



## Purely kinetic coupled gravity

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### ABSTRACT

Cosmic acceleration can be achieved not only with a sufficiently flat scalar field potential but through kinetic terms coupled to gravity. These derivative couplings impose a shift symmetry on the scalar field, aiding naturalness. We write the most general purely kinetic action not exceeding mass dimension 6 and obeying second order field equations. The result reduces to a simple form involving a coupling of the Einstein tensor with the kinetic term and can be interpreted as adding a new term to Galileon gravity in curved spacetime. We examine the cosmological implications of the effective dark energy and classify the dynamical attractor solutions, finding a quasistable loitering phase mimicking late time acceleration by a cosmological constant.

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### 1. Introduction

Modifications to the Einstein–Hilbert action for gravity can provide mechanisms for explaining cosmic acceleration, either the inflationary epoch in the early universe or the present dark energy era. These often give additional degrees of freedom which can be viewed as scalar fields. However, in the presence of a scalar field potential there is no particular need to modify the action; a canonical, minimally coupled scalar field can induce acceleration by itself. It is of interest, therefore, to consider purely kinetic, i.e. free field, theories where a noncanonical kinetic term or coupling to the metric is responsible for the physics of acceleration.

Recently, great interest has arisen in theories where the scalar field has a shift symmetry or a Galilean symmetry. These protect the theory against high energy radiative corrections, reducing the unnaturalness of rigidly translating a high energy Lagrangian into the same form at the drastically lower energies of the present universe. Such Galileon theories [1,2] indeed have noncanonical kinetic terms and sometimes coupling to metric quantities. Non-canonical kinetic models without coupling, such as *k*-essence and *k*-inflation theories [3,4], have also been successful at giving rise to cosmic acceleration and particularly inflationary scenarios.

One of the problems with adding a scalar degree of freedom, particularly with noncanonical kinetic terms and gravitational cou-

pling, is the appearance of higher order field equations that have an ill defined initial value problem and the presence of ghosts. Interesting early work on this was carried out by [5,6]. Here we revisit this from a different angle, coming to similar conclusions as a number of papers, e.g. [7–10], that a particular form of the coupling is essential. While these papers examine the impact on inflation, in this Letter we address late time cosmic acceleration, use a purely kinetic theory with shift symmetry, include matter fields, and calculate the dynamics of the effective dark energy. In particular we identify a “loitering” effective cosmological constant epoch and discuss extensions to the Galileon framework.

In Section 2 we consider the most general action involving derivative couplings and impose conditions on the order of the field equations and the mass dimension of the terms, giving a unique form. The equations of motion for the resulting action are derived in Section 3 and specialized to a cosmological background. We solve the equations and identify dynamical attractor solutions in Section 4 and discuss the cosmological implications for the cosmic expansion and acceleration.

We will adopt units such that  $\hbar = c = 1$ .

### 2. Action with derivative couplings

Scalar–tensor theories generally involve coupling of the field itself to the Ricci curvature appearing in the Einstein–Hilbert action, changing  $R/G_N$  to  $F(\phi)R$ , where  $G_N$  is Newton’s constant for the gravitational coupling and  $\phi$  is the scalar field value. Derivative couplings instead involve the covariant derivative  $\phi_{;\mu} \equiv \nabla_{\mu}\phi$

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coupled to functions of the metric  $g_{\mu\nu}$  in such a way to give a scalar quantity in the action.

Such derivative couplings arise in string theory and higher dimensional or massive gravity (see, e.g., [11,5,2,10] and references therein). One attractive motivation for considering derivative couplings is the idea that when the field freezes, e.g. in a de Sitter asymptotic state, then the couplings turn off and general relativity is restored. Such theories may in fact have an attractor toward an accelerating universe that looks like standard cosmological constant  $\Lambda$ CDM. Another motivation is that mentioned in the Introduction, that the theory is at least somewhat protected and natural. See [5,8] for some early work on kinetic coupling.

The most general action coupling functions of the metric with field derivatives, not exceeding mass dimension 6, is

$$S_c = \int d^4x \sqrt{-g} \left[ R \left( \frac{c_1}{M_p^2} \phi_\mu \phi_\nu g^{\mu\nu} + \frac{c_2}{M_p} \square \phi \right) + \frac{c_3}{M_p^2} R^{\mu\nu} \phi_\mu \phi_\nu + \frac{c_4}{M_p^2} R^{\alpha\beta\gamma\delta} \Phi_{\alpha\beta\gamma\delta} \right], \quad (1)$$

where the  $c_i$  are dimensionless coefficients. Note that this is in addition to the normal Einstein–Hilbert action and matter Lagrangian, i.e. we concentrate in this section on the difference. For simplicity we use the notation  $\phi_\mu = \nabla_\mu \phi$  and  $\Phi_{\alpha\beta\gamma\delta}$  is a function of the  $\phi_\mu$  discussed below.

