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Entropic accelerating universe

Damien A. Easson [a,b,c,*], Paul H. Frampton [a,d], George F. Smoot [a,e,f,g,h]

[a] Institute for the Physics and Mathematics of the Universe, University of Tokyo, Kashiwa, Chiba 277-8568, Japan
[b] Department of Physics & School of Earth and Space Exploration & Beyond Center, Arizona State University, Phoenix, AZ 85287-1504, USA
[c] Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106-4030, USA
[d] Department of Physics and Astronomy, University of North Carolina, Chapel Hill, NC 27599, USA
[e] Lawrence Berkeley National Lab, 1 Cyclotron Road, Berkeley, CA 94720, USA
[f] Physics Department, University of California, Berkeley, CA 94720, USA
[g] Institute for the Early Universe, Ewha Womans University & Advanced Academy, Seoul, Republic of Korea
[h] Chaire Blaise Pascale, Universite Paris Denis Diderot, Paris, France

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ABSTRACT

To accommodate the observed accelerated expansion of the universe, one popular idea is to invoke a driving term in the Friedmann–Lemaître equation of dark energy which must then comprise 70% of the present cosmological energy density. We propose an alternative interpretation which takes into account the entropy and temperature intrinsic to the horizon of the universe due to the information holographically stored there. Dark energy is thereby obviated and the acceleration is due to an entropic force naturally arising from the information storage on the horizon surface screen. We consider an additional quantitative approach inspired by surface terms in general relativity and show that this leads to the entropic accelerating universe.

1. Introduction

The most important observational advance in cosmology since the early studies of cosmic expansion in the 1920's was the dramatic and unexpected discovery, in the waning years of the twentieth century, that the expansion rate is accelerating. This was first announced in February 1998, based on the concordance of two groups' data on Supernovae Type 1A [1,2].

A plethora of subsequent experiments concerning the Cosmic Microwave Background (CMB), Large Scale Structure (LSS), and other measurements have all confirmed the 1998 claim for cosmic acceleration. There have been many attempts to avoid the conclusion of the cosmic acceleration. Typically they involve an ingenious ruse which assigns a special place to the Earth in the Universe, in a frankly Ptolemaic manner and in contradiction to the well-tested and time-honored cosmological principle at large distance. We find these to be highly contrived and *ad hoc*.

We therefore adopt the position that the accelerated expansion rate is an observed fact which we, as theorists, are behooved to interpret theoretically with the most minimal set of additional assumptions.

* Corresponding author at: Institute for the Physics and Mathematics of the Universe, University of Tokyo, Kashiwa, Chiba 277-8568, Japan.

E-mail addresses: easson@asu.edu (D.A. Easson), frampton@physics.unc.edu (P.H. Frampton), gfsmoot@lbl.gov (G.F. Smoot).

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2. Interpretation as dark energy

On the basis of general relativity theory, together with the cosmological principle of homogeneity and isotropy, the scale factor $a(t)$ in the FRW metric satisfies [3,4] the Friedmann–Lemaître equation

$$H(t)^2 = \left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi G}{3}\right)\rho \tag{1}$$

where we shall normalize $a(t_0) = 1$ at the present, time $t = t_0$, and ρ is an energy density source which drives the expansion of the universe. Two established contributions to ρ are ρ_m from matter (including dark matter) and ρ_γ radiation, so that

$$\rho \supseteq \rho_m + \rho_\gamma \tag{2}$$

with $\rho_m(t) = \rho_m(t_0)a(t)^{-3}$ and $\rho_\gamma(t) = \rho_\gamma(t_0)a(t)^{-4}$.

For the observed accelerated expansion, the most popular approach is to add to the sources, in Eq. (1), a dark energy term $\rho_{DE}(t)$ with

$$\rho_{DE}(t) = \rho_{DE}(t_0)a(t)^{-3(1+\omega)} \tag{3}$$

where $\omega = p/\rho c^2$ is the equation of state parameter. For the case $\omega = -1$, as for a cosmological constant, Λ, and discarding the matter and radiation terms which are relatively negligible we can easily integrate the Friedmann–Lemaître equation to find

$$a(t) = a(t_0)e^{Ht} \tag{4}$$

where $\sqrt{3}H = \sqrt{\Lambda} = \sqrt{8\pi G \rho_{DE}}$.

