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Massless versus massive Hawking radiation in AdS_2 spacetime

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Abstract

We study massless and massive Hawking radiations on a two-dimensional AdS spacetime. For the massless case, the quantum stress-energy tensor of a massless scalar field on the AdS background is calculated, and the expected null radiation is obtained. However, for the massive case, the scattering analysis is performed in order to calculate the absorption and reflection coefficients which are related to statistical Hawking temperature. On the contrary to the massless case, we obtain a nonvanishing massive radiation.

There has been a great interest in a lower-dimensional gravity since it is possible to construct a consistent and renormalizable quantum gravity without encountering some complexities of four-dimensional realistic models. In the Callan–Giddings–Harvey–Strominger (CGHS) model [1], the asymptotically flat black hole solution is obtained under a linear dilaton background and the quantum effect of the black hole can be described by Hawking radiation [2]. In general, a thermal equilibrium state of the black hole with the thermal bath is defined by the Hartle–Hawking (HH) vacuum [3,4].

From the CGHS model, the two-dimensional anti-de Sitter (AdS_2) solution can be obtained by assuming a constant dilaton background [5], which is in contrasted with the original CGHS solution in that the curvature scalar of AdS spacetime is constant and its asymptotic metric is no more Minkowski spacetime. This phenomenon has already appeared in the three-dimensional low energy string theory [6], so that the asymptotically flat black string solution is obtained on the presence of the dilaton charge while the asymptotically nonflat Bañados–Teitelboim–Zanelli (BTZ) [7] solution is derived for the constant dilaton background. On the other hand, in connection with the calculation of the statistical entropy of the extremal Reissner–Nordstrom black hole, the AdS_2 geometry has been intensively studied in Refs. [8,9].

On the other hand, it has been shown that the massless Hawking radiation on this AdS background does not appear [5,10] whereas it is possible to radiate if the dilaton field couples to the massless scalar field [11]. It is

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now natural to ask what happens for the massive case on this metric background. So we would like to study whether the massive radiation on the AdS_2 spacetime background is possible or not in terms of the scattering analysis, which was given for the massless case in Ref. [10] In this paper, for the massless case, the stress-energy tensor calculation will be done on the AdS_2 background and the null radiation will be found, which is essentially compatible with the result of the null radiation through the scattering analysis [11]. In this paper we mainly study the massive scalar field on the AdS_2 background and calculate the scattering amplitudes in order to obtain the Hawking temperature. We finally obtain some unexpected result on the Hawking temperature.

We first study the massless radiation on the AdS_2 background by calculating the stress-energy tensors of the conformal matter field. This confirms the expected vanishing Hawking radiation. Now the two-dimensional action for the conformal matter is given as

$$S_M = -\tfrac{1}{2}\int dx^2\sqrt{-g}\,(\nabla f)^2, \tag{1}$$

where f is a massless scalar field. The effective action of the scalar field is written as the Polyakov induced gravity action,

$$S_{\rm eff} = -\frac{1}{96\pi}\int dx^2\sqrt{-g}\,R\frac{1}{\Box}R, \tag{2}$$

and equally written as a local form of

$$S_{\rm eff} = -\frac{1}{96\pi}\int dx^2\sqrt{-g}\,[-\Psi\,\Box\Psi + 2\Psi R], \tag{3}$$

by introducing the auxiliary field Ψ satisfying

$$\Box\Psi = R. \tag{4}$$

The stress-energy tensor is

$$\langle T_{\mu\nu}\rangle = \frac{2\pi}{\sqrt{-g}}\frac{\delta S_{\rm eff}}{\delta g^{\mu\nu}} = -\tfrac{1}{48}\Big[2\nabla_\mu\nabla_\nu\Psi - \nabla_\mu\Psi\,\nabla_\nu\Psi - g_{\mu\nu}\big(2R - \tfrac{1}{2}(\nabla\Psi)^2\big)\Big] \tag{5}$$

where the background metric may be assumed to be in the form of $ds^2 = -g(r)dt^2 + \frac{1}{g(r)}dr^2$. In the light-cone coordinates, they are explicitly rewritten as

$$\langle T_{\pm\pm}(\sigma^+,\sigma^-)\rangle = \tfrac{1}{4}\big(\langle T_{tt}\rangle + g^2(r)\langle T_{rr}\rangle \pm 2g(r)\langle T_{rt}\rangle\big), \tag{6}$$

$$\langle T_{+-}(\sigma^+,\sigma^-)\rangle = \tfrac{1}{4}\big(\langle T_{tt}\rangle - g^2(r)\langle T_{rr}\rangle\big) \tag{7}$$

where $\sigma^{\pm} = t \pm r^*$ and $r^*(r) = \int dr\frac{1}{g(r)}$.