One of the interesting aspects of this action is that it is somewhat reminiscent of a Gauss–Bonnet action,  $g_1 R \cdot R + g_2 R_{\mu\nu} R^{\mu\nu} + g_3 R_{abcd} R^{abcd}$ . However here, instead of coupling gravity to itself we are coupling gravity to the scalar field derivatives.

Terms that do not appear in Eq. (1) include ones like  $\phi \square \phi$  – this is equivalent to the  $c_1$  term up to a total derivative when avoiding more than two derivatives of any quantity, and in any case does not obey shift symmetry – and ones involving more than two products of  $\phi_\mu$  (or more than one  $\phi_{\mu\nu}$ ) since these will exceed mass dimension 6 and so be suppressed by higher powers of the Planck mass. Coupling the field to the Gauss–Bonnet invariant would be equivalent to a term involving the logarithm of  $\phi_\mu \phi^\mu$  [12].

The quantity  $\Phi_{\alpha\beta\gamma\delta}$  must have the same symmetry properties as  $R^{\alpha\beta\gamma\delta}$  in order to give a contribution. There is only one possibility not exceeding mass dimension 6:

$$\Phi_{\alpha\beta\gamma\delta} = \phi_\alpha \phi_\gamma g_{\beta\delta} - \phi_\beta \phi_\gamma g_{\alpha\delta} - \phi_\alpha \phi_\delta g_{\beta\gamma} + \phi_\beta \phi_\delta g_{\alpha\gamma}, \quad (2)$$

but it contracts against  $R^{\alpha\beta\gamma\delta}$  to give  $4\phi_\alpha \phi_\gamma R^{\alpha\gamma}$  and so it is absorbed into the  $c_3$  term.

Another term that would have the correct symmetry properties is

$$\Phi_{\alpha\beta\gamma\delta} = \phi_{\alpha\gamma} \phi_{\beta\delta} - \phi_{\beta\gamma} \phi_{\alpha\delta}, \quad (3)$$

or also a  $\Phi_{\alpha\beta\gamma\delta}$  involving terms of the kind  $\phi_\alpha \phi_\gamma \phi_{\beta\delta}$ . But the first kind of  $\Phi_{\alpha\beta\gamma\delta}$  has mass dimension 8 and the other one, besides being more than quadratic in the fields, has mass dimension 9. Even if one wanted to consider these terms, one would find that they would be forced to vanish due to causing more than second order field equations, without canceling against any other term.

So we have no  $c_4$  terms allowed into  $S_c$ .

To find out the conditions on the coefficients  $c_1, c_2, c_3$  that avoid the appearance of higher than second order derivatives we have to look at the variation of the action  $\delta S_c / \delta g^{\mu\nu}$ .

$$\delta S_c = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} g_{\mu\nu} \delta g^{\mu\nu} R \left( \frac{c_1}{M_p^2} \phi_\alpha \phi_\beta g^{\alpha\beta} + \frac{c_2}{M_p} \square \phi \right) \right.$$

$$\left. + \delta R \left( \frac{c_1}{M_p^2} \phi_\alpha \phi_\beta g^{\alpha\beta} + \frac{c_2}{M_p} \square \phi \right) + \frac{c_2}{M_p} R \delta(\phi_{\mu\nu}) g^{\mu\nu} + R \left( \frac{c_1}{M_p^2} \phi_\mu \phi_\nu + \frac{c_2}{M_p} \phi_{\mu\nu} \right) \delta g^{\mu\nu} - \frac{1}{2} \frac{c_3}{M_p^2} g_{\mu\nu} \delta g^{\mu\nu} R^{\alpha\beta} \phi_\alpha \phi_\beta + \frac{c_3}{M_p^2} \delta(R_{\alpha\beta}) \phi^\alpha \phi^\beta + \frac{c_3}{M_p^2} R_{\alpha\beta} \phi_\mu \phi_\nu \delta(g^{\mu\alpha} g^{\nu\beta}) \right]. \quad (4)$$

Carrying out the variation one finds that the terms that could potentially lead to higher order derivatives in the equations of motion are:

$$\sqrt{-g} \delta(R_{\mu\nu}) \left[ g^{\mu\nu} \left( \frac{c_1}{M_p^2} \phi_\alpha \phi_\beta g^{\alpha\beta} + \frac{c_2}{M_p} \square \phi \right) + \frac{c_3}{M_p^2} \phi^\mu \phi^\nu \right]. \quad (5)$$

Writing explicitly the variations and integrating by parts it can be shown that the terms proportional to  $c_2$  do not vanish and are the only ones producing fourth order derivatives in the equations of motion, so that they cannot cancel out with anything. Thus we have to ask that  $c_2 = 0$ .