By differentiation of Eq. (4) with respect to time p times we obtain for the pth derivative

$$\left.\frac{d^p}{dt^p}a(t)\right|_{t=0} = (H)^p. \tag{5}$$

Therefore, if $\Lambda > 0$ is positive, as in a de Sitter geometry, not only is the acceleration ($p = 2$) positive and non-zero, but so are the jerk ($p = 3$), the snap ($p = 4$), the crackle ($p = 5$), the pop ($p = 6$) and all $p \geqslant 7$.

The insertion of the dark energy term (3) in Eq. (1) works very well as a part of the ΛCDM model. However, it is an *ad hoc* procedure which gives no insight into what dark energy is. Identifying the cosmological constant with vacuum energy leads to the infamous cosmological constant problem: the observed value of the cosmological constant $\rho_{\Lambda_{\rm obs}} \sim (10^{-3}\ \text{eV})^4$ and the theoretical prediction (assuming a UV cutoff at the Planck scale) $\rho_{\Lambda_{\rm th}} \sim (10^{18}\ \text{GeV})^4$, disagree by an embarrassing 120 orders of magnitude.

With this background, we shall now move to a different explanation for the accelerated expansion which obviates the need for any ambiguous dark energy component, including scalar fields. This approach (which does not directly solve the cosmological constant problem) even leads to new insight into the perplexing ratio $\rho_{\Lambda_{\rm obs}}/\rho_{\Lambda_{\rm th}} \sim 10^{-120}$ discussed above.

3. Interpretation as entropic force

We now adopt a different approach, with no dark energy, where instead the central role is played by the ideas of information and holography, entropy and temperature.[1]

The first and only assumption is that a horizon has both a temperature and entropy associated with it. This was first shown clearly to hold for black hole horizons with a temperature given by the Hawking temperature and an entropy given by the Bekenstein entropy. Here we take the apparent horizon of the universe.[2]

At this horizon, there is a horizon temperature, T_β, which we can estimate as

$$T_\beta = \frac{\hbar}{k_B}\frac{H}{2\pi} \sim 3 \times 10^{-30}\ \text{K}. \tag{6}$$

Such a temperature is closely related to the de Sitter temperature.[3] More relevant to the central question is the fact that the temperature of the horizon leads to the concomitant entropic force and resultant acceleration $a_{Horizon}$ of the horizon given by the Unruh [5] relationship

$$a_{Horizon} = \left(\frac{2\pi c k_B T_\beta}{\hbar}\right) = cH \sim 10^{-9}\ \text{m/s}^2. \tag{7}$$

When T_β is used in Eq. (7), we arrive at a cosmic acceleration essentially in agreement with the observation.

[1] The entropy of the universe has received some recent attention [9,10], in part because it relates to the feasibility of constructing a consistent cyclic model. For example, the cyclic model in [11], assuming its internal consistency will indeed be fully confirmed, may provide a solution to the difficult entropy question originally posed seventy-five years earlier by Tolman [12].

[2] In the following discussion we consider a flat $k = 0$ universe so the apparent horizon coincides with the Hubble radius. We relegate a more general discussion of $k \neq 0$ to Appendix A.

[3] We suspect, without rigor, that in the third law of thermodynamics the notion of absolute zero, $T = 0$, must be replaced by $T \geqslant T_\beta$, although this is not our present concern.

From this viewpoint, the ambiguous dark energy component is non-existent. Instead there is an entropic force contribution acting at the horizon and pulling outward towards the horizon to create the appearance of a dark energy component.[4] We emphasize again that because we have nothing new to say about the physics governing quantum fluctuations, the above argument does not, by itself, solve the cosmological constant problem. However, the interpretation of cosmic acceleration as due to entropic force helps to understand why the accelerating component is expected to be small today (of order H as in Eq. (7)), in contrast to the embarrassingly large value predicted by quantum field theory combined with general relativity.

