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Model independent tests of cosmic growth versus expansion

Arman Shafieloo,[1,2,3] Alex G. Kim,[4] and Eric V. Linder[2,4,5]

[1]*Asia Pacific Center for Theoretical Physics, Pohang, Gyeongbuk 790-784, Korea*
[2]*Institute for the Early Universe WCU, Ewha Womans University, Seoul 120-720, Korea*
[3]*Department of Physics, POSTECH, Pohang, Gyeongbuk 790-784, Korea*
[4]*Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*
[5]*Berkeley Center for Cosmological Physics, University of California, Berkeley, California 94720, USA*
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We use Gaussian processes to map the expansion history of the universe in a model-independent manner from the Union2.1 supernovae data and then apply these reconstructed results to solve for the growth history. By comparing this to Baryon Oscillation Spectroscopic Survey and WiggleZ large-scale structure data we examine whether growth is determined wholly by expansion, with the measured gravitational growth index testing gravity without assuming a model for dark energy. A further model-independent analysis looks for redshift-dependent deviations of growth from the general relativity solution without assuming the growth index form. Both approaches give results consistent with general relativity.

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I. INTRODUCTION

Cosmological surveys have advanced to provide precision determinations of the distance-redshift relation and, to a lesser extent, the growth-redshift relation for cosmic history at redshifts $z \lesssim 1$. These improvements allow not only the determination of cosmological parameters, but fundamental tests of the cosmological framework in a more model-independent manner as well. Rather than assuming a model with cold dark matter plus a cosmological constant (ΛCDM) or plus dark energy parametrized by a constant equation-of-state ratio w or time-varying $w(z) = w_0 + w_a z/(1+z)$, one might like to investigate the expansion history $H(z)$ and growth factor $D(z)$ or the growth rate $f(z) = d\ln D/d\ln(1+z)$ directly, with minimal assumptions.

Reconstruction of the expansion history, in terms of the inverse Hubble parameter $H^{-1}(z)$ or deceleration parameter $q(z) = -d\ln H^{-1}/d\ln(1+z) - 1$, can be carried out purely kinematically, without assuming a particular theory of gravity or field equations (i.e., Friedmann equations). Gaussian processes [1] prove to be an effective statistical technique for carrying out such a reconstruction from distance data, as done in Ref. [2].

The growth of matter density perturbations into large-scale structure, however, depends explicitly on the dynamics, i.e., the gravitational force law. Within general relativity (GR) (and pressureless matter being the only significantly clustering component), expansion and growth are locked together, either one determining the other. Given that recently growth data have advanced to cover a reasonable redshift range, $z \approx 0$–0.8, at $\sim 10\%$ precision, it is interesting to test whether this interrelation actually holds. We can enlarge the Gaussian process technique to do this in a model-independent manner (rather than assuming a dark energy parametrization), although somewhat less generally than the previous expansion history reconstruction in that we must separate out the matter density.

In Sec. II we briefly review the cosmographic reconstruction of the expansion history from distance data and describe the extraction of the growth history from large-scale structure data, in particular using redshift-space-distortion measurements. In Sec. III we carry out a likelihood analysis of the current data and derive confidence contours on the matter density and gravitational growth index. We outline future applications and conclude in Sec. IV.

II. GAUSSIAN PROCESS METHOD

A. From distances to expansion

Distance measurements play an essential role in our understanding of the history and contents of the universe. They have a linear relation to (through the integration of) the inverse Hubble parameter or expansion rate for a spatially flat Roberston-Walker universe, as we assume. This linearity is an important property for a Gaussian process (GP), since the derivative (or integral) of a GP is another GP, making error propagation particularly straightforward. Thus, for a luminosity distance

$$d_l(z) = (1+z)\eta(z) = (1+z)\int_0^z dz' H^{-1}(z'), \quad (1)$$

if we model d_l as a GP then the conformal distance η or inverse Hubble parameter H^{-1} is one as well.