As regards the other terms, proportional to  $c_1$  and  $c_3$ , they generate third order derivatives in the equations of motion, giving a contribution proportional to

$$\begin{aligned} & -\delta g_{\gamma\lambda} g^{\gamma\lambda} \left[ 2c_1 (\phi_{\mu\beta}^\beta \phi^\mu) + \frac{c_3}{2} (\phi_{\mu\beta}^\beta \phi^\mu + \phi_{\beta\mu}^\beta \phi^\mu) \right] \\ & + \delta g_{\gamma\lambda} [2c_1 \phi_\mu^{\gamma\lambda} \phi^\mu + c_3 \phi^{\lambda\gamma} \phi_\mu \phi^\mu] \\ & = -\delta g_{\gamma\lambda} g^{\gamma\lambda} \left[ (2c_1 + c_3) (\phi_{\mu\beta}^\beta \phi^\mu) + \frac{c_3}{2} g^{\beta\alpha} R^\rho_{\alpha\beta\mu} \phi_\rho \phi^\mu \right] \\ & + \delta g_{\gamma\lambda} [(2c_1 + c_3) \phi_\mu^{\gamma\lambda} \phi^\mu + c_3 g^{\lambda\alpha} g^{\gamma\beta} R^\rho_{\beta\alpha\mu} \phi_\rho \phi^\mu]. \quad (6) \end{aligned}$$

In order not to have third order derivatives in the equations of motion we require  $2c_1 + c_3 = 0$ . This condition has been recognized previously, e.g. [7–10], although generally starting from different approaches and with different applications.

Thus our conclusion is that to have no more than second order field equations one must have  $c_1 = -c_3/2$ ,  $c_2 = 0 = c_4$ . This puts the initial action equation (1) into the final form

$$\begin{aligned} S_c &= \frac{c_3}{M_p^2} \int d^4x \sqrt{-g} \left[ -\frac{1}{2} R g^{\mu\nu} + R^{\mu\nu} \right] \phi_\mu \phi_\nu \\ &= \frac{c_3}{M_p^2} \int d^4x \sqrt{-g} G^{\mu\nu} \phi_\mu \phi_\nu, \quad (7) \end{aligned}$$

where  $G^{\mu\nu}$  is the Einstein tensor.

The simplicity of this result is interesting. Such an action has been considered before, e.g. from disformal field theories [13] and Higgs inflation [10], and applied to inflation [9,14] and the cosmological constant [15], but we have come to it from a different direction.

This additional contribution to the action vanishes in flat space, so it does (trivially) obey Galilean symmetry in flat space as well as shift symmetry. In curved space, Galilean symmetry is broken as for the standard Galileon theories. Oddly, this term is not included in the usual treatments of Galileon gravity [1]. We suspect the reason is that in generalizing the flat spacetime Galileon theory to curved spacetime [2] the focus was on the  $\mathcal{L}_4$  and  $\mathcal{L}_5$  Galileon terms (of mass dimension 10 and 13 respectively), since only these could generate higher than second order derivatives in the field equations. Thus our term, which is like a covariantization of  $\mathcal{L}_2$  in curved spacetime, was not considered. It does appear to be an

interesting and healthy extension to general relativity, worthy of further exploration (also see, e.g., [16,17]).

Another view is to consider the field derivative as similar to a vector field, tying this to vector gravity theories. This analogy has been investigated by [18] for the case of compact objects rather than cosmology, and using a coupling to  $R^{\mu\nu}$  rather than  $G^{\mu\nu}$ .

### 3. Equations of motion

The final form of the action in the purely kinetic coupling theory is the sum of the canonical action and the coupling action,

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R - \frac{1}{2} g^{\mu\nu} \phi_\mu \phi_\nu + \frac{c_3}{M_p^2} G^{\mu\nu} \phi_\mu \phi_\nu - \mathcal{L}_m \right], \quad (8)$$

where  $M_p^2 = 1/(8\pi G)$  is the reduced Planck mass and  $\mathcal{L}_m$  is the matter Lagrangian.

#### 3.1. General considerations and implications

Note that the kinetic term in the Lagrangian was forced by the condition of second order field equations to a very special form, one with interesting implications. Let us write a general separable Lagrangian

$$\mathcal{L} = F(\phi_\mu \phi_\nu) E(g^{\mu\nu}), \quad (9)$$

where proper contraction of indices is understood. Then upon variation with respect to the field  $\phi$  we obtain

$$\begin{aligned} 0 &= \nabla_\mu \left( E \frac{\partial F}{\partial X} \nabla_\nu \phi \right) \\ &= (\nabla_\mu E) F_X \nabla_\nu \phi + E F_X \nabla_\mu \nabla_\nu \phi + E F_{XX} (\nabla_\mu \nabla_\mu \phi) \nabla_\nu \phi \nabla_\nu \phi. \end{aligned} \quad (10)$$

Here  $X = (1/2)g^{\mu\nu} \phi_\mu \phi_\nu$  is the canonical kinetic energy and  $F_X = dF/dX$ .

