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RAPID COMMUNICATIONS

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Will geometric phases break the symmetry of time in quantum cosmology?

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We investigate the Wheeler-DeWitt equation for a quantum minisuperspace cosmological model with minimal scalar fields. The adiabatic basis and the nonadiabatic basis of the scalar field Hamiltonian induce mode-dependent gauge potentials to the matrix effective graviational Hamiltonian equation in the projected minisuperspace. In particular, in the nonadiabatic basis the matrix effective gravitational Hamiltonian equation decouples except for a quadratic term of the gauge potential at the classical level. For a closed loop in the projected minisuperspace of the expanding and subsequently recollapsing universe, the mode-dependent geometric phases may break the symmetry of the cosmological time defined along classical trajectories of mode-dependent wave functions.

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For the last decade there has been an active investigation of quantum minisuperspace cosmological models in either the canonical approach based on the Wheeler-DeWitt (WD) equation or the path integral approach. The Hartle-Hawking no-boundary proposal [1] and Vilenkin's proposal [2] for the quantum creation of the Universe are such an attempt to prescribe an initial condition and to describe the evolution of the Universe. Finding the wave functions for quantum minisuperspace cosmological models is a very difficult task largely due to the coupling nature of gravitational and matter fields, except for some simple models, and several methods have already been developed to obtain approximate wave functions. The complex contour integral [3] is such a typical method in the path integral approach and the semiclassical asymptotic expansion [4] of the WD in the Planck mass whose lowest expansion is nothing but the Einstein-Hamilton-Jacobi equation is such a typical method in the canonical approach. In the context of cosmology, one important and long-standing question has been how to explain the cosmological time emerges. It is generally agreed that the wave functions themselves contain all the information of the Universe, in spite of the disagreements on the interpretation, probability measure, square integrability, etc., of the wave functions. There was an attempt to define the cosmological time from the wave functions of the Universe [5].

In the macroscopic world there are three arrows (asymmetries) of time: the thermodynamic arrow of time as an increase of entropy, the psychological arrow of time as a distinction between the past and future, and the cosmological arrow of time as an expansion and subsequent recollapse of the Universe. At the classical level, there were the pros by Gold [6] and cons by Penrose [7] for the connection between the thermodynamic and cosmological arrows of time on whether they point in the same direction. At the quantum level, Hawking [8] proposed the connections among arrows of time based on the Hartle-Hawking no-boundary wave function. According to his argument the thermodynamic arrow of time should reverse in the recollapsing universe, because the wave function is *CPT* invariant. Hawking's argument was immediately refuted by Page [9] who pointed out that the *CPT* theorem does not exclude a time asymmetric wave function in which entropy increases monotonically throughout an expansion and subsequent recollapse of the universe. Hawking thereafter withdrew his argument. There were similar arguments [10].

In this Rapid Communication we shall consider an asymmetry of cosmological time in the context of the canonical approach to a quantum minisuperspace cosmological model for an inhomogeneous or anisotropic gravity minimally coupled to scalar fields. The WD equation has two different mass scales, one of which is the Planck mass scale for gravitational fields and the other is the mass scale for scalar fields. The gravitational fields behave as heavy nuclei and the scalar fields behave as light electrons of a molecular system, in spite of the fundamental disparity that quantum cosmological models obey the WD equation, a relativistic functional wave equation, whereas molecular systems obey the Schrödinger equation. Just as in molecular systems, the scalar fields induce mode-dependent gauge potentials (Berry connections) to the gravitational field background, which in turn gives rise to geometric phases [11]. It is proposed that in a quantum cosmological model the geometric phase may break the symmetry of the cosmological time defined along each classical trajectory along which the wave function is peaked when the universe recollapses to the same gravitational configuration. The same result was obtained in nonadiabatic and adiabatic path integral approaches [12].