Gaussian processes provide a robust statistical method for using stochastic data measured at certain points (redshifts) and reconstructing the full function (distance-redshift relation or inverse Hubble parameter) describing the underlying relation, complete with covariances and without assuming a specific model for the relation.

See Ref. [1] for a detailed explanation of their general application, and Refs. [2–5] for the specific application to dark energy and cosmology (also see Refs. [6,7], though they fix several aspects of the GP and distance model, and Ref. [8] for a genetic algorithm approach).

Given data $\mathbf{y}$ at a set of points Z we reconstruct the underlying function $\mathbf{f}$, or its derivatives, at any set of points Z_1. The probability distribution functions are Gaussians described by a mean function $\mathbf{m}(\mathbf{Z})$ and a covariance matrix $k(Z_i, Z_j)$:

$$\begin{bmatrix} \mathbf{y} \\ \mathbf{f} \\ \mathbf{f}' \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mathbf{m}(\mathbf{Z}) \\ \mathbf{m}(\mathbf{Z_1}) \\ \mathbf{m}'(\mathbf{Z_1}) \end{bmatrix}, \begin{bmatrix} \Sigma_{00}(Z,Z) & \Sigma_{00}(Z,Z_1) & \Sigma_{01}(Z,Z_1) \\ \Sigma_{00}(Z_1,Z) & \Sigma_{00}(Z_1,Z_1) & \Sigma_{01}(Z_1,Z_1) \\ \Sigma_{10}(Z_1,Z) & \Sigma_{10}(Z_1,Z_1) & \Sigma_{11}(Z_1,Z_1) \end{bmatrix}\right), \tag{2}$$

where

$$\Sigma_{\alpha\beta} = \frac{d^{(\alpha+\beta)}k(Z_i, Z_j)}{dz_i^\alpha dz_j^\beta}, \tag{3}$$

and a prime indicates d/dz.

The inferred mean and covariance of the functions are given by

$$\begin{bmatrix} \bar{\mathbf{f}} \\ \bar{\mathbf{f}}' \end{bmatrix} = \begin{bmatrix} \mathbf{m}(\mathbf{Z_1}) \\ \mathbf{m}'(\mathbf{Z_1}) \end{bmatrix} + \begin{bmatrix} \Sigma_{00}(Z_1, Z) \\ \Sigma_{10}(Z_1, Z) \end{bmatrix} \Sigma_{00}^{-1}(Z,Z)\mathbf{y}, \tag{4}$$

$$\begin{aligned} \mathrm{Cov}\left(\begin{bmatrix} \mathbf{f} \\ \mathbf{f}' \end{bmatrix}\right) &= \begin{bmatrix} \Sigma_{00}(Z_1,Z_1) & \Sigma_{01}(Z_1,Z_1) \\ \Sigma_{10}(Z_1,Z_1) & \Sigma_{11}(Z_1,Z_1) \end{bmatrix} \\ &\quad - \begin{bmatrix} \Sigma_{00}(Z_1,Z) \\ \Sigma_{10}(Z_1,Z) \end{bmatrix} \Sigma_{00}^{-1}(Z,Z) \\ &\quad \times [\Sigma_{00}(Z,Z_1), \Sigma_{01}(Z,Z_1)]. \end{aligned} \tag{5}$$

For the GP covariance function we use a common form, the squared exponential:

$$k(z, z') = \sigma_f^2 \exp\left(-\frac{|z-z'|^2}{2l^2}\right), \tag{6}$$

where σ_f defines the overall amplitude of the correlation and l measures the coherence length of the correlation. The parameters σ_f^2 and l are hyperparameters in the fit.

