Standard Model as a Double Field Theory

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We show that, without any extra physical degree introduced, the standard model can be readily reformulated as a double field theory. Consequently, the standard model can couple to an arbitrary stringy gravitational background in an $\mathbf{O}(4,4)$ T-duality covariant manner and manifest two independent local Lorentz symmetries, $\mathbf{Spin}(1,3)\times\mathbf{Spin}(3,1)$. While the diagonal gauge fixing of the twofold spin groups leads to the conventional formulation on the flat Minkowskian background, the enhanced symmetry makes the standard model more rigid, and also stringy, than it appeared. The CP violating θ term may no longer be allowed by the symmetry, and hence the strong CP problem can be solved. There are now stronger constraints imposed on the possible higher order corrections. We speculate that the quarks and the leptons may belong to the two different spin classes.

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Symmetry dictates the structure of the standard model (SM). It determines the way physical degrees should enter the Lagrangian. The conventional (continuous) symmetries of the SM are the Poincaré symmetry and the $\mathbf{SU}(3)\times\mathbf{SU}(2)\times\mathbf{U}(1)$ gauge symmetry. This set of symmetries does not forbid the CP violating θ term to appear in the Lagrangian. But, there is no experimentally known violation of the CP symmetry in strong interactions. This is the strong CP problem.

Especially when coupled to gravity, the description of the fermions in the SM calls for a vierbein or tetrad, $e_\mu{}^a$. All the spinorial indices in the SM Lagrangian are then subject to the *local* Lorentz group, $\mathbf{Spin}(1,3)$. Physically this gauge symmetry amounts to the freedom to choose a locally inertial frame arbitrarily at every spacetime point. Then to couple the derivatives of the fermions to gamma matrices and to construct a spin connection, it is necessary to employ the tetrad. Yet, restricted on the flat Minkowskian background, one can of course choose the trivial gauge of the tetrad, $e_\mu{}^a\equiv\delta_\mu{}^a$. However, *a priori* fermions live on the locally inertial frame and it should be always possible to revive the local Lorentz symmetry and to couple the SM to gravity.

A recent development in string theory, under the name double field theory (DFT) [1–3] (cf., Ref. [4]), generalizes the Einstein gravity and reveals hidden symmetries, such as T duality. Especially, the maximal supersymmetric $D=10$ DFT has been constructed to the full order in fermions which exhibits *twofold* local Lorentz symmetries [5] (cf., Ref. [6]). Since the twofold spin removes the chirality difference between IIA and IIB, the theory unifies the IIA and IIB supergravities. To manifest all the known as well as the hidden symmetries simultaneously, it is also necessary to employ a novel differential geometry which goes beyond Riemann and to some extent "generalized geometry" [7–9].

Physically, the doubling of the local Lorentz symmetries indicates a genuine stringy character that there are two separate locally inertial frames for each left and right string mode [10,11]. By comparison, there should be only one locally inertial frame for a point particle.

In this Letter, we show that, without introducing any additional physical degree, it is possible to reformulate the standard model as a sort of DFT, such that it couples to the gravitational DFT and manifests all the existing symmetries at once, which include $\mathbf{O}(4,4)$ T duality, diffeomorphism invariance, and a pair of local Lorentz symmetries, $\mathbf{Spin}(1,3)\times\mathbf{Spin}(3,1)$.

The enhanced symmetry then seems to forbid the θ term, and hence the strong CP problem can be solved rather naturally, without introducing any extra particles, e.g., axions [12,13]. Further, the doubling of the local Lorentz symmetries has an immediate phenomenological consequence that the spin of the standard model can be twofold, and every SM fermion should choose one of the twofold spin groups for its own spinorial representation. The enlarged symmetry also puts stronger constraints now on the possible higher order corrections.