We shall next amplify on the distinction, and study more the entropy and surface screen considerations, showing that even the present fraction of the critical energy associated with acceleration can thereby be understood. The next portion derives an expression for the pressure, which is negative and thus a tension in the direction of the screen. The following results rely on simple principles of entropy and thermodynamics and are not dependent on any specific model.

The entropy on the Hubble horizon, e.g. the Hubble radius $R_H = c/H$, is

$$\begin{aligned} S_H &= \frac{k_B c^3}{G\hbar}\frac{A}{4} = \frac{k_B c^3}{G\hbar}\pi R_H^2 \\ &= \frac{k_B c^3}{G\hbar}\pi\left(\frac{c}{H}\right)^2 \sim (2.6 \pm 0.3) \times 10^{122} k_B. \end{aligned} \tag{8}$$

Increasing the radius R_H, by Δr, increases the entropy by ΔS_H according to

$$\begin{aligned} \Delta S_H &= \frac{k_B c^3}{G\hbar}2\pi R_H \Delta r \\ &= \frac{k_B c^3}{G\hbar}2\pi\left(\frac{c}{H}\right)\Delta r \sim (2.6 \pm 0.3) \times 10^{122} k_B \Delta r/R_H. \end{aligned} \tag{9}$$

The entropic force is simply

$$\begin{aligned} F_r &= -\frac{dE}{dr} = -T\frac{dS}{dr} = -T_\beta\frac{dS_H}{dr} \\ &= -\frac{\hbar}{k_B}\frac{H}{2\pi}\frac{k_B c^3}{G\hbar}2\pi\left(\frac{c}{H}\right) = -\frac{c^4}{G} \end{aligned} \tag{10}$$

where the minus sign indicates pointing in the direction of increasing entropy or the screen, which in this case is the horizon.

The pressure from entropic force exerted is

$$\begin{aligned} P &= \frac{F_r}{A} = -\frac{1}{A}T\frac{dS}{dr} = -\frac{1}{A}\frac{c^4}{G} = -\frac{1}{4\pi c^2/H^2}\frac{c^4}{G} \\ &= -\frac{c^2H^2}{4\pi G} = -\frac{2}{3}\rho_{critical}c^2 \end{aligned} \tag{11}$$

where $\rho_{critical}$ is the critical energy density $\rho_{critical} \equiv 3H^2/8\pi G$.

This is close to the value of the currently measured dark energy/cosmological constant negative pressure (tension). In this case the tension does not arrive from the negative pressure of dark energy but from the entropic tension due to the entropy content of the horizon surface. This is equivalent to the outward acceleration $a_H = cH$ of Eq. (7).

If we chose to put the information screens at smaller radii, then, associating entropy with information, we would have found a

[4] The possibility that cosmic acceleration can be described by an entropic force should be distinguished from the idea that gravity itself is an entropic force [6,7]; although the two ideas are not *prima facie* incompatible.

proportionally smaller pressure, and an acceleration that decreases linearly with the radius in accordance with our expected Hubble law. Thus, the acceleration of the universe simply arises as a natural consequence of the entropy on the horizon of the universe.

4. Acceleration from the entropy and surface terms

In this section, we present a specific phenomenological model inspired by surface terms usually ignored in general relativity and show that this also leads to an accelerating universe. While the surface terms source our motivation, the model is best viewed as a phenomenological model which we introduce here without rigorous derivation. We consider the least additional assumption is that general relativity is correct, and that it can be easily understood and derived from a variational principle using the action.

We show that, under reasonable assumptions, the new terms lead to an acceleration term in the Friedmann–Lemaître equations. There is a solution to the acceleration equation that evolves from a decelerating to an accelerating phase. Our discussion of surface terms presents a specific class of models that give rise to cosmological acceleration; however, we note that our main result, the derivation of cosmological acceleration as an entropic force presented in the previous section, is model independent.