If $f(z)$ is the reconstructed distance then f' is the reconstructed inverse Hubble parameter. Within general relativity, H^{-1} also determines the linear growth history of large-scale structure. While the growth factor or growth rate will not be linear functions of H^{-1}, and so are not GPs themselves, the error propagation is still direct. The basic approach is that supernova (SN) distance data allow for a (model-independent) GP reconstruction of $H^{-1}(z)$, with its covariances between redshifts, as in Ref. [2], and then this can be propagated to predictions of growth. These can then be compared to growth data from galaxy redshift surveys.

B. From expansion to growth

The linear growth factor is difficult to measure directly, free from astrophysical effects such as galaxy bias. Weak gravitational lensing data, which does not involve galaxy bias, is not currently sufficiently accurate to be useful for the desired reconstruction. Therefore we use galaxy redshift survey measurements of the growth rate through redshift-space distortions, whose anisotropic angular dependence allows separation from galaxy bias.

Redshift-space distortions arise as follows. The matter density perturbations forming large-scale structure induce gravitational potential inhomogeneities, and these in turn give rise to motions of the matter, or peculiar velocities. These velocities add to the galaxy redshift due to cosmic expansion, causing an anisotropic observed density field in redshift space. Since the peculiar velocities are proportional to the growth rate $f = d\ln D/d\ln a$, where the scale factor $a = 1/(1+z)$, then these redshift-space distortions can be used as a cosmological probe [9]. In the linear perturbation limit, Ref. [10] showed that the observed (redshift-space) galaxy power spectrum is related to the isotropic real-space density power spectrum by

$$P^s(k, \mu) = (b + f\mu^2)^2 P^r(k), \tag{7}$$

where k is the wavemode of the density perturbation, μ is the cosine of its angle with respect to the line of sight, and b is the galaxy bias.

Since the power spectrum is proportional to the square of the mass fluctuation amplitude, $P^r(k,a) \propto \sigma_8^2(a) \propto D^2(a)$, then the redshift-space distortion observable is $f\sigma_8 \propto dD/d\ln a$ (see, e.g., Ref. [11]). Normalized to the present mass fluctuation amplitude $\sigma_{8,0}$, an excellent approximation to the cosmological and gravitational dependence of this quantity is

$$\phi(a) \equiv \frac{f\sigma_8}{\sigma_{8,0}} = \Omega_m(a)^\gamma e^{\int_a^1 d\ln a' [\Omega_m(a')^\gamma - 1]}, \tag{8}$$

where γ is a constant called the gravitational growth index [12]. For general relativity and ΛCDM, $\gamma = 0.55$. The gravitational growth index form has been shown to be accurate at the 0.1% level for a wide variety of dark energy and gravity models [12,13], so long as the gravitational strength remains scale-independent and no strong clustering of dark energy occurs.

This form immediately allows us to carry out a test of gravity without choosing a dark energy model or parametrizing the Hubble expansion, since

$$\Omega_m(a)^\gamma = [\Omega_{m,0}a^{-3}(H^{-2}/H_0^{-2})]^\gamma. \tag{9}$$

The normalization relative to today, i.e., the $\sigma_{8,0}$ in ϕ, ensures that there is no dependence on $H(z)$ for redshifts higher than the highest growth measurements (and hence distance data, since these extend further). We can thus use the model-independent GP reconstruction of $H^{-1}(a)/H_0^{-1}$ from the SN distance data, propagate it to predictions of the growth relation, and by comparing to the growth data fit for the matter density today $\Omega_{m,0}$ and γ, the latter testing gravity.

In a second approach, one can actually enhance the model independence by writing

$$\phi(a) = \phi_{\rm GR}(a) + \delta\phi(a), \tag{10}$$

where $\phi_{\rm GR}$ fixes $\gamma = 0.55$ as from general relativity, and use Gaussian processes to reconstruct the function $\delta\phi$ without assuming a functional form given by the growth index γ. (Mathematically, one uses the GR relation as the mean function in the GP and sees if the hyperparameter σ_f giving the amplitude of deviations is consistent with zero.) This allows for the exploration of a wider variety of extended gravity theories. We use both approaches in the next section.