The method that expands the WD equation by the adiabatic basis of the scalar field Hamiltonian to result in a matrix effect gravitational equation has already been introduced for the quantum Friedmann-Robertson-Walker cosmology minimally coupled to power-law scalar fields in Ref. [13]. The adiabatic basis was chosen real and nondegenerate so that the induced gauge potential consists only of an off-diagonal matrix, behaves as a coupling matrix among different modes of the matrix effective gravitational equation, and leads to the superadiabatic expansion of wave functions. This is the very reason why geometric phases were not discussed there. The non-Abelian gauge potential of the degenerate adiabatic basis coming from the level crossing of eigenvalues of the scalar field Hamiltonian was treated [14] and related to the nonunitarity of quantized fields of the Universe [15]. The main difference of this paper from others is that the adiabatic basis now may be complex to induce a nontrivial gauge potential and there is always a nontrivial gauge potential even for the real nonadiabatic basis.

We take the super-Hamiltonian constraint for a quantum minisuperspace cosmological model:

$$H(\zeta,\phi)=\left[\frac{1}{2m_p}F^{ab}(\zeta)\pi_a\pi_b+V_g(\zeta)+\frac{1}{2m_s}G^{kl}(\zeta)p_kp_l+V_s(\phi,\zeta)\right]=0\ , \quad (1)$$

where F^{ab} and G^{kl} are the inverse supermetrics of the supermetrics F_{ab} and G_{kl} on the extended minisuperspace of the three-geometry plus scalar fields with signatures $\eta_{ab}=(-1,1,\ldots,1)$ and $\delta_{kl}=(1,\ldots,1)$. We shall interpret below the scalar fields as a kind of fiber defined on the projected minisuperspace $\wp$ of the three-geometry only rather than as a part of the super-Hamiltonian on the extended minisuperspace. This interpretation allows us to regard the bases of orthonormal eigenstates of the scalar field Hamiltonian

$$H_{\text{matter}}(\phi,\zeta)=\frac{1}{2m_s}G^{kl}(\zeta)p_kp_l+V_s(\phi,\zeta) \quad (2)$$

as a fiber bundle of frames.

Choosing a basis (complete set) of orthonormal eigenstates of some Hermitian operator $h(\phi,\zeta)$ on $\wp$,

$$h(\phi,\zeta)|u_\lambda(\phi,\zeta)\rangle=\lambda(\zeta)|u_\lambda(\phi,\zeta)\rangle\ ,$$

$$\langle u_{\lambda'}(\phi,\zeta)|u_\lambda(\phi,\zeta)\rangle=\delta_{\lambda'\lambda}\ , \quad (3)$$

and denoting a column vector of $|u_\lambda(\phi,\zeta)\rangle$ by $\mathbf{U}_u(\phi,\zeta)$, we may define a gauge potential (Berry connection) [16,17]

$$A_{u,a}(\zeta)=i\mathbf{U}_u^*(\phi,\zeta)\frac{\partial}{\partial\zeta^a}\mathbf{U}_u^T(\phi,\zeta)\ , \quad (4)$$

where the asterisk and the superscript T denote dual and transpose operations. The gauge potential is a Hermitian matrix: $A_{u,a}^\dagger(\zeta)=A_{u,a}(\zeta)$. We may expand the wave function of the WD equation by the basis $\mathbf{U}_u(\phi,\zeta)$:

$$\Psi(\zeta,\phi)=\sum_\lambda\Psi_\lambda(\zeta)|u_\lambda(\phi,\zeta)\rangle=\mathbf{U}_u^T(\phi,\zeta)\mathbf{\Psi}(\zeta)\ . \quad (5)$$

It is then straightforward to see that

$$\frac{\partial}{\partial\zeta^a}\Psi(\zeta,\phi)=\mathbf{U}_u^T(\phi,\zeta)\left[\frac{\partial}{\partial\zeta^a}-iA_{u,a}(\zeta)\right]\mathbf{\Psi}(\zeta)\ . \quad (6)$$

The wave function $\Psi(\zeta,\phi)$ when expanded by $\mathbf{U}_u(\phi,\zeta)$ defines a fiber $\mathbf{\Psi}(\zeta)$ of the amplitudes of frame of the eigenstates on $\wp$, and so the action of $\partial/\partial\zeta^a$ on $\Psi(\zeta,\phi)$ induces a covariant derivative $D/D\zeta^a=\partial/\partial\zeta^a-iA_{u,a}(\zeta)$ acting only on $\mathbf{\Psi}(\zeta)$. Equation (6) implies that the operator π_a should be substituted by $\pi_a-A_{u,a}(\zeta)$ when acted on $\mathbf{\Psi}(\zeta)$.