If there is no coupling, so  $E = 1$ , then we have the  $k$ -essence modified Klein–Gordon equation. See [3,4,19] for details of  $k$ -essence, its accelerating cosmological solutions, and the purely kinetic  $k$ -essence behavior.

However, if we have  $X$  (really  $\phi_\mu \phi_\nu$ ) appearing linearly in the action, as required from our mass dimension constraint, then  $F_X = 1$ ,  $F_{XX} = 0$  and the last term in the equation of motion vanishes. But in fact the first term also vanishes due to our requirement of second order field equations; the action was forced to a particular form of

$$E^{\mu\nu} = -\frac{1}{2} g^{\mu\nu} + \frac{c_3}{M_p^2} G^{\mu\nu}, \quad (11)$$

and both  $g^{\mu\nu}$  and  $G^{\mu\nu}$  are covariantly conserved so that  $\nabla_\mu E^{\mu\nu} = 0$ . Thus, the form of the Klein–Gordon equation is

$$E^{\mu\nu} \nabla_\mu \nabla_\nu \phi = 0. \quad (12)$$

This resembles the unmodified form of the Klein–Gordon equation if one use a deformed metric  $\tilde{g}^{\mu\nu} = g^{\mu\nu} - (2c_3/M_p^2)G^{\mu\nu}$  (such a transformation is sometimes called disformal, to distinguish it from conformal transformations, and has interesting implications [6,13]) to contract the indices (but not in the derivatives). One can turn this around and say that if one insists that the form of the field equation of motion appears “conserved” in this way under such a deformed metric then one is led to the purely kinetic coupling action of Eq. (8). That is, the special form involving

only the combination  $G^{\mu\nu}$  ensures that the deformed metric remains covariantly conserved. Also note that the kinetic coupling still induces modifications through the contribution to the expansion history  $H$  within the covariant derivatives.

A final point of interest is that another solution exists where  $E^{\mu\nu} = 0$ . This implies

$$g^{\mu\nu} - \frac{2c_3}{M_p^2} G^{\mu\nu} = 0, \quad (13)$$

which has a solution in de Sitter space. This determines the coupling constant

$$c_3 = -\frac{M_p^2}{6H^2} \quad (\text{de Sitter}). \quad (14)$$

Such a solution is quite interesting as it indicates that a possible fixed point exists for the theory giving an accelerating universe asymptotically approaching a cosmological constant state – without any explicit cosmological constant, or even potential, in the theory. We discuss this further after we derive the full equations of motion.

#### 3.2. Field equations

The dynamics of the system described by the action of Eq. (8) is found by minimizing the action with respect to variations of the metric and variations of the field. Variation with respect to the metric gives the spacetime dynamics, i.e. the expansion as a function of time, while variation with respect to the field gives the evolution equation for the field. Of course, since the action (8) couples the metric and the field, we will have a set of coupled differential equations, describing the interplay between the expansion evolution and the field dynamics. Some dynamical analysis, focused on de Sitter or superacceleration states, has been touched on in [7,20].

The variation of the action with respect to the metric can be found by evaluating  $\delta R$  etc., and integrating by parts as needed. Requiring that the variation vanishes,  $\delta S/\delta g^{\mu\nu} = 0$ , the dynamical equations for the metric are

$$\begin{aligned} \frac{M_p^2}{2} \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) - \frac{1}{2} \phi_\mu \phi_\nu \\ + \frac{1}{4} g_{\mu\nu} \phi_\alpha \phi_\beta g^{\alpha\beta} - \frac{c_3}{2M_p^2} [G_{\mu\nu} \phi_\alpha \phi_\beta g^{\alpha\beta} \\ + R \phi_\mu \phi_\nu + g_{\mu\nu} R^{\alpha\beta} \phi_\alpha \phi_\beta - 4R_{\nu\beta} \phi_\mu \phi_\alpha g^{\alpha\beta} \\ - 2\phi_{\lambda\mu} \phi^\lambda{}_\nu + 2R^\alpha{}_{\nu\mu\beta} \phi_\alpha \phi^\beta + 2\phi_{\nu\mu} \phi^\kappa{}_\kappa \\ + g_{\mu\nu} \phi_\alpha \phi_\beta g^{\alpha\beta} - g_{\mu\nu} \phi^\kappa g^{\sigma\tau} R^\rho{}_{\tau\sigma\kappa} \phi_\rho \\ - g_{\mu\nu} \phi^\kappa{}_\kappa \phi^\sigma{}_\sigma] = 0. \end{aligned} \quad (15)$$

In the following we will assume that the background geometry is described by the flat Robertson–Walker metric  $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$ , and we will derive the corrections to the Friedmann equations due to the presence of the coupled scalar field. The components of the equations for the metric dynamics that are of interest for the cosmic evolution are the ones with  $\mu = \nu = 0$  (leading to the modified first Friedmann equation) and  $\mu = \nu = i$  (leading to the modified second Friedmann equation).