III. RESULTS FROM DISTANCE AND GROWTH

We apply GP to the Union2.1 compilation of supernova distance data [14] and incorporate the $f\sigma_8(z)$ data from the Baryon Oscillation Spectroscopic Survey [15], SDSS DR7 [16], WiggleZ [17], 2dF [18], and 6dF [19] galaxy surveys. Note that one must be careful to use the growth data values derived without assuming a specific expansion model. Through a scan over the likelihood surface, marginalizing over hyperparameters, we can derive confidence contours for $\Omega_{m,0}$ and γ, or study the hyperparameters themselves.

Figure 1 shows the joint two-dimensional contours on $\Omega_{m,0}$-γ for two different values of $\sigma_{8,0}$. The results are consistent with the GR value of $\gamma = 0.55$, and the variation of $\sigma_{8,0}$ slides the contours in $\Omega_{m,0}$ with little effect on the probability distribution of γ.

The GP method, without assuming any dark energy model, indicates that γ can take values in a considerable range, though general relativity, i.e., $\gamma = 0.55$, is right near the peak of the likelihood. In particular, even when fixing $\Omega_{m,0} = 0.28$, say, values of γ as low as those characteristic of scalar-tensor theories such as $f(R)$ gravity [20] or as high as that of Dvali-Gabadadze-Porrati gravity [21], 0.42 and 0.68, respectively, are allowed at 95% CL.

Current data therefore does not have the leverage to look for more subtle redshift-dependent deviations from GR that might not be captured by γ, at least not without assuming a specific model. We quantify the reach of current data in two ways. Figure 2 shows the GP reconstructions of the growth rate as a function of redshift, with data from Baryon Oscillation Spectroscopic Survey, SDSS DR7, WiggleZ, 2dF, and 6dF overplotted. The light green band is composed of samples of reconstructions with $\Delta\chi^2 < 3$ relative to the best fit, when fixing $\gamma = 0.55$, while the dark red band allows γ to float. In both of these cases we have fixed $\sigma_{8,0} = 0.801$.

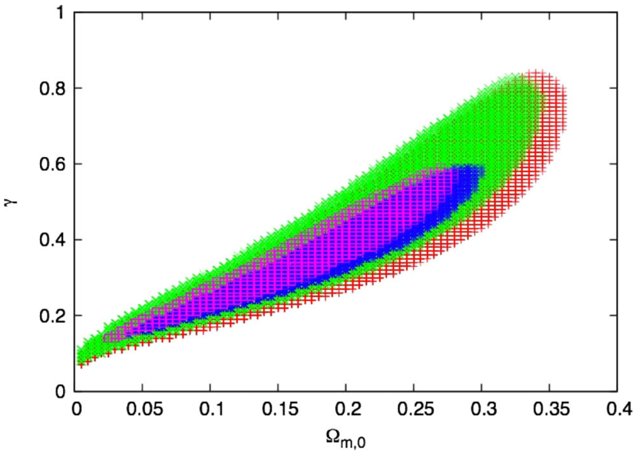

FIG. 1 (color online). 68 and 95% joint confidence limits on γ and $\Omega_{m,0}$ are shown derived without assuming a dark energy model, using current supernovae distance and galaxy clustering growth data. The left contour of each pair has $\sigma_{8,0} = 0.801$, the WMAP7 concordance value [22], and the right has $\sigma_{8,0} = 0.78$ to show the effect of a small shift.

The second method involves taking the even more model-independent approach of fitting for an arbitrary time-dependent correction to the general relativity growth rate, $\delta\phi(a) = \phi(a) - \phi_{\rm GR}(a)$. That is, we take GR to provide the mean function for the GP and let the data constrain the amplitude of the deviations given by the hyperparameter σ_f. Figure 3 shows the two-dimensional bound in the σ_f-$\Omega_{m,0}$ plane. The data prefers no deviation from general relativity, i.e., $\sigma_f = 0$ is within 68% CL.