Convention: Unbarred and barred small letters, α,β,a,b and $\bar{\alpha},\bar{\beta},\bar{a},\bar{b}$, are for the spin groups, $\mathbf{Spin}(1,3)$ and $\mathbf{Spin}(3,1)$ respectively: the spinor indices are Greek and the vector indices are Latin subject to the flat metrics, $\eta_{ab}=\mathrm{diag}(-+++)$ and $\bar{\eta}_{\bar{a}\bar{b}}=\mathrm{diag}(+---)$.

The gamma matrices are accordingly twofold,

$$\begin{aligned}(\gamma^a)^\alpha{}_\beta\colon\ \{\gamma^a,\gamma^b\}&=2\eta^{ab},\\ (\bar{\gamma}^{\bar{a}})^{\bar{\alpha}}{}_{\bar{\beta}}\colon\ \{\bar{\gamma}^{\bar{a}},\bar{\gamma}^{\bar{b}}\}&=2\bar{\eta}^{\bar{a}\bar{b}}.\end{aligned}\tag{1}$$

Capital Latin letters, $A,B,\cdots$, denote the $\mathbf{O}(4,4)$ vector indices which can be freely raised or lowered by the $\mathbf{O}(4,4)$ invariant constant metric, $\mathcal{J}_{AB}=\left(\begin{smallmatrix}0&1\\1&0\end{smallmatrix}\right)$.

It is crucial to note that different types of indices cannot be contracted. This distinction of the indices will essentially forbid many types of higher order interactions.

Gravitational DFT: stringy differential geometry.—In order to reformulate the SM as a DFT, we adopt the stringy differential geometry explored in Refs. [14,15]. The key characteristic is the *semicovariant derivative* which can be completely covariantized by certain projections.

Doubled-yet-gauged coordinate system: For the description of the four-dimensional spacetime, we employ a doubled eight-dimensional coordinate system, $\{x^A\}$, which should be yet gauged subject to an equivalence relation [16,17],

$$x^A \sim x^A + \varphi_1 \partial^A \varphi_2. \tag{2}$$

Here φ_1, φ_2 denote arbitrary fields in DFT. They include all the physical fields, gauge parameters, and their arbitrary derivatives. Each equivalence class or a gauge orbit in the doubled coordinate space represents a single physical point.

The equivalence relation Eq. (2) is realized by enforcing that all the fields in the DFT are invariant under the coordinate gauge symmetry shift [18],

$$\varphi(x + \Delta) = \varphi(x), \qquad \Delta^A = \varphi_1 \partial^A \varphi_2. \tag{3}$$

This invariance is in fact equivalent (i.e., necessary [16] and sufficient [17]) to the section condition [2],

$$\partial_A \partial^A = 0\colon\ \partial_A \partial^A \varphi = 0, \qquad \partial_A \varphi_1 \partial^A \varphi_2 = 0. \tag{4}$$

The diffeomorphism on the doubled-yet-gauged spacetime is generated by a generalized Lie derivative [3],

$$\begin{aligned}\hat{\mathcal{L}}_X T_{A_1\cdots A_n} \coloneqq{}& X^B \partial_B T_{A_1\cdots A_n} + \omega_T \partial_B X^B T_{A_1\cdots A_n} \\ &+ \sum_{i=1}^{n} 2\partial_{[A_i} X_{B]} T_{A_1\cdots A_{i-1}}{}^{B}{}_{A_{i+1}\cdots A_n},\end{aligned} \tag{5}$$

where ω_T denotes the weight. Only the $\mathbf{O}(4,4)$ indices are explicitly denoted above and other types of indices are suppressed.