To derive the Hartle–Hawking temperature for the massless scalar field on the two dimensional AdS_2 background, we consider the AdS black hole metric given as a solution of the CGHS model with the constant dilaton background [5]

$$(ds)^2 = -\left(-M + \frac{r^2}{\ell^2}\right)dt^2 + \left(-M + \frac{r^2}{\ell^2}\right)^{-1}dr^2 \tag{8}$$

where the horizon is located at $r_H = \sqrt{M}\ell$. The auxiliary field in Eq. (4) is exactly solved as

$$\Psi(r,t) = \frac{a\ell}{2\sqrt{M}}\ln\left(1 - \frac{2\sqrt{M}\ell}{r+\sqrt{M}\ell}\right) - \ln\left(-M + \left(\frac{r}{\ell}\right)^2\right) + b + ct \tag{9}$$

on the background (8). The integration constants a,b,c will be determined by some boundary conditions. By using Eqs. (8) and (9), the explicit expression of the stress-energy tensors (5) in terms of the Schwarzschild coordinate is given as

$$\langle T_{tt}\rangle = \tfrac{1}{96}\left(c^2 + a^2 + \frac{4r^2}{\ell^4} - \frac{8M}{\ell^2}\right), \quad \langle T_{rr}\rangle = \tfrac{1}{96}\left(\frac{r^2}{\ell^2} - M\right)^{-2}\left(c^2 + a^2 - \frac{4r^2}{\ell^4}\right),$$

$$\langle T_{rt}\rangle = \frac{ac}{48}\left(\frac{r^2}{\ell^2} - M\right)^{-1}, \tag{10}$$

and in the light-cone coordinate they are simply

$$\langle T_{\pm\pm}\rangle = \tfrac{1}{192}\left((c \pm a)^2 - \frac{4M}{\ell^2}\right), \tag{11}$$

$$\langle T_{+-}\rangle = \frac{1}{48\ell^2}\left(\frac{r^2}{\ell^2} - M\right). \tag{12}$$

Now we impose the Hartle–Hawking boundary condition on the stress-energy tensors. The black hole embedded in the thermal bath satisfies the equilibrium condition which is described as $\langle T_{\pm\pm}\rangle|_{r\to r_H} = 0$ together with $c = 0$ [3,4,12], and then the parameters are fixed as $a = \pm\frac{2\sqrt{M}}{\ell}$ and $b = 0$. If we make the auxiliary field finite at the horizon, we can choose $a = \frac{2\sqrt{M}}{\ell}$. This boundary condition determines the behavior of the stress-energy tensors which are expressed as

$$\langle T_{\pm\pm}\rangle_{\mathrm{HH}} = 0, \tag{13}$$

$$\langle T_{+-}\rangle_{\mathrm{HH}} = \frac{1}{48\ell^2}\left(\frac{r^2}{\ell^2} - M\right). \tag{14}$$

As expected, from the relation, $T_H = \frac{1}{\pi}T_{--}$ [1], the Hartle–Hawking state gives the vanishing temperature of $T_H = 0$ for the massless case, and this fact is compatible with the scattering analysis given in Ref. [10].

Let us now study the massive radiation from the AdS_2 black hole. Unfortunately, we do not have an exact form of the effective action unlike the massless case and the calculation of the stress-energy tensor of the massive scalar is not straightforward, which is recently discussed in Ref. [9]. Instead, we want to study the Hawking radiation for the massive scalar field following the scattering procedure which is successful for the massless field case in Refs. [10,11].