To summarize the essential quantities:  $g_{00} = g^{00} = -1$ ,  $g_{ii} = a(t)^2 = (g^{ii})^{-1}$  and the nonzero Christoffel symbols are

$$\Gamma^i{}_{0i} = \frac{\dot{a}}{a} \equiv H(t), \quad \Gamma^0{}_{ii} = a\dot{a}. \quad (16)$$

The nonzero components of the Riemann tensor are

$$R^0_{i0i} = a\ddot{a}, \quad R^i_{0i0} = -\frac{\ddot{a}}{a} \equiv \dot{H} + H^2, \quad R^i_{jij} = \dot{a}^2 \quad (17)$$

while the ones for the Ricci tensor are

$$R_{00} = -3\frac{\ddot{a}}{a}, \quad R_{ii} = a\ddot{a} + 2\dot{a}^2 \quad (18)$$

and the Ricci scalar is given by

$$R = 6\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right]. \quad (19)$$

Taking all this into account we get the  $\mu = \nu = 0$  component of the metric evolution equation to be

$$\begin{aligned} & 3\frac{M_p^2}{2}\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{4}\dot{\phi}^2 - \frac{1}{4}a^{-2}\sum_i(\partial_i\phi)^2 \\ & - \frac{c_3}{2M_p^2}\left[-\left(\frac{\dot{a}}{a^2}\right)^2\sum_i(\partial_i\phi)^2 + 9\left(\frac{\dot{a}}{a}\right)^2\dot{\phi}^2\right. \\ & - a^{-4}\sum_{ij}(\partial_i\partial_j\phi)^2 - 4a^{-3}\dot{\phi}\sum_i\partial_i\partial_i\phi \\ & \left. + a^{-4}\left(\sum_i\partial_i\partial_i\phi\right)^2\right] = \frac{1}{2}\rho_m, \end{aligned} \quad (20)$$

where a dot denotes a standard time derivative,  $\partial_i$  denotes the derivative with respect to the comoving spatial coordinate  $i = \{x, y, z\}$ , and  $\rho_m$  is the energy density associated with the matter fields.

The  $\mu = \nu = i$  component of the equation for the metric evolution is

$$\begin{aligned} & \frac{M_p^2}{2}(-2\ddot{a}a - \dot{a}^2) - \frac{1}{2}\partial_i\phi\partial_i\phi + \frac{1}{4}a^2\left(-\dot{\phi}^2 + a^{-2}\sum_j(\partial_j\phi)^2\right) \\ & - \frac{c_3}{2M_p^2}\left[-\dot{a}^2\dot{\phi}^2 - 2a\ddot{a}\dot{\phi}^2 - \left(\frac{\dot{a}}{a}\right)^2\sum_j(\partial_j\phi)^2 + 2(\partial_i\dot{\phi})^2\right. \\ & + 2\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right)(\partial_i\phi)^2 - 4\partial_i\dot{\phi}\frac{\dot{a}}{a}\partial_i\phi - 2a^{-2}\sum_j(\partial_j\partial_i\phi)^2 \\ & - 2\frac{\dot{a}}{a}\dot{\phi}\partial_i\partial_i\phi - 2\ddot{\phi}\partial_i\partial_i\phi - 4\dot{\phi}\ddot{a}a + 2(\partial_i\partial_i\phi)a^{-2}\sum_j(\partial_j\partial_j\phi) \\ & - 2\sum_j\left[(\partial_j\dot{\phi})^2 - 2\partial_j\dot{\phi}\frac{\dot{a}}{a}\partial_j\phi\right] + a^{-2}\sum_{j,k}(\partial_j\partial_k\phi)^2 \\ & \left. + \left(2\frac{\dot{a}}{a}\dot{\phi} + 2\ddot{\phi}\right)\sum_j\partial_j\partial_j\phi - a^{-2}\left(\sum_j\partial_j\partial_j\phi\right)^2\right] \\ & = \frac{a^2P_m}{2}, \end{aligned} \quad (21)$$

where  $P_m$  is the pressure of the matter fields.

The equation for the field evolution is given by the variation of the action with respect to the field, requiring that  $\delta S/\delta\phi = 0$ . So from the action of Eq. (8), and using  $\nabla_\mu G^{\mu\nu} = 0$ , we get

$$-g^{\mu\nu}\nabla_\mu\nabla_\nu\phi + \frac{2c_3}{M_p^2}G^{\mu\nu}\nabla_\mu\nabla_\nu\phi = 0, \quad (22)$$

as predicted in Eqs. (11)–(12).