Gravitational fields: The whole massless NS-NS sector in string theory enters DFT as geometric objects, in terms of a dilaton, d, and a pair of vielbeins, V_{Aa}, $\bar{V}_{A\bar{a}}$. While the vielbeins are weightless, the dilaton gives rise to the $\mathbf{O}(4,4)$ invariant integral measure after exponentiation, e^{-2d}, carrying unit weight. The vielbeins satisfy four defining properties [15,19]:

$$\begin{aligned}&V_{Aa} V^A{}_b = \eta_{ab}, \qquad \bar{V}_{A\bar{a}} \bar{V}^A{}_{\bar{b}} = \bar{\eta}_{\bar{a}\bar{b}}, \\ &V_{Aa} \bar{V}^A{}_{\bar{b}} = 0, \qquad V_{Aa} V_B{}^a + \bar{V}_{A\bar{a}} \bar{V}_B{}^{\bar{a}} = \mathcal{J}_{AB}.\end{aligned} \tag{6}$$

Namely, they are orthogonal and complete. The vielbeins are covariant $\mathbf{O}(4,4)$ vectors as their indices indicate. As a solution to Eq. (6), they can be, if desired, parametrized in terms of a pair of ordinary tetrads and a two-form gauge potential, in various ways up to $\mathbf{O}(4,4)$ rotations and field redefinitions, e.g., Eq. (22). Yet, in the present covariant formulation, no particular parametrization needs to be assumed. The defining properties, Eq. (6), suffice.

The vielbeins naturally generate a pair of symmetric, orthogonal, and complete two-index projectors,

$$\begin{aligned}&P_{AB} = P_{BA} = V_A{}^a V_{Ba}, \qquad \bar{P}_{AB} = \bar{P}_{BA} = \bar{V}_A{}^{\bar{a}} \bar{V}_{B\bar{a}}, \\ &P_A{}^B P_B{}^C = P_A{}^C, \qquad \bar{P}_A{}^B \bar{P}_B{}^C = \bar{P}_A{}^C, \\ &P_A{}^B \bar{P}_B{}^C = 0, \qquad P_A{}^B + \bar{P}_A{}^B = \delta_A{}^B,\end{aligned}$$

which, with the dilaton, d, constitute the "metric" formulation of the bosonic DFT. (The difference between the projectors is known as "generalized metric" [3]).

Semicovariant derivative and complete covariantization: The semicovariant derivative [14,15] is defined by

$$\begin{aligned}\nabla_C T_{A_1 A_2\cdots A_n} \coloneqq{}& \partial_C T_{A_1 A_2\cdots A_n} - \omega_T \Gamma^B{}_{BC} T_{A_1 A_2\cdots A_n} \\ &+ \sum_{i=1}^{n} \Gamma_{CA_i}{}^B T_{A_1\cdots A_{i-1} B A_{i+1}\cdots A_n}.\end{aligned} \tag{7}$$

By analogy with the Christoffel symbol, the connection can be uniquely chosen [15],

$$\begin{aligned}\Gamma_{CAB} ={}& 2(P\partial_C P\bar{P})_{[AB]} + 2(\bar{P}_{[A}{}^D \bar{P}_{B]}{}^E - P_{[A}{}^D P_{B]}{}^E)\partial_D P_{EC} \\ &- \frac{4}{3}(\bar{P}_{C[A}\bar{P}_{B]}{}^D + P_{C[A}P_{B]}{}^D)(\partial_D d + (P\partial^E P\bar{P})_{[ED]}),\end{aligned} \tag{8}$$

such that it satisfies torsionless conditions, $\Gamma_{C(AB)} = 0$, $\Gamma_{[CAB]} = 0$, and makes the semicovariant derivative compatible with the $\mathbf{O}(4,4)$ invariant metric, the projectors and the dilaton,

$$\begin{aligned}&\nabla_A \mathcal{J}_{BC} = 0, \qquad \nabla_A P_{BC} = 0, \qquad \nabla_A \bar{P}_{BC} = 0, \\ &\nabla_A d = -\frac{1}{2} e^{2d} \nabla_A (e^{-2d}) = \partial_A d + \frac{1}{2}\Gamma^B{}_{BA} = 0.\end{aligned} \tag{9}$$

These are all analogous to the Riemannian Einstein gravity. However, unlike the Christoffel symbol, the diffeomorphism [Eq. (5)] cannot transform the connection [Eq. (8)] to vanish pointwise. This can be viewed as the failure of the equivalence principle applied to an extended object, i.e., string.

