Spin polarized states in strongly asymmetric nuclear matter

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The possibility of appearance of spin polarized states in strongly asymmetric nuclear matter is analyzed within the framework of a Fermi liquid theory with the Skyrme effective interaction. The zero temperature dependence of the neutron and proton spin polarization parameters as functions of density is found for *SLy*4 and *SLy*5 effective forces. It is shown that at some critical density strongly asymmetric nuclear matter undergoes a phase transition to the state with the oppositely directed spins of neutrons and protons while the state with the same direction of spins does not appear. In comparison with neutron matter, even small admixture of protons strongly decreases the threshold density of spin instability. It is clarified that protons become totally polarized within a very narrow density domain while the density profile of the neutron spin polarization parameter is characterized by the appearance of long tails near the transition density.

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I. INTRODUCTION

The spontaneous appearance of spin polarized states in nuclear matter is the topic of a great current interest due to relevance in astrophysics. In particular, the effects of spin correlations in the medium strongly influence the neutrino cross section and neutrino luminosity. Hence, depending on whether nuclear matter is spin polarized or not, drastically different scenarios of supernova explosion and cooling of neutron stars can be realized. Another aspect relates to pulsars, which are considered to be rapidly rotating neutron stars, surrounded by strong magnetic field. There is still no general consensus regarding the mechanism to generate such strong magnetic field of a neutron star. One of the hypotheses is that magnetic field can be produced by a spontaneous ordering of spins in the dense stellar core.

The possibility of a phase transition of normal nuclear matter to the ferromagnetic state was studied by many authors. In the gas model of hard spheres, neutron matter becomes ferromagnetic at $\varrho \approx 0.41$ fm^{-3} [1]. It was found in Refs. [2,3] that the inclusion of long-range attraction significantly increases the ferromagnetic transition density (e.g., up to $\varrho \approx 2.3$ fm^{-3} in the Brueckner theory with a simple central potential and hard core only for singlet spin states [3]). By determining magnetic susceptibility with Skyrme effective forces, it was shown in Ref. [4] that the ferromagnetic transition occurs at $\varrho \approx 0.18 - 0.26$ fm^{-3}. The Fermi liquid criterion for the ferromagnetic instability in neutron matter with the Skyrme interaction is reached at $\varrho \approx 2 - 4\varrho_0$ [5], where $\varrho_0 = 0.16$ fm^{-3} is the nuclear matter saturation density. The general conditions on the parameters of neutron-neutron interaction, which result in a magnetically ordered state of neutron matter, were formulated in Ref. [6]. Spin correlations in dense neutron matter were studied within the relativistic Dirac-Hartree-Fock approach with the effective nucleon-meson Lagrangian in Ref. [7], predicting the ferromagnetic transition at several times nuclear matter saturation density.

The importance of the Fock exchange term in the relativistic mean-field approach for the occurrence of ferromagnetism in nuclear matter was established in Ref. [8]. The stability of strongly asymmetric nuclear matter with respect to spin fluctuations was investigated in Ref. [9], where it was shown that the system with localized protons can develop a spontaneous polarization, if the neutron-proton spin interaction exceeds some threshold value. This conclusion was confirmed also by calculations within the relativistic Dirac-Hartree-Fock approach to strongly asymmetric nuclear matter [10]. Competition between ferromagnetic (FM) and antiferromagnetic (AFM) spin ordering in symmetric nuclear matter with the Skyrme effective interaction was studied in Ref. [11], where it was clarified that a FM spin state is thermodynamically preferable over a AFM one for all relevant densities.

For the models with realistic nucleon-nucleon (*NN*) interaction, the ferromagnetic phase transition seems to be suppressed up to densities well above ϱ_0 [12–14]. In particular, no evidence of ferromagnetic instability has been found in recent studies of neutron matter [15] and asymmetric nuclear matter [16] within the Brueckner-Hartree-Fock approximation with realistic Nijmegen II, Reid93 and Nijmegen NSC97e *NN* interactions. The same conclusion was obtained in Ref. [17], where magnetic susceptibility of neutron matter was calculated with the use of the Argonne v_{18} two-body potential and Urbana IX three-body potential.

