bmatrix}0 & A\\ A & 0\end{bmatrix} \ .$$

Among the 91 SO(14) generators

$$S_{k,l}=\frac{1}{2i}\Gamma_k\Gamma_l,\quad k\neq l \ ,$$

there are seven mutually commuting diagonal elements,

$$S_{2k-1,2k},\quad k=1,\ldots,7 \ . \tag{2}$$

The $SU(3)_c$ diagonal generators are $(S_{3,4}-S_{1,2})$ and $(S_{5,6}-S_{3,4})$, and the $SU(2)_W$ third component is

$$I_3=\tfrac{1}{2}(S_{9,10}-S_{7,8}) \ . \tag{3}$$

The crucial point of our theory lies in the definition of the electromagnetic charge operator Q given by

$$Q=\tfrac{1}{3}(S_{1,2}+S_{3,4}+S_{5,6})+S_{9,10} \ , \tag{4}$$

which combines with I_3 to form the electroweak hypercharge

$$Y\equiv Q-I_3=\tfrac{1}{3}(S_{1,2}+S_{3,4}+S_{5,6})+\tfrac{1}{2}(S_{7,8}+S_{9,10}) \ . \tag{5}$$

Another important point is a new operator C defined by

$$C=\tfrac{1}{2}(S_{13,14}-S_{11,12}) \ , \tag{6}$$

which will be used for family classification and other purposes.

III. FERMION CONTENTS

The 64-dimensional spinor representation $\zeta=(64)$ accomodates four families of quarks and leptons. Their $SU(3)_c\times SU(2)_W\times U(1)_Y\times U(1)_C$ contents are shown in Table I. All fermions are left-handed (LH). The SU(7) decompositions are also given for convenience.

From the table one can see the role of the new quantum number C. First it classfies the fermions by three types. ($C=0$: u,d,c,s), ($C=-\frac{1}{2}$: b,t), ($C=\frac{1}{2}$: h,f) for quarks and ($C=0$: e,μ,ν_1,ν_2),

TABLE I. Fermion classification. The SU(7) indices are included for convenience. a,b,c are color indices and i denotes weak index. C, classification charge; Y, hypercharge; Q, electromagnetic charge.

C	$SU(3)_c \times SU(2)_W$	Y	Q	Particles	SU(7)
0	(3,2)	$\frac{1}{6}$	$(\frac{2}{3},-\frac{1}{3})$	$(u,d)_L$	(ai)
				$(c,s)_L$	$(abi)^*$
	$(3^*,1)$	$-\frac{2}{3}$	$-\frac{2}{3}$	u_L^c	$(a)^*$
				c_L^c	$(a67)^*$
		$\frac{1}{3}$	$\frac{1}{3}$	d_L^c	(ab)
				s_L^c	$(a45)^*$
$-\frac{1}{2}$	(3, 1)	$\frac{2}{3}$	$\frac{2}{3}$	t_L	$(a7)$
		$-\frac{1}{3}$	$-\frac{1}{3}$	b_L	$(ab6)^*$
$\frac{1}{2}$	$(3^*,2)$	$-\frac{1}{6}$	$(-\frac{2}{3},\frac{1}{3})$	$(t^c,b^c)_L$	$(ai7)^*$
$\frac{1}{2}$	(3, 1)	$-\frac{2}{3}$	$\frac{2}{3}$	f_L	$(a6)$
		$-\frac{1}{3}$	$-\frac{1}{3}$	h_L	$(ab7)^*$
$-\frac{1}{2}$	$(3^*,2)$	$-\frac{1}{6}$	$(-\frac{2}{3},\frac{1}{3})$	$(f^c,h^c)_L$	$(ai6)^*$
0	(1,2)	$-\frac{1}{2}$	$(-1,0)$	$(e,\nu_1)_L$	$(i)^*$
				$(\mu,\nu_2)_L$	$(i67)^*$
	(1,1)	1	1	e_L^c	singlet
				μ_L^c	(67)
		0	0	ν_{1L}^c	(45)
				ν_{2L}^c	$(abc)^*$
$-\frac{1}{2}$	(1,1)	-1	-1	τ_L	$(6)^*$
		0	0	ν_{3L}	$(456)^*$
$\frac{1}{2}$	(1,2)	$\frac{1}{2}$	(1,0)	$(\tau^c,\nu_3^c)_L$	$(i6)$
$\frac{1}{2}$	(1,1)	-1	-1	ϵ_L	$(7)^*$
		0	0	ν_{4L}	$(457)^*$
$-\frac{1}{2}$	(1,2)	$\frac{1}{2}$	(1,0)	$(\epsilon^c,\nu_4^c)_L$	$(i7)$

$(C=-\frac{1}{2}$: $\tau,\nu_3)$, $(C=\frac{1}{2}$: $\epsilon,\nu_4)$ for leptons.

With conserved C, it is possible to suppress flavor-changing neutral currents.[5] The C-breaking mass scale M_C should be larger than but could be of the same order of M_W, weak symmetry-breaking scale.

Secondly, we note that the spinor representation is real and complex with respect to $G_W=SU(3)_c \times SU(2)_W \times U(1)_Y$ and $G_C=SU(3)_c \times SU(2)_W \times U(1)_Y \times U(1)_C$, respectively. Therefore, if $M_C \sim O(M_W)$ the fermions are protected from having superheavy masses. Effectively we follow the reasoning behind Georgi's second law of grand unification, "the representation of the LH fermions should be complex with respect to G_W."[6]

For the first point M_C is better to be safely larger than M_W, while the second point requires M_C as close as possible to M_W. In a specific realization of symmetry breaking we find that M_C is a free parameter, and can only be determined experimentally.