Then the massive scalar field equation is

$$(\Box + m^2) f(r,t) = 0, \tag{15}$$

where m is a mass parameter. It is written as a spatial equation by using the ansatz, $f(r,t) = R(r)e^{-i\omega t}$, which is given as

$$(r^2 - r_H^2)\partial_r^2 R(r) + 2r\partial_r R(r) + \frac{1}{(r^2 - r_H^2)}\left[\omega^2\ell^4 + m^2\ell^2(r^2 - r_H^2)\right]R(r) = 0. \tag{16}$$

By use of the change of variable $z = \frac{r - r_H}{r + r_H}$ $(0 < z < 1)$, the field Eq. (16) is written as

$$z(1-z)\partial_z^2 R(z) + (1-z)\partial_z R(z) + \left[\frac{\omega^2\ell^4}{4r_H^2}\left(\frac{1}{z} - 1\right) + \frac{m^2\ell^2}{1-z}\right]R(z) = 0. \tag{17}$$

To eliminate two singularities at $z=0$ and $z=1$, we set $R(z)=z^{\alpha}(1-z)^{\beta}g(z)$ and then the wave equation becomes

$$z(1-z)\partial_z^2 g(z)+\left[1+2\alpha-(1+2\alpha+2\beta)z\right]\partial_z g(z)+\left[\frac{1}{z}\left(\alpha^2+\frac{\omega^2\ell^4}{4r_H^2}\right)+\frac{1}{1-z}\left(\beta^2-\beta+m^2\ell^2\right)-\beta(\beta+2\alpha)\right]g(z)=0. \tag{18}$$

Here we choose $\alpha^2=-\frac{\omega^2\ell^4}{4r_H^2}$ and $\beta=(1-\sqrt{1-4m^2\ell^2})/2$, and then get the final form of the field equation,

$$z(1-z)\partial_z^2 g(z)+\left[1+2\alpha-(1+2\alpha+2\beta)z\right]\partial_z g(z)-\beta(\beta+2\alpha)g(z)=0. \tag{19}$$

Actually, we have two roots satisfying $\beta^2-\beta+m^2\ell^2=0$, however, we take the negative sign because there exists massless limit($\beta=0$). Note that we assume our semiclassical approximation is valid only for the case that the energy of our test field is properly small, and we need not consider the back reaction of the geometry. The curvature of the AdS geometry is proportional to $\frac{1}{\ell^2}$, and we assume m^2 is less than $\frac{1}{\ell^2}$, β is approximately $m^2\ell^2$.

The field Eq. (19) can be solved as

$$R^{\text{bulk}}(z)=C_{\text{out}}z^{\alpha}(1-z)^{\beta}F(\beta,\beta+2\alpha,1+2\alpha,z)+C_{\text{in}}z^{-\alpha}(1-z)^{\beta}F(\beta,\beta-2\alpha,1-2\alpha,z). \tag{20}$$

This hypergeometric solution is symmetric under the exchange of $\alpha\to-\alpha$, so we may take the positive value of α for convenience. In the near horizon limit, the solution (20) is explicitly written as

$$R^{\text{bulk}}_{\text{near}}(r)=C_{\text{out}}e^{i\frac{\omega\ell 2}{2r_H}\ln\left(\frac{r-r_H}{r+r_H}\right)}+C_{\text{in}}e^{-i\frac{\omega\ell 2}{2r_H}\ln\left(\frac{r-r_H}{r+r_H}\right)}. \tag{21}$$

In Eq. (21), it is independent of β and the asymptotic form of the field solution is remarkably same with that of the massless scalar field [10]. This fact is plausible in that the wave frequency is ultraviolet shift in the near horizon limit and the mass term can be negligible compared to the wave number in the dispersion relation. So in this region, the scalar field can be regard as a massless conformal field.

Next, we can find the far region solution by using the $z\to 1-z$ transformation of the hypergeometric function [13],

$$F(a,b,c;z)=\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}F(a,b,a+b-c+1;1-z)+(1-z)^{c-a-b}\frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}F(c-a,c-b,c-a-b+1;1-z). \tag{22}$$