Specializing to a scalar field smooth on the Hubble scale or below, as usually considered, we can neglect the spatial derivatives

relative to the time derivatives and obtain the following modified Friedmann equations:

$$\frac{3M_p^2}{2}H^2 = \frac{1}{2}\rho_m + \frac{1}{4}\dot{\phi}^2 + \frac{9c_3}{2M_p^2}H^2\dot{\phi}^2, \quad (23)$$

$$\begin{aligned} M_p^2\left(\frac{\ddot{a}}{a} + \frac{1}{2}H^2\right) &= -\frac{1}{2}P_m - \frac{1}{4}\dot{\phi}^2 \\ &+ \frac{c_3}{2M_p^2}\left(H^2\dot{\phi}^2 + 2\frac{\ddot{a}}{a}\dot{\phi}^2 + 4H\dot{\phi}\ddot{\phi}\right). \end{aligned} \quad (24)$$

In addition there is the equation of motion for the field  $\phi$ , the modified Klein–Gordon equation

$$0 = \ddot{\phi}\left(1 + \frac{6c_3}{M_p^2}H^2\right) + 3H\dot{\phi}\left[1 + \frac{4c_3}{M_p^2}\left(\frac{\ddot{a}}{a} + \frac{1}{2}H^2\right)\right], \quad (25)$$

although as usual only two of the three equations are independent.

#### 4. Cosmological dynamics and attractors

Given the evolution equations we can now examine the cosmological dynamics and look for fixed point solutions or attractors for the dynamics insensitive to initial conditions. The first Friedmann equation has a contribution modified from that of a canonical scalar field by a factor proportional to  $C = c_3H^2/M_p^2$ . So  $C$  will be a key parameter. We can in fact write the dynamics as an autonomous system of coupled first order differential equations

$$\frac{dC}{dN} = -C(1 + w\Omega_\phi), \quad (26)$$

$$\frac{d\Omega_\phi}{dN} = -3w\Omega_\phi(1 - \Omega_\phi), \quad (27)$$

using the definition of the total equation of state  $w_{\text{tot}} \equiv -1 - (1/3)d\ln H^2/dN = w\Omega_\phi$ , where the second equality holds for the barotropic fluid having zero pressure, as for matter. We will consider a matter plus scalar field universe for the following calculations.

Examining the system of dynamical equations, we can immediately see several possible solutions of interest. There may be fixed points at 1)  $\Omega_\phi = 0$ ,  $C = 0$ , 2)  $w = 0$ ,  $C = 0$ , 3)  $\Omega_\phi = 1$ ,  $C = 0$ , and 4)  $\Omega_\phi = 1$ ,  $w = -1$ . We must examine the equations further to assess which are physical and stable, but the last in particular is of interest since it gives a cosmological constant solution despite having only kinetic terms in the action. Recall there is no potential at all.

The first Friedmann equation acts as a constraint, which we can write as

$$1 = \Omega_m + \frac{\dot{\phi}^2}{6M_p^2H^2} + \frac{3c_3\dot{\phi}^2}{M_p^4}. \quad (28)$$

The last two terms define the effective dark energy density

$$\Omega_\phi = \frac{\dot{\phi}^2}{6M_p^2H^2}(1 + 18C). \quad (29)$$

To prevent ghosts we insist that the kinetic energy is nonnegative, imposing the requirement

$$C \geq -1/18. \quad (30)$$

Using the second modified Friedmann equation we can define the effective pressure of the dark energy and hence its equation of state ratio  $w = P_\phi/\rho_\phi$ . The result is

$$w = \frac{1 + 30C + (P_m/\rho_\phi)6\Omega_\phi C(1 - 18C)}{1 + (24 - 6\Omega_\phi)C + 108(1 + \Omega_\phi)C^2}, \quad (31)$$

and as stated above we will take the barotropic pressure  $P_m = 0$ . We can investigate when it is possible to have  $w = -1$ , say. There are two solutions, with  $C = -1/6$  as mentioned earlier in Eq. (14) and  $C = -1/18$ . The first violates the condition in Eq. (30) while the second one saturates it.

Now let us return to examination of the four possible fixed points. The first one corresponds to both the noncanonical nature, i.e. the kinetic coupling, and the dark energy as a whole fading away. (The dark energy fading follows from the kinetic coupling vanishing since canonical kinetic energy redshifts away as  $a^{-6}$ .) The second possible fixed point does not actually occur because  $C = 0$  in Eq. (31) implies  $w = +1$ , i.e. a (canonical) kinetic dominated evolution, not  $w = 0$ . The third possibility also gives  $w = 1$ , in which case  $\Omega_\phi = 1$  is only valid transiently unless there is no other component, so this is expected to be an unstable point.