Then the WD equation becomes a matrix effective gravitational Hamiltonian equation

$$H(\zeta,\phi)\Psi(\zeta,\phi)=\mathbf{U}^T(\phi,\zeta)\left[\frac{1}{2m_P}F^{ab}(\zeta)[\pi_a-A_{u,a}(\zeta)][\pi_b-A_{u,b}(\zeta)]+V_g(\zeta)+H_{\text{matter},u}(\zeta)\right]\mathbf{\Psi}(\zeta)=0\ , \quad (7)$$

where

$$H_{\text{matter},u}(\zeta)=\mathbf{U}_u^*(\phi,\zeta)H_{\text{matter}}(\phi,\zeta)\mathbf{U}_u^T(\phi,\zeta)\ , \tag{8}$$

is a back reaction of the scalar fields to the gravitational fields. Notice that $H_{\text{matter},u}(\zeta)$ is not necessarily a diagonal matrix.

If two bases are related by a unitary transformation

$$\begin{aligned}\mathbf{U}_u(\phi,\zeta)&=S^T(\zeta)\mathbf{U}_v(\phi,\zeta)\ ,\\ S^\dagger(\zeta)S(\zeta)&=S(\zeta)S^\dagger(\zeta)=I\ ,\end{aligned} \tag{9}$$

then we can show that $A_u(\zeta)$ transforms as a true gauge potential

$$A_{u,a}(\zeta)=S^\dagger(\zeta)A_{v,a}(\zeta)S(\zeta)+iS^\dagger(\zeta)\frac{\partial}{\partial\zeta^a}S(\zeta)\ , \tag{10}$$

where $A_v(\zeta)$ is a gauge potential defined by the basis $\mathbf{U}_v(\phi,\zeta)$

$$A_{v,a}(\zeta)=i\mathbf{U}_v^*(\phi,\zeta)\frac{\partial}{\partial\zeta^a}\mathbf{U}_v^T(\phi,\zeta)\ . \tag{11}$$

Each basis $\mathbf{U}_u(\phi,\zeta)$ on $\wp$ defines a unitary frame over $\wp$, i.e., a unitary frame bundle. Since there is an arbitrariness in the choice of basis, the whole set of complex bases defines an SU(N) principal bundle and that of real oriented bases defines as SO(N) principal bundle over $\wp$, N being the dimension of bases [18]. In general, N is ∞, and so there is an SU(∞) symmetry for complex bases and an SO(∞) symmetry for real bases [16].

Among the infinite set of bases there are two distinguished bases. The first choice is the basis that diagonalizes $H_{\text{matter}}(\phi,\zeta)$, i.e., the adiabatic basis of orthonormal eigenstates:

$$\begin{aligned}H_{\text{matter}}(\phi,\zeta)|w_\lambda(\phi,\zeta)\rangle&=\lambda(\zeta)|w_\lambda(\phi,\zeta)\rangle\ ,\\ \langle w_{\lambda'}(\phi,\zeta)|w_\lambda(\phi,\zeta)\rangle&=\delta_{\lambda'\lambda}\ .\end{aligned} \tag{12}$$

This basis, however, does not diagonalize the gauge potential

$$A_{w,a}(\zeta)=i\mathbf{U}_w^*(\phi,\zeta)\frac{\partial}{\partial\zeta^a}\mathbf{U}_w^T(\phi,\zeta)\ , \tag{13}$$

because there are off-diagonal entries

$$A_{w,a,\lambda'\lambda}(\zeta)=i\frac{\left\langle w_{\lambda'}(\phi,\zeta)\left|\frac{\partial}{\partial\zeta^a}H_{\text{matter}}(\phi,\zeta)\right|w_\lambda(\phi,\zeta)\right\rangle}{\lambda(\zeta)-\lambda'(\zeta)}\ , \tag{14}$$