Note that our transformed solution has no massless limit $\beta=0$ because the hypergeometric function of the transformed solution is not defined for $\beta=0$. Thus this solution is only valid for the case of the massive scalar field and it is meaningless at this stage to take the massless limit. If r goes to infinity, from Eqs. (20) and (22), the far solution can be expanded as

$$\begin{aligned}R^{\text{bulk}}_{\text{far}}(r)=&(2r_H)^{1-\beta}\left[C_{\text{out}}\frac{\Gamma(1+2\alpha)\Gamma(2\beta-1)}{\Gamma(\beta+2\alpha)\Gamma(\beta)}+C_{\text{in}}\frac{\Gamma(1-2\alpha)\Gamma(2\beta-1)}{\Gamma(\beta-2\alpha)\Gamma(\beta)}\right]\frac{1}{r^{1-\beta}}\\&+(2r_H)^{\beta}\left[C_{\text{out}}\frac{\Gamma(1+2\alpha)\Gamma(1-2\beta)}{\Gamma(1-\beta+2\alpha)\Gamma(1-\beta)}+C_{\text{in}}\frac{\Gamma(1-2\alpha)\Gamma(1-2\beta)}{\Gamma(1-\beta-2\alpha)\Gamma(1-\beta)}\right]\frac{1}{r^{\beta}}\\&+\mathscr{O}\left(\frac{1}{r^{2-\beta}},\frac{1}{r^{\beta+1}}\right).\end{aligned} \tag{23}$$

Note that this asymptotic solution was derived from the exact bulk solution. At this stage, we should carefully consider the boundary geometry of this AdS black hole because it is nontrivial in contrast to the asymptotically flat black hole. The background geometry of the usual black hole at the asymptotically far region is happened to be that of the massless limit of the black hole geometry. So, in that case, the far region limit means the massless limit of the black hole geometry. In our model, however, this is not the case. Therefore, we should take a boundary metric by defining $M=0$ in Eq. (16). Then the equation of motion at the boundary is given as

$$r^2\partial_r^2 R(r) + 2r\partial_r R(r) + \frac{1}{r^2}(\omega^2\ell^4 + m^2\ell^2 r^2)R(r) = 0 \tag{24}$$

by setting $r_H = 0$ from Eq. (16). This Eq. (24) yields the solution of a linear combination of the Bessel functions J_λ and $J_{-\lambda}$,

$$R_{\text{boundary}}(r) = \frac{1}{\sqrt{r}}\left(A_1 J_{-\lambda}\left(\frac{\omega\ell^2}{r}\right) + A_2 J_\lambda\left(\frac{\omega\ell^2}{r}\right)\right), \tag{25}$$

where λ is defined as $\lambda \equiv \frac{1}{2} - \beta$. For a large r, we can rewrite this solution (25) as

$$R_{\text{boundary}}(r) = B_1\frac{1}{r^\beta} + B_2\frac{1}{r^{1-\beta}}. \tag{26}$$

To decompose this boundary solution into ingoing and outgoing modes, we define our coefficients B_1 and B_2 in Eq. (26) in terms of new coefficients B_{in} and B_{out},

$$B_1 \equiv B_{\text{in}} + B_{\text{out}}, \quad B_2 \equiv \frac{i}{\pi}(2r_H)^{1-2\beta}(B_{\text{in}} - B_{\text{out}}). \tag{27}$$

Then the boundary solution can be written as

$$R_{\text{boundary}}(r) = \frac{1}{r^\beta}\left[B_{\text{in}}\left(1 + \frac{i}{\pi}\left(\frac{2r_H}{r}\right)^{1-2\beta}\right) + B_{\text{out}}\left(1 - \frac{i}{\pi}\left(\frac{2r_H}{r}\right)^{1-2\beta}\right)\right]. \tag{28}$$

Now we match Eq. (23) with Eq. (28) to get the relations between far coefficients and boundary coefficients. Then two relations between these coefficients are obtained as