The Einstein–Hilbert action including the surface term and a matter action is (schematically)

$$I = \int_M (R + \mathcal{L}_m) + \frac{1}{8\pi} \oint_{\partial M} K \tag{12}$$

where R is the scalar curvature, $\mathcal{L}_m$ is the matter and field Lagrangian, and K is the trace of the extrinsic curvature of the boundary [8]. The application of variational procedures then produces the usual Einstein equations for general relativity with the addition of a surface energy term:

Curvature of Space–Time *proportional* to the Stress–Energy Content + Surface Terms

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} + \text{Surface Terms}. \tag{13}$$

Typically the surface terms are neglected though they make a significant appearance when a horizon is present. This would in the case of spherical symmetry and homogeneity lead to the Friedmann–Lemaître equations:

Scale factor acceleration = Energy Content deceleration + acceleration from Surface Terms

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) + a_{surface}/d_H. \tag{14}$$

Now, there are a number of approaches to determine the form of the new terms. Here we consider a simple possibility. From our surface term motivation, we would anticipate that the integral of the trace of the intrinsic curvature would be of order $6(2H^2 + \dot{H})$ so that the term would be approximately $\frac{6(2H^2+\dot{H})}{8\pi} \sim \frac{3}{2\pi}(H^2 + \dot{H}/2)$. We can also take an approach motivated by entropic ideas and see that these naturally lead to a slowly expanding, late time accelerating universe.

For a horizon, it is well known that there is an associated curvature and temperature, and that these two quantities are related. The temperature T is given by the Unruh, de Sitter, or Hawking temperature prescriptions (except for a pesky factor of two difference between the Hawking temperature and the other two due to location of evaluation of the temperature – at the horizon or remote). We can then associate the surface entropy or surface term with its temperature and its acceleration. Using the relations we find (see Eq. (7)) for the horizon acceleration

$$a_{surface} = a_{entropic} = cH \tag{15}$$

where $H = \dot{a}/a$ is the Hubble expansion rate. If we have a scale, which is naturally and necessarily the Hubble horizon scale, $d_H = c/H$, for our cosmological treatment, then we can complete the Friedmann–Lemaître acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) + H^2. \tag{16}$$

This is remarkably like the surface term order of magnitude estimate except for the $3/2\pi$ factor. With the Hawking temperature description the coefficient would have been $1/2$. There is some freedom here and we chose the value that leads to nice equations in the two limiting cases. It is easy to show that if the H^2 is highly dominant over the $\frac{4\pi G}{3}(\rho + 3P/c^2)$, the solution to the equation is simply a de Sitter space with scale factor $a(t) = a(t_0)e^{H(t-t_0)}$.

The alternate equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) + \frac{3}{2\pi} H^2 + \frac{3}{4\pi}\dot{H} \tag{17}$$

may provide a better fit to the data (see Section 4.1) or a rigorous derivation but does not have the simplicity of Eq. (16).

In order to adopt a broader approach, we generalized our discussion by considering

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) + C_H H^2 + C_{\dot{H}}\dot{H} \tag{18}$$

where we anticipate the coefficients to be bounded by $\frac{3}{2\pi} \lesssim C_H \leqslant 1$ and $0 \leqslant C_{\dot{H}} \lesssim \frac{3}{4\pi}$.

4.1. Comparison with supernova data

We conclude with a demonstration that the entropic acceleration mechanism inspired by surface terms can provide a surprisingly remarkable fit the supernova data, assuming the simple form for the acceleration equation (17). Because we are using a metric theory of gravity, we may use the standard formula for the luminosity distance:

$$d_L\big(z; H(z), H_0\big) = \frac{c(1+z)}{H_0} \int_0^z \frac{dz'}{H(z')} \tag{19}$$

where z is the redshift defined by $z + 1 \equiv a_0/a$. For ΛCDM, the luminosity distance can be written [1]:

$$d_L(z; \Omega_M, \Omega_\Lambda, H_0) = \frac{c(1+z)}{H_0\sqrt{|\kappa|}} \mathcal{S}\left(\sqrt{|\kappa|} \int_0^z dz' \left[\left(1+z'\right)^2\left(1+\Omega_M z'\right) - z'\left(2+z'\right)\Omega_\Lambda\right]^{-\frac{1}{2}}\right) \tag{20}$$

where $\mathcal{S}(x) \equiv \sin(x)$ and $\kappa = 1 - \Omega_{tot}$ for $\Omega_{tot} > 1$ while $\mathcal{S}(x) \equiv \sinh(x)$ with $\kappa = 1 - \Omega_{tot}$ for $\Omega_{tot} < 1$ while $\mathcal{S}(x) \equiv x$ and $\kappa = 1$, for $\Omega_{tot} = 1$. For the ΛCDM models we take $\Omega_{tot} = \Omega_M + \Omega_\Lambda$. Here we have defined $\Omega_M \equiv \rho_M/\rho_c = 8\pi G\rho_M/3H^2$ and $\Omega_\Lambda = \Lambda/3H^2$ where ρ_c is the critical energy density. The results are that the entropic acceleration models we consider can provide excellent fits