Here we continue the study of spin polarizability of nuclear matter with the use of an effective *NN* interaction. As a framework of consideration, a Fermi liquid (FL) description of nuclear matter is chosen [18,19]. As a potential of *NN* interaction, we use the Skyrme effective interaction, utilized earlier in a number of contexts for nuclear matter calculations [20–23]. The main emphasis will be laid on strongly asymmetric nuclear matter and neutron matter as its limiting case. We explore the possibility of FM and AFM phase transitions in nuclear matter, when the spins of protons and neutrons are aligned in the same direction or in the opposite

direction, respectively. In contrast with the approach based on the calculation of magnetic susceptibility, we obtain the self-consistent equations for the FM and AFM spin order parameters and find their solutions at zero temperature. This allows us to determine not only the critical density of instability with respect to spin fluctuations, but also to establish the density dependence of the order parameters and to clarify the question of thermodynamic stability of various phases.

Note that we consider the thermodynamic properties of spin polarized states in nuclear matter up to the high density region relevant for astrophysics. Nevertheless, we take into account the nucleon degrees of freedom only, although other degrees of freedom, such as pions, hyperons, kaons, or quarks could be important at such high densities.

II. BASIC EQUATIONS

The normal states of nuclear matter are described by the normal distribution function of nucleons $f_{\kappa_1\kappa_2} = \mathrm{Tr}\, \varrho a_{\kappa_2}^{+} a_{\kappa_1}$, where $\kappa \equiv (\mathbf{p}, \sigma, \tau)$, $\mathbf{p}$ is momentum, $\sigma(\tau)$ is the projection of spin (isospin) on the third axis, and ϱ is the density matrix of the system. The energy of the system is specified as a functional of the distribution function f, $E = E(f)$, and determines the single particle energy

$$\varepsilon_{\kappa_1\kappa_2}(f) = \frac{\partial E(f)}{\partial f_{\kappa_2\kappa_1}}. \qquad (1)$$

The self-consistent matrix equation for determining the distribution function f follows from the minimum condition of the thermodynamic potential [18] and is

$$f = \{\exp(Y_0\varepsilon + Y_4) + 1\}^{-1} \equiv \{\exp(Y_0\xi) + 1\}^{-1}. \qquad (2)$$

Here the quantities ε and Y_4 are matrices in the space of κ variables, with $Y_{4\kappa_1\kappa_2} = Y_{4\tau_1}\delta_{\kappa_1\kappa_2}$ $(\tau_1 = n, p)$, $Y_0 = 1/T$, $Y_{4n} = -\mu_n^0/T$, and $Y_{4p} = -\mu_p^0/T$ being the Lagrange multipliers, μ_n^0 and μ_p^0 being the chemical potentials of neutrons and protons, and T being the temperature. Since it is assumed to consider a nuclear system with an excess of neutrons, the positive isospin projection is assigned to neutrons. This is different from the formalism of Ref. [11], aimed to investigate symmetric nuclear matter. Further we shall study the possibility of formation of various types of spin ordering (ferromagnetic and antiferromagnetic) in nuclear matter.

The normal distribution function can be expanded in the Pauli matrices σ_i and τ_k in spin and isospin spaces

$$f(\mathbf{p}) = f_{00}(\mathbf{p})\sigma_0\tau_0 + f_{30}(\mathbf{p})\sigma_3\tau_0 + f_{03}(\mathbf{p})\sigma_0\tau_3 + f_{33}(\mathbf{p})\sigma_3\tau_3. \qquad (3)$$

For the energy functional invariant with respect to rotations in spin and isospin spaces, the structure of the single particle energy is similar to the structure of the distribution function f:

$$\varepsilon(\mathbf{p}) = \varepsilon_{00}(\mathbf{p})\sigma_0\tau_0 + \varepsilon_{30}(\mathbf{p})\sigma_3\tau_0 + \varepsilon_{03}(\mathbf{p})\sigma_0\tau_3 + \varepsilon_{33}(\mathbf{p})\sigma_3\tau_3. \qquad (4)$$