$$\begin{aligned}
B_{\text{in}} = \tfrac{1}{2}\Bigg[&(2r_H)^\beta\left(C_{\text{out}}\frac{\Gamma(1+2\alpha)\Gamma(1-2\beta)}{\Gamma(1-\beta+2\alpha)\Gamma(1-\beta)} + C_{\text{in}}\frac{\Gamma(1-2\alpha)\Gamma(2\beta-1)}{\Gamma(\beta-2\alpha)\Gamma(\beta)}\right)\\
&-i\pi(2r_H)^\beta\left(C_{\text{out}}\frac{\Gamma(1+2\alpha)\Gamma(2\beta-1)}{\Gamma(\beta+2\alpha)\Gamma(\beta)} + C_{\text{in}}\frac{\Gamma(1-2\alpha)\Gamma(2\beta-1)}{\Gamma(\beta-2\alpha)\Gamma(\beta)}\right)\Bigg],\\
B_{\text{out}} = \tfrac{1}{2}\Bigg[&(2r_H)^\beta\left(C_{\text{out}}\frac{\Gamma(1+2\alpha)\Gamma(1-2\beta)}{\Gamma(1-\beta+2\alpha)\Gamma(1-\beta)} + C_{\text{in}}\frac{\Gamma(1-2\alpha)\Gamma(2\beta-1)}{\Gamma(\beta-2\alpha)\Gamma(\beta)}\right)\\
&+i\pi(2r_H)^\beta\left(C_{\text{out}}\frac{\Gamma(1+2\alpha)\Gamma(2\beta-1)}{\Gamma(\beta+2\alpha)\Gamma(\beta)} + C_{\text{in}}\frac{\Gamma(1-2\alpha)\Gamma(2\beta-1)}{\Gamma(\beta-2\alpha)\Gamma(\beta)}\right)\Bigg].
\end{aligned} \tag{29}$$

The ingoing mode(outgoing mode) at the boundary can be represented as ingoing and outgoing modes at the bulk [14]. From the following definition of the flux,

$$F \equiv \frac{2\pi}{i}\left(\frac{r^2 - r_H^2}{\ell^2}\right)\left[R^*(r)\partial_r R(r) - R(r)\partial_r R^*(r)\right] \tag{30}$$

we explicitly calculate the radiation flux at the boundary as

$$F_{\text{boundary}}^{\text{in}} = -\frac{4(2r_H)^{1-2\beta}}{\ell^2}\sqrt{1-4m^2\ell^2}\,|B_{\text{in}}|^2, \quad F_{\text{boundary}}^{\text{out}} = \frac{4(2r_H)^{1-2\beta}}{\ell^2}\sqrt{1-4m^2\ell^2}\,|B_{\text{out}}|^2 \tag{31}$$

where we imposed the appropriate boundary condition that there does not exist the outgoing mode near the black hole horizon, i.e., $C_{\rm out}=0$ [15]. Then the reflection coefficient is represented by the ratio of the ingoing and the outgoing amplitude,

$$R=\left|\frac{F^{\rm in}_{\rm boundary}}{F^{\rm out}_{\rm boundary}}\right| \tag{32}$$

$$=\left|\frac{B_{\rm in}}{B_{\rm out}}\right|^2 \tag{33}$$

and this formal expression can be explicitly evaluated in the small mass compared to the given curvature scale of AdS geometry, which is plausible approximation in that we have not considered the back reaction of the geometry. The useful formulas for some expansions of the gamma functions are summarized as follows,

$$\begin{aligned}
&\frac{\Gamma(1-2a)}{\Gamma(1-a)}=1+\gamma a+\mathscr{O}(a^2),\\
&\frac{\Gamma(2a-1)}{\Gamma(a)}=-\tfrac{1}{2}+\left(-1+\frac{\gamma}{2}\right)a+\mathscr{O}(a^2),\\
&\frac{1}{\Gamma(1+a+ib)\Gamma(1+a-ib)}=\frac{1}{\Gamma(1+ib)\Gamma(1-ib)}\left[1+(\psi(1-ib)+\psi(1+ib))a+\mathscr{O}(a^2)\right],\\
&\frac{1}{\Gamma(a+ib)\Gamma(a-ib)}=\frac{1}{\Gamma(ib)\Gamma(-ib)}\left[1-(\psi(-ib)+\psi(ib))a+\mathscr{O}(a^2)\right],\\
&\psi(1\pm a)=-\gamma\pm\frac{\pi^2}{3}a-4\zeta(3)a^2+\mathscr{O}(a^3),\\
&\psi(\pm a)=\mp\frac{1}{2a}+\psi(1\pm a),
\end{aligned} \tag{34}$$

where γ is an Euler's constant, $\psi(z)$ is a digamma function, and $\zeta(3)$ is the Riemann zeta function $\zeta(3)=\sum_{k=1}^{\infty}k^{-3}=1.20205\cdots$. Then the reflection coefficient is simply given as

$$R=\frac{4\left(r_H^2+2\zeta(3)\,\omega^2m^2\ell^6\right)+\pi^2\omega^2\ell^4(1+4m^2\ell^2)-4\pi r_H\,\omega\ell^2(1+2m^2\ell^2)}{4\left(r_H^2+2\zeta(3)\,\omega^2m^2\ell^6\right)+\pi^2\omega^2\ell^4(1+4m^2\ell^2)+4\pi r_H\,\omega\ell^2(1+2m^2\ell^2)}. \tag{35}$$