In order to take care of not only the diffeomorphism [Eq. (5)] but also the twofold local Lorentz symmetries, we need to further set the master semicovariant derivative [19,20],

$$\mathcal{D}_A \coloneqq \nabla_A + \Phi_A + \bar{\Phi}_A = \partial_A + \Gamma_A + \Phi_A + \bar{\Phi}_A, \tag{10}$$

which includes the spin connections, Φ_A, $\bar{\Phi}_A$, for each local Lorentz group, $\mathbf{Spin}(1,3)$, $\mathbf{Spin}(3,1)$, respectively. By definition, in addition to the dilaton as Eq. (9), it is compatible with the vielbeins,

$$\begin{aligned}\mathcal{D}_A V_{Ba} &= \partial_A V_{Ba} + \Gamma_{AB}{}^C V_{Ca} + \Phi_{Aa}{}^b V_{Bb} = 0,\\ \mathcal{D}_A \bar{V}_{B\bar{a}} &= \partial_A \bar{V}_{B\bar{a}} + \Gamma_{AB}{}^C \bar{V}_{C\bar{a}} + \bar{\Phi}_{A\bar{a}}{}^{\bar{b}} \bar{V}_{B\bar{b}} = 0.\end{aligned} \tag{11}$$

The spin connections are then fixed by the diffeomorphism connection, Γ (8),

$$\Phi_{Aab} = V^B{}_a \nabla_A V_{Bb}, \qquad \bar{\Phi}_{A\bar{a}\bar{b}} = \bar{V}^B{}_{\bar{a}} \nabla_A \bar{V}_{B\bar{b}}. \tag{12}$$

The master semicovariant derivative is also compatible with all the constant metrics, $\mathcal{J}_{AB}, \eta_{ab}, \bar{\eta}_{\bar{a}\bar{b}}$, as well as the gamma matrices, $(\gamma^a)^\alpha{}_\beta$, $(\bar{\gamma}^{\bar{a}})^{\bar{\alpha}}{}_{\bar{\beta}}$. Consequently, the standard relation between the spinorial and the vectorial representations of the spin connections holds, such as $\Phi_A{}^\alpha{}_\beta = \frac{1}{4}\Phi_{Aab}(\gamma^{ab})^\alpha{}_\beta$ and $\bar{\Phi}_A{}^{\bar{\alpha}}{}_{\bar{\beta}} = \frac{1}{4}\bar{\Phi}_{A\bar{a}\bar{b}}(\bar{\gamma}^{\bar{a}\bar{b}})^{\bar{\alpha}}{}_{\bar{\beta}}$.

The characteristic of the (master) semicovariant derivative is that, although it may not be fully diffeomorphic covariant by itself, it can be completely covariantized after being appropriately contracted with the projectors or the vielbeins. The completely covariant derivatives, relevant to the present work, are from Ref. [15],

$$\begin{aligned}&\bar{P}_C{}^D P_{A_1}{}^{B_1}\cdots P_{A_n}{}^{B_n}\mathcal{D}_D T_{B_1\cdots B_n} \Leftrightarrow \mathcal{D}_{\bar{a}} T_{b_1\cdots b_n},\\ &P_C{}^D \bar{P}_{A_1}{}^{B_1}\cdots \bar{P}_{A_n}{}^{B_n}\mathcal{D}_D T_{B_1\cdots B_n} \Leftrightarrow \mathcal{D}_a T_{\bar{b}_1\cdots \bar{b}_n}.\end{aligned} \tag{13}$$

Further, acting on the $\mathbf{Spin}(1,3)$ spinor, ψ^α (unprimed), or the $\mathbf{Spin}(3,1)$ spinor, $\psi'^{\bar{\alpha}}$ (primed), both of which are $\mathbf{O}(4,4)$ scalars, the completely covariant Dirac operators, with respect to both diffeomorphisms and local Lorentz symmetries, are from [19–21],

$$\gamma^a \mathcal{D}_a \psi = \gamma^A \mathcal{D}_A \psi, \qquad \bar{\gamma}^{\bar{a}} \mathcal{D}_{\bar{a}} \psi' = \bar{\gamma}^A \mathcal{D}_A \psi'. \tag{14}$$

For the full list of the completely covariant derivatives, we refer readers to Refs. [15,19,21] (cf., Ref. [22]).