for $\lambda'\neq\lambda$. For degenerate eigensubspaces $\lambda'=\lambda$, $A_{w,a,\lambda\lambda}(\zeta)$ cannot be determined by Eq. (14), whereas there is no diagonal matrix of the guage potential, $A_{w,a,\lambda\lambda}(\zeta)=0$, for nondegenerate real eigenstates. The WD equation is approximated by the adiabatic gravitational Hamiltonian equations

$$\left[\frac{1}{2m_p}F^{ab}(\zeta)[\pi_a-A_{w,a,\lambda\lambda}(\zeta)][\pi_b-A_{w,b,\lambda\lambda}(\zeta)]+V_g(\zeta)+H_{\text{matter},w,\lambda\lambda}(\zeta)\right]\Psi_\lambda(\zeta)\approx 0\ . \tag{15}$$

The second choice is the nonadiabatic basis for the Hermitian invariant of $H_{\text{matter}}(\phi,\zeta)$ defined by the invariant equation

$$\frac{dI_{\text{matter}}(\phi,\zeta)}{dt}=\frac{\partial I_{\text{matter}}(\phi,\zeta)}{\partial\zeta^a}\dot\zeta^a-i[I_{\text{matter}}(\phi,\zeta),H_{\text{matter}}(\phi,\zeta)]=0\ , \tag{16}$$

with the orthonormal eigenstates

$$\begin{aligned}I_{\text{matter}}(\phi,\zeta)|z_\lambda(\phi,\zeta)\rangle&=\lambda|z_\lambda(\phi,\zeta)\rangle\ ,\\ \langle z_{\lambda'}(\phi,\zeta)|z_\lambda(\phi,\zeta)\rangle&=\delta_{\lambda'\lambda}\ .\end{aligned} \tag{17}$$

The eigenvalues λ are independent of ζ. In this basis there is a remarkable decoupling theorem valid up to linear order of the gauge potential at the classical level. At the classical level the so-called matrix nonadiabatic gravitational Hamiltonian equation

$$\left[\frac{1}{2m_p}F^{ab}(\zeta)[\pi_a-A_{z,a}(\zeta)][\pi_b-A_{z,b}(\zeta)]+V_g(\zeta)+H_{\text{matter},z}(\zeta)\right]\Psi(\zeta)=0 \tag{18}$$

has the off-diagonal terms

$$\begin{aligned}-\frac{1}{2m_p}F_{ab}(\zeta)[-\pi_a A_{z,b}(\zeta)-\pi_b A_{z,a}(\zeta)+A_{z,a}(\zeta)A_{z,b}(\zeta)]\\ =-A_{z,a}(\zeta)\dot\zeta^a+\frac{1}{2m_p}F^{ab}(\zeta)A_{z,a}(\zeta)A_{z,b}(\zeta)\ ,\end{aligned} \tag{19}$$

where we used $\pi_a=m_pF_{ac}\dot\zeta^c$. Use a well-known result [19] of the invariant

$$A_{z,a,\lambda'\lambda}(\zeta)\dot\zeta^a=H_{\text{matter},z,\lambda'\lambda}(\zeta)\ , \tag{20}$$

for $\lambda'\neq\lambda$; then there is left only the quadratic term of the gauge potential of the order of

$$\frac{F^{ab}\left\langle\frac{\partial I_{\text{matter}}}{\partial\zeta^a}\right\rangle\left\langle\frac{\partial I_{\text{matter}}}{\partial\zeta^b}\right\rangle}{2m_p(\Delta\lambda)^2}\ , \tag{21}$$

where $\Delta\lambda$ is the minimum separation of eigenvalues. As a result of the above decoupling theorem, the WD equation is quite well approximated by the nonadiabatic gravitational Hamiltonian equations

$$\left[\frac{1}{2m_p}F^{ab}(\zeta)[\pi_a-A_{z,a,\lambda\lambda}(\zeta)][\pi_b-A_{z,b,\lambda\lambda}(\zeta)]+V_g(\zeta)+H_{\text{matter},z,\lambda\lambda}(\zeta)\right]\Psi_\lambda(\zeta)\approx 0\ . \tag{22}$$