We can obtain the Hawking temperature from the following relation,

$$\langle 0|N|0\rangle=\frac{R}{1-R}=\frac{1}{e^{\frac{\omega}{T_H}}-1} \tag{36}$$

where N is a number operator [14]. The Hawking temperature is simply expressed as

$$T_H=-\frac{\omega}{\ln R} \tag{37}$$

in terms of the reflection coefficient. Note that we expand the reflection coefficient with respect to the frequency ω and the mass m, and $\ln R$ term in the denominator is proportional to ω, which yields the Hawking temperature

$$T_H=\frac{r_H}{2\pi\ell^2}\left[1-2m^2\ell^2\left(1+\frac{\pi^2\ell^2}{24r_H^2}\right)\right]+\mathscr{O}(m^3). \tag{38}$$

It would be interesting to note that the massless limit does not recover the well-known null temperature. As was shown in Refs. [5,10] there does not exist the massless radiation on AdS_2 background, and naturally the corresponding Hawking temperature vanishes. For the massless case, the transformation rule of the hypergeometric function (22) can not be used because it is singular and the transformation rule for the massless case is somewhat different. In this massive case, the nonvanishing result is in fact unexpected. One might think that the massive Hawking radiation does not appear since the massless radiation does not occur on the AdS background. However, as was discussed in Ref. [11], even for the massless case the Hawking radiation is possible if the massless scalar field couples to some background fields, for example, the dilaton field. So in the present massive case, the mass term seems to play a role of the constant background field.

The final point to be mentioned is that the asymptotic behaviors for the cases of the massless field, the dilaton coupled massless field, and the present massive field. In the near horizon, the asymptotic behaviors of the three cases are coincident, which means that the dilaton coupled massless field and the massive field behave like a conformal type massless field. At the infinite boundary, however, three cases show different aspects unlike those of the near the horizon. First, for the massless field case, the boundary solution is given as

$$R^{\text{boundary}}_{\text{massless}}(r) = A_{\text{out}} e^{-i\frac{\omega\ell^2}{r}} + A_{\text{in}} e^{i\frac{\omega\ell^2}{r}}. \tag{39}$$

If r goes to the asymptotic infinity, the boundary solution becomes a constant. Next, for the massless field with the dilaton background, the boundary solution becomes

$$R^{\text{boundary}}_{\text{dilaton}}(r) = A_{\text{out}}\left(1 - i\frac{r_H^2}{\pi r^2}\right) + A_{\text{in}}\left(1 + i\frac{r_H^2}{\pi r^2}\right). \tag{40}$$

Note that this solution is also constant at the boundary. For the massive field case, the boundary solution is given as

$$R^{\text{boundary}}_{\text{massive}}(r) = A_{\text{out}}\frac{1}{r^\beta}\left(1 - \frac{i}{\pi}\left(\frac{2r_H}{r}\right)^{1-2\beta}\right) + A_{\text{in}}\frac{1}{r^\beta}\left(1 + \frac{i}{\pi}\left(\frac{2r_H}{r}\right)^{1-2\beta}\right), \tag{41}$$

where it vanishes at the infinity due to the effect of the field mass. Therefore the massive field is confined within the infinite boundary. On the other hand, the key ingredient of the null radiation for the massless case is due to the asymptotic forms of the field solutions. For the massless case, the boundary solution is the same with the far region solution in the AdS bulk, which means that the modes mixing does not occur, whereas both for the dilaton coupled and the massive case, the boundary solution is drastically different of the far region solution from the bulk so that the bulk solution can be decomposed into the ingoing and the outgoing modes at the boundary. Therefore, this fact gives the nonvanishing Hawking temperature in the latter two cases. We conclude that in two dimensions the massive Hawking radiation of AdS black hole is possible since the mass term in the field equation plays a role of a constant background field, which is similar to the dilaton coupled case.

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