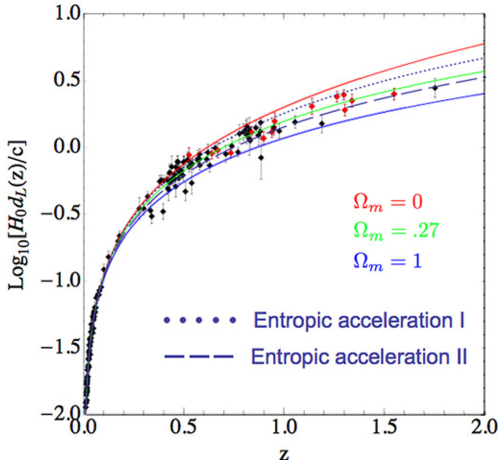

Fig. 1. Comparison of entropic acceleration and several ΛCDM models. The supernova data points are plotted with error bars and the data is taken from [13]. The luminosity distance d_L for the entropic models *I* (Eq. (16)) and *II* (Eq. (17)) are denoted by the dotted and dashed (blue in the online version) curves respectively. The theoretical predictions for ΛCDM are represented by the solid curves.

to the data as can be seen from Fig. 1.[5] The entropic models move smoothly from a decelerating to an accelerating phase sometime near a redshift of $z = 0.5$, analogous to the ΛCDM models. We suspect a complete model may be further constrained by consideration of Big-Bang nucleosynthesis and possibly by precision data relating to the equivalence principle.

5. Conclusions

We have proposed a theory underlying the accelerated expansion of the universe based on entropy and entropic force. This approach, while admittedly heuristic, provides a physical understanding of the acceleration phenomenon which was lacking in the description as dark energy. The evidence and general arguments supporting our hypothesis were presented in Section 3. In addition we considered an interesting phenomenological model, loosely based on surface terms in Section 4, and showed the models are capable of providing a good fit to the supernova data.

Following the above arguments to their logical conclusion, the accelerated expansion rate is the inevitable consequence of the entropy associated with the holographic information storage on a surface screen placed at the horizon of the universe. An interesting question [14] is: how does this entropic viewpoint of cosmic acceleration impact on inflationary theory?

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Appendix A. The Friedmann equations from the first law

In this appendix, we generalize our discussion to include the possibility that $\Omega \neq 1$. We include a curvature term and use the apparent event horizon r_A as our preferred screen rather than the Hubble horizon r_H – these are equivalent for the case of vanishing curvature. We have

$$r_A = \frac{c}{\sqrt{H^2 + k/a^2}} \quad \text{and} \quad \dot{r}_A = -\frac{H r_A^3}{c^2}\left(\dot{H} - \frac{k}{a^2}\right). \tag{A.1}$$

The energy flow across the apparent horizon is $-dE = dM c^2 + P\,dV = (\rho c^2 + P)\,dV$

$$\begin{aligned} -dE &= \left(\rho + P/c^2\right)c^2\,dV = \left(\rho + P/c^2\right)c^2 A_A v\,dt \\ &= \left(\rho + P/c^2\right)c^2 A_A H r_A\,dt \end{aligned} \tag{A.2}$$

where $A_A = 4\pi r_A^2$. Assuming that the apparent horizon has an associate entropy S and approximate temperature T given by

$$S = \frac{k_B c^3}{\hbar G}\frac{A}{4}, \qquad T = \frac{\hbar H}{2\pi k_B} = \frac{\hbar}{2\pi k_B}\frac{c}{r_A}, \tag{A.3}$$

we can use the first law of thermodynamics $-dE = T\,dS$ to find

$$\begin{aligned} -dE &= A\left(\rho + P/c^2\right)c^2 H r_A\,dt = T\,dS \\ &= T\frac{k_B c^3}{4\hbar G}\frac{dA_A}{dt}\,dt = T\frac{k_B c^3}{4\hbar G} 2 \times 4\pi r_A \dot{r}_A\,dt. \end{aligned} \tag{A.4}$$