Without the gauge potentials, the wave functions determined from the adiabatic gravitational Hamiltonian equations (15) or the nonadiabatic gravitational Hamiltonian equations (22) have the WKB approximations

$$\Psi_\lambda(\zeta)\approx\exp[iS_\lambda(\zeta)]\ , \tag{23}$$

which are peaked along classical trajectories in oscillatory regions. It is known that these classical trajectories are the solutions of the Einstein equation with scalar fields [20]. The cosmological time may be defined along each classical trajectory through a tangent vector [21]

$$\frac{\partial}{\partial t_\lambda}=\frac{F^{ab}}{m_p}\frac{\partial}{\partial \zeta^a}S_\lambda(\zeta)\frac{\partial}{\partial \zeta^b} . \tag{24}$$

The cosmological time is symmetric with respect to an expansion and subsequent recollapse, because the gravitational potential and the back reaction of the scalar fields are symmetric.

With the gauge potentials, however, the symmetry of the cosmological time may be broken, since if the Universe expands and subsequently recollapses, and so the classical trajectory does make a closed loop L in $\wp$, then the recollapsing wave functions $\Psi_\lambda(\zeta,\phi)$ differ from the expanding wave functions by the geometric phases given by the Wilson loop operators (holonomies)

$$W_L(A_w)=P\exp\left[i\oint_L A_{w,a}(\zeta)d\zeta^a\right] , \tag{25}$$

for the adiabatic basis and

$$W_L(A_z)=P\exp\left[i\oint_L A_{z,a}(\zeta)d\zeta^a\right] , \tag{26}$$

for the nonadiabatic basis. Thus individual wave functions in the expanding period

$$\Psi_\lambda^E(\zeta,\phi)=\Psi_\lambda(\zeta)|u_\lambda(\phi,\zeta)\rangle , \tag{27}$$

have the wave functions in the recollapsing period

$$\Psi_\lambda^R(\zeta,\phi)=W_L(A_u)\Psi_\lambda(\zeta)|u_\lambda(\phi,\zeta)\rangle , \tag{28}$$

where $u=w,z$ for the adiabatic and nonadiabatic basis, respectively. This means that the recollapsing wave functions need not be the same as the expanding wave functions even for the identical gravitational configuration. Then the cosmological time in the recollapsing period should be defined according to (24) now with respect to the total action including the geometric phases

$$\Psi_\lambda^R(\zeta)=W_L(A_u)\Psi_\lambda(\zeta)\approx\exp[iS_{\text{total},\lambda}(\zeta)] . \tag{29}$$

So we have the history (classical trajectory) dependent arrow of cosmological time which is not symmetric with respect to the expansion and subsequent recollapse.

As a simple model, we consider a scalar field with a variable mass squared

$$H_{\text{matter}}(\phi,\zeta)=\frac{1}{2m_s}p^2+\frac{1}{2}m_s\omega^2(\zeta)\phi^2 . \tag{30}$$

The Hamiltonian describes an inhomogeneous and anisotropic oscillator on $\wp$. Using the Lie algebra so(2,1) of a harmonic oscillator, we may find the invariant of the form

$$I_{\text{matter}}(\phi,\zeta)=g_1(\zeta)\frac{1}{2}p^2+g_2(\zeta)\frac{1}{2}(p\phi+\phi p)+g_3(\zeta)\frac{1}{2}\phi^2 . \tag{31}$$

There are many methods [22] to find the invariant for a time-dependent harmonic oscillator. The explicit form of the invariant (31) will not be concerned here. First, we transform canonically the momentum and position as

$$P=p+\frac{g_2(\zeta)}{g_1(\zeta)}\phi, \quad Q=\phi . \tag{32}$$

Then by introducing ζ-dependent creation and annihilation operators

$$\begin{aligned} P&=\frac{i}{\sqrt{2}}\left[\frac{\omega_0}{g_1(\zeta)}\right]^{1/2}[C^\dagger(\zeta)-C(\zeta)] ,\\ Q&=\frac{1}{\sqrt{2}}\left[\frac{g_1(\zeta)}{\omega_0}\right]^{1/2}[C^\dagger(\zeta)+C(\zeta)] , \end{aligned} \tag{33}$$