Case 4 has two branches: we have seen that the  $C = -1/6$  root is invalid, leaving the  $C = -1/18$  root. When  $C = -1/18$ , saturating the ghost protection condition, then by Eq. (29) the energy density  $\Omega_\phi = 0$  not 1, showing that this is not a fixed point. However, if while  $C$  is near  $-1/18$  simultaneously  $\phi^2$  gets large, then a finite  $\Omega_\phi$  is possible. This indeed turns out to be an unstable critical point, a saddle point with interesting quasi-attractor properties. Thus we expect two main solutions: the dark energy fades away along with the kinetic coupling, but there may be an intermediate “loitering” phase with cosmological constant behavior dominating the universe.

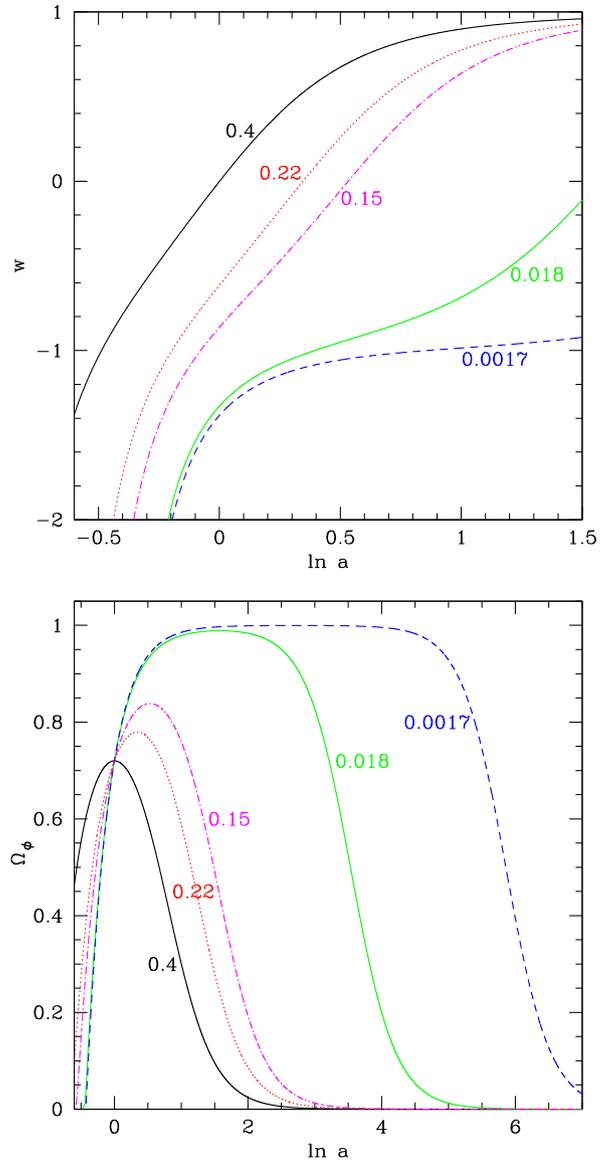
Because of Eq. (31) giving the relation  $w(\Omega_\phi, C)$ , and the first order nature of the coupled dynamical equations, if we specify  $\Omega_\phi$  and  $w$ , say, at some particular time then the evolutionary trajectory is determined for all times. We can define the present by  $\Omega_\phi = 0.72$  and generate a one parameter family of curves corresponding to different  $w_0$ , the value of the equation of state today, or different  $C_0$ .

Fig. 1 shows the dynamical evolution of the purely kinetic coupled gravity cosmology for various values for  $C_0$ . We define the fractional deviation  $\delta = (C_0 - C_*)/C_*$  away from the critical value  $C_* = -1/18$  as the key parameter. As this gets small, the evolution of  $w$  begins to loiter around the cosmological constant value  $w = -1$ . The loitering can last for several e-folds of expansion as  $\delta$  gets very small. This certainly involves a fine tuning, but so does a true cosmological constant.

In the past the dark energy fades away very quickly, as  $w \rightarrow -\infty$ . This means that all early universe (in fact  $z \gtrsim 1$ ) cosmology is unchanged from a standard matter (or earlier radiation) dominated universe. In the asymptotic future, dark energy fades away as well, restoring a matter dominated universe. Thus cosmic acceleration is a transient phenomenon in the purely kinetic coupled gravity scenario.

We see that for moderate values of  $\delta$ , the evolution of  $w$  is rapid and dark energy domination lasts for less than two e-folds of expansion; indeed for much of that time the dark energy does not act in an accelerating manner. Large values of  $\delta > 0.4$  are not permitted because the universe never reaches a sufficient dark energy fraction  $\Omega_\phi = 0.72$  observed today. However as  $\delta$  gets small the quasistable attractor to  $w = -1$  gets stronger and simultaneously the dark energy almost completely dominates the energy density for several e-folds. We can thus get a period of cosmological constant dominated behavior, despite the absence of any cosmological constant (or potential of any sort), with only kinetic coupling terms.