IV. CHARACTERISTICS OF THE MODEL

(1) There are no exotic fermions. (2) A single irreducible representation includes four families of quarks and leptons classified to three types by C. (3) The model has left-right symmetry, and lepton number can be viewed as a fourth color as proposed by Pati and Salam.[4] (4) There are two neutral leptons ν_1^c, ν_2^c which are singlet under G_C. These particles can have superheavy mass. Therefore, it is possible to make electron and muon neutrinos very light. (5) The model is anomaly free automatically and asymptotically free. (6) The bare Weinberg angle is $\sin^2\theta_W^0=\frac{3}{8}$. (7) The C-breaking mass scale must be $O(M_W)$.

V. SYMMETRY-BREAKING PATTERN

The following scheme of symmetry breaking is only one of many possibilities. The full stages are

$$SO(14) \underset{M_{14}}{\rightarrow} SO(10) \times SO(4)_W \underset{M_{10}}{\rightarrow} SO(6) \times SO(4)_C \times SO(4)_W \underset{M_P}{\rightarrow} G_C \underset{M_C}{\rightarrow} G_W \underset{M_W}{\rightarrow} G_e \ , \tag{7}$$

where $G_e = SU(3)_c \times U(1)_{em}$. By the renormalization-group method the effective coupling constants at M_W and renormalized Weinberg angles are related to the mass scales M_{14}, M_{10}, M_P as follows:

$$2\ln\frac{M_P}{M_W} = \eta + \tfrac{1}{6}(2\xi - \eta) \ , \tag{8}$$

$$3\ln\frac{M_{14}}{M_{10}} + \ln\frac{M_{10}}{M_P} = \tfrac{5}{24}(2\xi - \eta) \ , \tag{9}$$

where

$$\xi = \frac{6\pi}{11}\left(\frac{\sin^2\theta_W}{\alpha_{em}} - \frac{1}{\alpha_c}\right) \ ,$$
$$\eta = \frac{6\pi}{11}\frac{1}{\alpha_{em}}\left(\frac{3\cos^2\theta_W}{5} - \sin^2\theta_W\right) \ . \tag{10}$$

Before plunging into numerical examples, we note that

$$2\ln\frac{M_{GG}}{M_W} = \eta \ , \tag{11}$$

where we denote the Georgi-Glashow SU(5) mass scale by M_{GG}.[7] Then we have

$$M_P = M_{GG}\exp\left(\frac{2\xi - \eta}{12}\right) \ . \tag{12}$$

For $\alpha_{em} = 1/128.5$, $\sin^2\theta_W = 0.20$, $\alpha_c = 0.23$ (0.15), we find $(2\xi - \eta)/12 = 0.96$ (0.30). This gives

$$M_P = 2.61 M_{GG} \ (1.35 M_{GG}) \ ,$$

$$3\ln\frac{M_{14}}{M_{10}} + \ln\frac{M_{10}}{M_P} = 2.4 \ (0.75) \ .$$

In brief, for reasonable values of α_{em}, α_c, and $\sin^2\theta_W$, we have $2\xi - \eta \simeq 0$ and $M_{14} \simeq M_{10} \simeq M_P \simeq M_{GG} \sim 10^{15}$ GeV.

An important point is that M_C does not appear in (8) and (9), and therefore can only be determined by other aspects of the model, such as the neutral-current parameters.

VI. DISCUSSIONS

The present model has remarkable similarity to $SU(5)_{GG}$ in its phenomenological aspects, which is not *a priori* manifest at all. The bare Weinberg angle is the same and the renormalized angles are very close to each other. The proton lifetime could become of the same order and the gauge hierarchy problem is almost identical ($M_P/M_W \approx M_{GG}/M_W$) for both models. Even the magnetic-monopole problems are similarly present.

Group theoretically our model has more affinity to the Pati-Salam model.[4] At the M_P scale except for $SO(4)_C$ factor, the $SO(6) \times SO(4)_W$ is the left-right symmetry and the realization of lepton as a fourth color.

Proton-decay modes, *b*-meson decay characteristics, and neutral-current phenomenology are important subjects to clarify the structure of the theory. Mass of fermions, mixing angles, and other theoretical details depend upon the Higgs sector and Yukawa couplings, which deserve further study.

Finally, we note that the group structure at M_P has *C-W* symmetry $SO(4)_W \times SO(4)_C$. Especially the opeators I_3 and C have similar forms [see Eqs. (3) and (6)]. It is also possible to use $C^+ = \frac{1}{2}(S_{13,14} + S_{11,12})$ instead of C. But it is not so useful for particle assignment.

More precision experiments and/or higher-energy (M_W) neutral-current experiments[8] may reveal group structure beyond $SU(2)_W \times U(1)_Y$. Frequently ambidextrous nature at higher mass scales has been suggested.[9] Our theory suggests another symmetric nature, i.e., *C-W* symmetry, which is a new idea that could produce testable predictions because $M_C \sim O(M_W)$.

A detailed presentation of the systematic search that led to the particular model and other breaking patterns will be given elsewhere.

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