The Fock space can be constructed as the nonadiabatic basis of the invariant

$$I_{\text{matter}}(\phi,\zeta)|n,\zeta\rangle=\omega_0\left[n+\frac{1}{2}\right]|n,\zeta\rangle , \tag{34}$$

where the frequency of the invariant, $\omega_0=\sqrt{g_1(\zeta)g_3(\zeta)-g_2^2(\zeta)}$, is a constant as a consequence of the invariant equation (16). After some algebra, we find the induced gauge potential from the nonadiabatic basis

$$\begin{aligned} A_{z,a}(\zeta)=&\,i\beta_a(\zeta)\left[C^\dagger(\zeta)C(\zeta)+\frac{I}{2}\right]\\ &+\frac{i}{2}[\gamma_a(\zeta)C^2(\zeta)-\gamma_a^*(\zeta)C^{\dagger 2}(\zeta)] , \end{aligned} \tag{35}$$

where

$$\begin{aligned} \beta_a(\zeta)&=\frac{1}{2i}\frac{g_1(\zeta)}{\omega_0}\frac{\partial}{\partial\zeta^a}\left[\frac{g_2(\zeta)}{g_1(\zeta)}\right] ,\\ \gamma_a(\zeta)&=-\frac{1}{2}\frac{1}{g_1(\zeta)}\frac{\partial g_1(\zeta)}{\partial\zeta^a}+\beta_a(\zeta) . \end{aligned} \tag{36}$$

It is the diagonal part of the gauge potential that gives rise to the mode-dependent geometric phases $W_L[A(z)]\approx\exp\{-\oint_L\beta_a(\zeta)[C^\dagger(\zeta)C(\zeta)+I/2]d\zeta^a\}$, $\beta_a(\zeta)$ being pure imaginary.

In summary, the WD equation for the quantum minisuperspace cosmological model has the Planck mass scale for gravitational fields and the mass scale for scalar fields. In either the adiabatic or nonadiabatic basis the scalar fields give the diagonal back reaction and induce the diagonal mode-dependent gauge potential to the matrix effective gravitational Hamiltonian equation. Thus a geometric phase is gained from the gauge potential when the Universe traces a closed loop in the minisuperspace. The cosmological time defined along each classical trajectory near which the wave function *without the gauge potential* is peaked is symmetric. *With the gauge potential* the recollapsing wave functions differ from the expanding wave functions by the geometric phases when the expanding universe recollapses to the identical gravitational configuration. Our result supports Page's argument that the individual wave functions need not be CPT invariant. We proposed that the geometric phases be an origin of the asymmetry of the cosmological time. Furthermore, the correlation defined as the degree of constructive interference may be lost in the recollapsing period for an expanding wave function $\Psi^E(\zeta,\phi)=\sum_\lambda c_\lambda \Psi_\lambda(\zeta)|u_\lambda(\phi,\zeta)\rangle$ of a linear superposition of individual wave functions in phase giving constructive interference, because the recollapsing wave function

$$\Psi^R(\zeta,\phi)=\sum_\lambda c_\lambda W_L(A_u)\Psi_\lambda(\zeta)|u_\lambda(\phi,\zeta)\rangle$$

may be out of phase due to the geometric phases to result in destructive interference. The quantum interference may have a fundamental effect on the relation between the cosmological arrow of time and the thermodynamic arrow of time.

After completion of this paper we learned that Hawking *et al.* [23] recently published a paper in which they argued that density perturbations starting small grow larger and become nonlinear as the Universe expands and recollapses, and give rise to a thermodynamic arrow of time that would not reverse even at the point of the maximum expansion and subsequent recollapse. Admitting that their argument may be right, our result is quite different from theirs in that we have ascribed an origin of time asymmetry to geometric phases resulting from an induced gauge potential of scalar fields when the universe expands and subsequently recollapses to form a closed loop in the projected minisuperspace for an inhomogeneous or anisotropic universe.

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