Fig. 2 shows the  $\Omega_\phi$ - $w$  phase space, where the time coordinate (scale factor) runs along the trajectories. We clearly see the cases

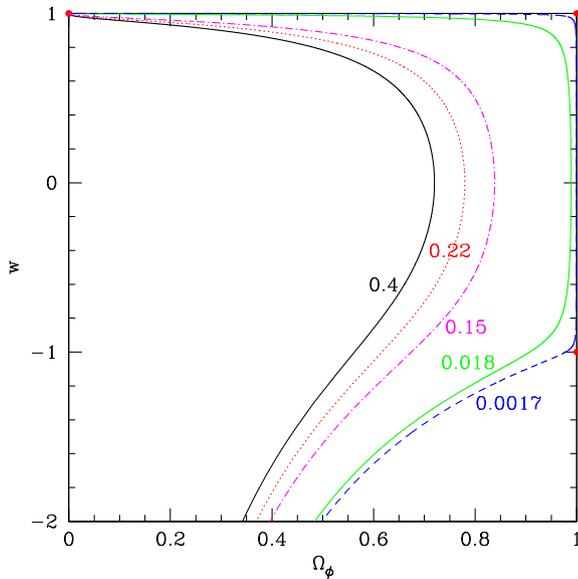


**Fig. 1.** In the purely kinetic coupled gravity cosmology the evolution of the dark energy equation of state  $w$  (top panel) is rapid, becoming relevant only at  $z < 1$ . From very negative values it then asymptotically approaches  $w = +1$ , though it can loiter near  $w = -1$  for several e-folds of expansion if  $\delta = (C_0 - C_*)/C_*$  is small enough. Curves are labeled with  $\delta$ . During the loitering phase, the fractional dark energy density  $\Omega_\phi$  (bottom panel) can approach 1, giving a cosmological constant dominated universe without a cosmological constant.

with the small values of  $\delta$  approach the quasistable fourth critical point at  $(1, -1)$  but then depart (to see the length of the loitering one must look at Fig. 1). These trajectories then move toward the unstable third critical point at  $(1, 1)$  but again are driven away, this time toward the stable first critical point at  $(0, 1)$  where dark energy has faded away. For large enough  $\delta$  though the trajectories immediately go to the stable critical point.

## 5. Conclusions

We have considered the implications for the expansion history of the universe of modifying the standard general relativity action at the lowest possible order in the Planck mass (or equivalently in the Newton constant) through the addition of a coupling between functions of the metric and kinetic terms of a free scalar field. This uses a purely kinetic approach, without any potential



**Fig. 2.** In the phase space diagram for  $\Omega_\phi$ - $w$ , one can clearly see the behavior of the trajectories near the three physical critical points (solid red dots). The two critical points with  $\Omega_\phi = 1$  are unstable, but trajectories corresponding to  $\delta = (C_0 - C_*)/C_* \ll 1$  (curves are labeled with  $\delta$ ) can approach them closely, in fact loitering several e-folds near  $w = -1$ ,  $\Omega_\phi = 1$ . Eventually, however, the dynamics leads to the stable attractor at  $\Omega_\phi = 0$ ,  $w = 1$  and cosmic acceleration ceases and dark energy fades away. Note that curves with  $\delta > 0.4$  never achieve  $\Omega_\phi = 0.72$  and so do not describe our universe.

that would suffer high energy quantum corrections, and also obeys a shift symmetry.

Starting from the most general form for the coupling we demonstrate that only one particular combination of all the possible terms is admissible to give no higher than second order field equations, if we impose the additional requirement of not having more than two derivatives of the field in the dynamical equations. The resulting theory is characterized by only one parameter setting the strength of the coupling between the metric and the derivatives of the scalar field, and so has no more parameters than a cosmological constant model.

The simple term involving the Einstein tensor and field kinetic term adds to Galileon gravity, often characteristic of higher dimension gravity theories, and also has ties to vector or Einstein aether gravity and disformal theories with a deformed metric.

We have studied the evolution of the universe under the dynamics described by this action without a cosmological constant. The range of variation of the free parameter of the theory is con-

strained on one side by the requirement of having positive kinetic energy of the field, and on the other side by the necessity of producing the actually observed value of the dark energy density fraction today. The fine tuning is no worse than that of the cosmological constant. Despite the absence of a potential, a wide variety of dark energy equations of state is possible, ranging from phantom to stiff behaviors. We delineate and show the phase space evolution of the dynamics.

Most interestingly, we find that for certain values of the one parameter the universe goes through a quasistable loitering phase that mimics a cosmological constant, without the necessity of adding any potential for the field. At high redshift the universe has standard matter (and radiation) dominated behavior before the effective dark energy dominates. The loitering cosmological constant phase can last several e-folds before the effective dark energy fades away leaving the universe once again matter dominated.

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