Standard model double field theory (SM-DFT).—The SM-DFT consists of gauge bosons, Higgs bosons, and three generations of quarks and leptons. We now turn to their DFT descriptions.

$\mathbf{SU}(3)\times\mathbf{SU}(2)\times\mathbf{U}(1)$ gauge bosons: For each gauge symmetry in the SM, we assign a Lie algebra valued Yang-Mills potential, $\mathcal{A}_B$, which should be a diffeomorphism covariant $\mathbf{O}(4,4)$ vector.

We introduce the *gauged* master semicovariant derivative,

$$\tilde{\mathcal{D}}_B = \mathcal{D}_B - i\sum_{\mathcal{A}} \mathcal{R}[\mathcal{A}_B], \tag{15}$$

where the sum is over all the gauge symmetries, and $\mathcal{R}[\mathcal{A}_B]$ denotes the appropriate representation (depending on the quark or lepton, left or right, or Higgs boson) which also includes the corresponding coupling constant, $g_{\mathcal{A}}$.

We consider the semicovariant field strength defined in terms of the semicovariant derivative [23],

$$\mathcal{F}_{AB} \coloneqq \nabla_A \mathcal{A}_B - \nabla_B \mathcal{A}_A - i g_{\mathcal{A}}[\mathcal{A}_A, \mathcal{A}_B]. \tag{16}$$

Unlike the Riemannian case, the Γ connection inside the semicovariant derivatives are not canceled out. After, in fact only after contractions with the "orthogonal" vielbeins, like Eq. (13), it can be completely covariantized to take the form [23]

$$\mathcal{F}_{a\bar{b}} = V^A{}_a \bar{V}^B{}_{\bar{b}} \mathcal{F}_{AB}. \tag{17}$$

Carrying no $\mathbf{O}(4,4)$ index, $\mathcal{F}_{a\bar{b}}$ is a diffeomorphism scalar. The Yang-Mills gauge symmetry is realized by

$$\begin{aligned}&\mathcal{A}_A \to \mathbf{g}\mathcal{A}_A\mathbf{g}^{-1} - i\frac{1}{g_{\mathcal{A}}}(\partial_A \mathbf{g})\mathbf{g}^{-1},\\ &\mathcal{F}_{a\bar{b}} \to \mathbf{g}\mathcal{F}_{a\bar{b}}\mathbf{g}^{-1}.\end{aligned} \tag{18}$$

It is crucial to note that the completely covariant Yang-Mills field strength, $\mathcal{F}_{a\bar{b}}$ (17), carries "opposite" vector indices, one unbarred $\mathbf{Spin}(1,3)$ and the other barred $\mathbf{Spin}(3,1)$. Clearly then, with $\mathcal{F}_{a\bar{b}}$ alone, it is impossible to write the topological θ term. Hence, the strong *CP* problem is naturally solved within the above DFT setup. On the other hand, the kinetic term of the gauge bosons, along with that of the Higgs boson and its potential, read

$$\sum_{\mathcal{A}} \mathrm{Tr}(\mathcal{F}_{a\bar{b}}\mathcal{F}^{a\bar{b}}) - (P^{AB} - \bar{P}^{AB})(\tilde{\mathcal{D}}_A\phi)^\dagger \tilde{\mathcal{D}}_B\phi - V(\phi). \tag{19}$$

It is worthwhile to note that $P^{AB} - \bar{P}^{AB}$ above corresponds to the "generalized metric".