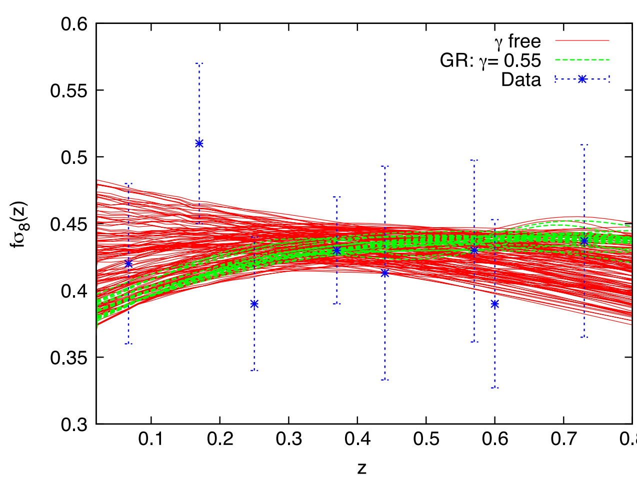

FIG. 2 (color online). Reconstruction of the growth rate $f\sigma_8(z)$ is shown for the case when fixing $\gamma = 0.55$ (light green curves) or allowing it to float (dark red curves). Current growth data is overplotted and we have fixed $\sigma_{8,0} = 0.801$.

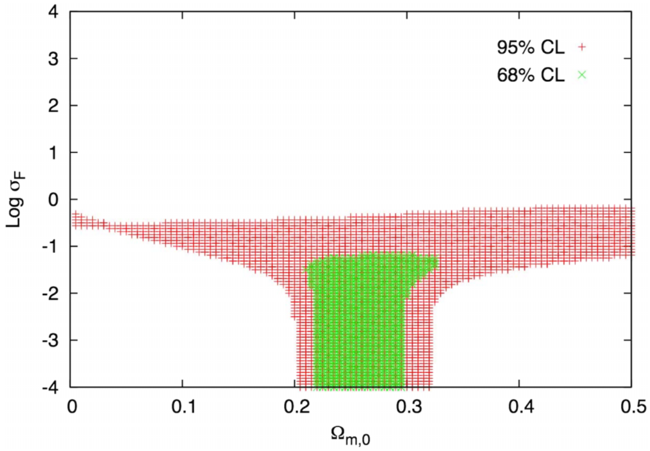

FIG. 3 (color online). Limits on the amplitude of the deviation σ_f from the GR growth relation (with model-independent dark energy) are shown in two-dimensional joint confidence contours with the present matter density.

IV. CONCLUSIONS

We have demonstrated a method to solve for the expansion and growth histories of the universe simultaneously, without assuming any model or parametrization for dark energy. This is a key test of the cosmological framework since within Einstein gravity one determines the other. Using the results we derived from Type Ia supernovae and large-scale structure data, we tested for and quantified deviations from general relativity in two ways.

Gaussian processes provide a useful statistical technique for such model-independent analyses. The GP reconstruction of the expansion history was juxtaposed with growth rate data from redshift-space distortion measurements in galaxy surveys to obtain probability distributions involving the gravitational growth index γ. The general relativity value was found to be a good fit, although due to uncertainty in the matter density $\Omega_{m,0}$ and to a lesser extent the mass fluctuation amplitude $\sigma_{8,0}$ a wide range of values is tenable within current constraints.

We further extended the model independence by looking for any deviation in the growth rate, without using the growth index formalism. Building on the GP reconstruction we tested the growth data for deviations from the prediction of general relativity as a function of redshift. The results are again consistent with standard gravity, tested without assuming any particular model of dark energy. Stringent exploration of the laws of gravity, however, requires more accurate future growth and distance data. Ongoing and future surveys will greatly enhance our ability to carry out a model-independent investigation of the cosmological framework.

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