Dividing by $c^2\,dt$ and using $T = \frac{\hbar}{2\pi k_B}\frac{c}{r_A}$ one finds, after substitution for $\dot{r}_A$,

$$A\left(\rho + P/c^2\right)H r_A = \frac{c^2}{G}\dot{r}_A = -\frac{1}{G}H r_A^3\left(\dot{H} - \frac{k}{a^2}\right). \tag{A.5}$$

Dividing through by $H r_A/G$ and using $A = 4\pi r_A^2$ we have

$$4\pi G\left(\rho + P/c^2\right) = -\left(\dot{H} - \frac{k}{a^2}\right). \tag{A.6}$$

Rearranging one has one form of the standard Friedmann acceleration equation

$$\dot{H} - \frac{k}{a^2} = -4\pi G\left(\rho + P/c^2\right). \tag{A.7}$$

If one takes for the continuity (conservation) equation for the perfect cosmological fluids,

$$\dot{\rho} + 3H\left(\rho + P/c^2\right) = 0. \tag{A.8}$$

We can substitute $H(\rho + P/c^2) = -\dot{\rho}/3$ into the equation for $\dot{H}$ and integrate

$$H\left(\dot{H} - \frac{k}{a^2} = -4\pi G\left(\rho + P/c^2\right)\right) = \frac{4\pi G}{3}\dot{\rho}, \tag{A.9}$$

$$\int H\dot{H} - k\int\frac{\dot{a}}{a^3} = \frac{4\pi G}{3}\int\dot{\rho} \tag{A.10}$$

which yields

$$\frac{1}{2}\left(H^2 + \frac{k}{a^2}\right) = \frac{4\pi G}{3}\rho + \text{constant}/2 \tag{A.11}$$

[5] In solving the equations for the fitting we have assumed the standard scaling behavior for matter and radiation. This scaling behavior is now determined only in part by the Friedmann–Lemaître equation and we do not propose any novel form for this constraint yet; rather, we assume that the standard scaling applies and should provide an adequate approximation for the purposes of Fig. 1.

or

$$H^2 + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho + \text{constant}$$
$$\to H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}. \tag{A.12}$$

This is simply the usual second Friedmann energy equation. We can find the other form of the acceleration equation by using this equation and the relationship $\dot{H} = \ddot{a}/a - H^2$ to substitute into Eq. (A.7) to obtain

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{P}{c^2}\right) + \text{constant}$$
$$\to \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{P}{c^2}\right) + \frac{\Lambda c^2}{3}. \tag{A.13}$$

We can see from this derivation that the Friedmann equations naturally arise from the first law of thermodynamics and the association of entropy and temperature to the apparent horizon (or Hubble horizon for a flat universe). We also see there is a place for curvature (currently observed to be small) and for a constant of integration Λ.

A.1. Continuity equation from the first law

The continuity equation is derived assuming the simple form of the first law of thermodynamics.

$$dE = -P\,dV,$$
$$dM\,c^2 = -P\,dV,$$
$$d\big(V_0\rho a^3\big)c^2 = -Pd\big(V_0 a^3\big),$$
$$d\rho\, c^2 a^3 + da\,\rho c^2 3a^2 = -3a^2 P\,da,$$
$$d\rho = -3\left(\rho + \frac{P}{c^2}\right)\frac{1}{a}\,da,$$
$$\frac{d\rho}{dt} = -3\left(\rho + \frac{P}{c^2}\right)\frac{1}{a}\frac{da}{dt},$$
$$\dot{\rho} + 3H\left(\rho + \frac{P}{c^2}\right) = 0. \tag{A.14}$$

This suggests that our assumption of the standard continuity equation is appropriate for the comparison to data and in deriving the standard Friedmann equations.

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