Apparently there appear doubled off-shell degrees of freedom in the eight-component gauge potential. In order to halve them, as in Refs. [17,24,25], we may impose the following "gauged" section condition:

$$(\partial_A - i\mathcal{A}_A)(\partial^A - i\mathcal{A}^A) = 0, \tag{20}$$

which, with Eq. (4), implies $\mathcal{A}_A\partial^A = 0$, $\partial_A\mathcal{A}^A = 0$, $\mathcal{A}_A\mathcal{A}^A = 0$. For consistency, this condition is preserved under all the symmetry transformations: $\mathbf{O}(4,4)$ rotations, diffeomorphism [Eq. (5)] and the Yang-Mills gauge symmetry [Eq. (18)], see Ref. [26] for demonstration.

Quarks and leptons: Since the spin group is twofold, we need to decide which spin class each lepton and quark belongs to. The nondiagonal Yukawa couplings to the Higgs doublet suggest that all the quarks should belong to the same spin class. This is separately true for the leptons as well. Hence, there are two logical possibilities: the leptons and the quarks share the same spin group, or they belong to the two distinct spin classes. Yet, the absence of experimental evidence of the proton decay might indicate that they might belong to the different spin classes. In this case, the quarks and the leptons enter the SM-DFT Lagrangian through the kinetic terms as well as the Yukawa couplings to the Higgs boson as follows:

$$\sum_{\psi}\bar{\psi}\gamma^{a}\tilde{\mathcal{D}}_{a}\psi+\sum_{\psi'}\bar{\psi}'\bar{\gamma}^{\bar{a}}\tilde{\mathcal{D}}_{\bar{a}}\psi'+y_{d}\bar{q}\cdot\phi d + y_{u}\bar{q}\cdot\tilde{\phi}u+y_{e}\bar{l}'\cdot\phi e', \tag{21}$$

where, without loss of generality, we have assigned the spin group, $\mathbf{Spin}(1,3)$ to the quarks, $\psi=(q,u,d)$, and the other $\mathbf{Spin}(3,1)$ to the leptons, $\psi'=(l',e')$ [27]. Of course, if the quarks and the leptons should belong to the same spin class, we need to remove the primes.

Higher order corrections: Possible higher order corrections have been classified in, e.g., Refs. [28,29]. However, these did not take into account the enhanced symmetry we have been considering. The classification needs to be further constrained. For example, the completely covariant field strength, $\mathcal{F}_{a\bar{b}}$ (17), is no longer able to couple to the skew-symmetric bifermionic tensors, $\bar{\psi}\gamma^{ab}\psi$ nor $\bar{\psi}'\bar{\gamma}^{\bar{a}\bar{b}}\psi'$, to form a dimension-5 operator. Further, if the quarks and the leptons indeed belong to the two different spin classes, one cannot form a bifermion, one from a quark and the other from a lepton. One cannot also contract a biquark vector with a bilepton vector, i.e., $\bar{\psi}\gamma^{a}\psi$ and $\bar{\psi}'\bar{\gamma}^{\bar{a}}\psi'$, which would form a dimension-6 operator. Experimental observation of this kind of suppression will confirm (or disprove) our conjecture that the quarks and the leptons may belong to the two distinct spin classes.

Riemannian reduction: With the decompositions of the doubled coordinates, $x^{A}=(\tilde{x}_{\mu},x^{\nu})$, $\partial_{A}=(\tilde{\partial}^{\mu},\partial_{\nu})$, and the gauge potential, $\mathcal{A}_{A}=(\tilde{A}^{\mu},A_{\nu})$, we can solve the section conditions, Eqs. (4) and (20), explicitly. Up to $\mathbf{O}(4,4)$ rotations, the most general solution is given by simply setting $\tilde{\partial}^{\mu}\equiv 0$ and $\tilde{A}^{\mu}\equiv 0$, such that $\partial_{A}\equiv(0,\partial_{\nu})$ and $\mathcal{A}_{A}\equiv(0,A_{\nu})$. It is then instructive to parametrize the vielbeins in terms of the Kalb-Ramond two-form potential, $B_{\mu\nu}$, and a pair of tetrads, ${e_{\mu}}^{a},{\bar{e}_{\mu}}^{\bar{a}}$, as follows [15]:

$$V_{Aa}=\frac{1}{\sqrt{2}}\begin{pmatrix}(e^{-1})_{a}{}^{\mu}\\ (B+e)_{\nu a}\end{pmatrix},\qquad \bar{V}_{A\bar{a}}=\frac{1}{\sqrt{2}}\begin{pmatrix}(\bar{e}^{-1})_{\bar{a}}{}^{\mu}\\ (B+\bar{e})_{\nu\bar{a}}\end{pmatrix}, \tag{22}$$

where we set $B_{\mu a}=B_{\mu\nu}(e^{-1})_{a}{}^{\nu}$, $B_{\mu\bar{a}}=B_{\mu\nu}(\bar{e}^{-1})_{\bar{a}}{}^{\nu}$ and the twofold tetrads must give the same Riemannian metric to solve Eq. (6): ${e_{\mu}}^{a}{e_{\nu}}^{b}\eta_{ab}=-{\bar{e}_{\mu}}^{\bar{a}}{\bar{e}_{\nu}}^{\bar{b}}\bar{\eta}_{\bar{a}\bar{b}}\equiv g_{\mu\nu}$.

The above setting will then, with the price of breaking the $\mathbf{O}(4,4)$ covariance, reduce the SM-DFT characterized by Eqs. (19) and (21), to undoubled, i.e., literally four dimensional, SM equipped with two copies of the tetrads which take care of the twofold spin groups (see, e.g., section 3.2 of Ref. [23] and Appendix A.4 of Ref. [21]). Finally, on the flat background ($B\equiv 0$), the full gauge fixing, ${e_{\mu}}^{a}\equiv{\delta_{\mu}}^{a}$, ${\bar{e}_{\mu}}^{\bar{a}}\equiv{\delta_{\mu}}^{\bar{a}}$, will completely reduce the SM-DFT to the conventional formulation of the SM.

Concluding remarks.—String theory has not yet succeeded in deriving the precise form of the standard model. Yet, T duality and the twofold spin structure are genuine stringy effects, which even survive [30] after the Scherk-Schwarz dimensional reductions of $D=10$ DFT [31,32]. We have shown that, without any extra physical degree introduced, the standard model can be readily reformulated as a double field theory, such that it can couple to an arbitrary stringy gravitational background in an $\mathbf{O}(4,4)$ T-duality covariant manner and manifest two independent local Lorentz symmetries, $\mathbf{Spin}(1,3)\times\mathbf{Spin}(3,1)$. We have further pointed out the possibility that the quarks and the leptons may belong to the two different spin classes. The lacking of experimental observation of the proton decay seems to support this conjecture. We urge experimentalists to test this.

Our formulation of the standard model as a DFT is limited to the classical level. Exploration of the quantum aspects require further analyses. For example, one might worry about the chiral anomaly cancelation in our formulation of the standard model. Since the same gauge potential, $\mathcal{A}_{B}$ as the physical quanta, is minimally coupled to the quarks and the leptons through the contractions, $\mathcal{A}_{a}=\mathcal{A}_{B}V^{B}{}_{a}$ and $\mathcal{A}_{\bar{a}}=\mathcal{A}_{B}\bar{V}^{B}{}_{\bar{a}}$, we expect that the anomaly cancelation of the triangular Feynman diagrams should still work.

It will be interesting to decide the spin class of the dark matter as well. If the quarks and the leptons should ever share the same spin group, it seems reasonable to expect that the dark matter spin may be the different one. But here we can only speculate.

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[27] Respectively, $q, u/d, l', e'$ denote the quark doublets, the up- or down-type quark singlets, the lepton doublets, and the electron-type singlets. Flavor indices are suppressed. Further, $\tilde{\phi}$ is the $\mathbf{SU}(2)$ doublet conjugation of the Higgs.
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