Using Eqs. (2) and (4), one can express evidently the distribution functions $f_{00}, f_{30}, f_{03}, f_{33}$ in terms of the quantities ε:

$$f_{00} = \tfrac{1}{4}\{n(\omega_{+,+}) + n(\omega_{+,-}) + n(\omega_{-,+}) + n(\omega_{-,-})\},$$

$$f_{30} = \tfrac{1}{4}\{n(\omega_{+,+}) + n(\omega_{+,-}) - n(\omega_{-,+}) - n(\omega_{-,-})\},$$

$$f_{03} = \tfrac{1}{4}\{n(\omega_{+,+}) - n(\omega_{+,-}) + n(\omega_{-,+}) - n(\omega_{-,-})\},$$

$$f_{33} = \tfrac{1}{4}\{n(\omega_{+,+}) - n(\omega_{+,-}) - n(\omega_{-,+}) + n(\omega_{-,-})\}. \qquad (5)$$

Here $n(\omega) = [\exp(Y_0\omega) + 1]^{-1}$ and

$$\omega_{+,+} = \xi_{00} + \xi_{30} + \xi_{03} + \xi_{33},$$

$$\omega_{+,-} = \xi_{00} + \xi_{30} - \xi_{03} - \xi_{33},$$

$$\omega_{-,+} = \xi_{00} - \xi_{30} + \xi_{03} - \xi_{33},$$

$$\omega_{-,-} = \xi_{00} - \xi_{30} - \xi_{03} + \xi_{33},$$

where

$$\xi_{00} = \varepsilon_{00} - \mu_{00}^0, \qquad \xi_{30} = \varepsilon_{30},$$

$$\xi_{03} = \varepsilon_{03} - \mu_{03}^0, \qquad \xi_{33} = \varepsilon_{33},$$

$$\mu_{00}^0 = \frac{\mu_n^0 + \mu_p^0}{2}, \qquad \mu_{03}^0 = \frac{\mu_n^0 - \mu_p^0}{2}.$$

As follows from the structure of the distribution functions f, the quantity $\omega_{\pm,\pm}$, being the exponent in the Fermi distribution function n, plays the role of the quasiparticle spectrum. We consider the case when the spectrum is fourfold split due to the spin and isospin dependence of the single particle energy $\varepsilon(\mathbf{p})$ in Eq. (4). The branches $\omega_{\pm,+}$ correspond to neutrons with spin up and spin down, and the branches $\omega_{\pm,-}$ correspond to protons with spin up and spin down.

The distribution functions f should satisfy the normalization conditions

$$\frac{4}{\mathcal{V}}\sum_{\mathbf{p}} f_{00}(\mathbf{p}) = \varrho, \qquad (6)$$

$$\frac{4}{\mathcal{V}}\sum_{\mathbf{p}} f_{03}(\mathbf{p}) = \varrho_n - \varrho_p \equiv \alpha\varrho, \qquad (7)$$

$$\frac{4}{\mathcal{V}}\sum_{\mathbf{p}} f_{30}(\mathbf{p}) = \varrho_\uparrow - \varrho_\downarrow \equiv \Delta\varrho_{\uparrow\uparrow}, \qquad (8)$$

$$\frac{4}{\mathcal{V}}\sum_{\mathbf{p}} f_{33}(\mathbf{p}) = (\varrho_{n\uparrow} + \varrho_{p\downarrow}) - (\varrho_{n\downarrow} + \varrho_{p\uparrow}) \equiv \Delta\varrho_{\uparrow\downarrow}. \qquad (9)$$

Here α is the isospin asymmetry parameter, $\varrho_{n\uparrow}, \varrho_{n\downarrow}$ and $\varrho_{p\uparrow}, \varrho_{p\downarrow}$ are the neutron and proton number densities with spin up and spin down, respectively; $\varrho_\uparrow = \varrho_{n\uparrow} + \varrho_{p\uparrow}$ and $\varrho_\downarrow = \varrho_{n\downarrow} + \varrho_{p\downarrow}$ are the nucleon densities with spin up and spin down. The quantities $\Delta\varrho_{\uparrow\uparrow}$ and $\Delta\varrho_{\uparrow\downarrow}$ may be regarded as FM and AFM spin order parameters. Indeed, in symmetric

nuclear matter, if all nucleon spins are aligned in one direction (totally polarized FM spin state), then $\Delta\varrho_{\uparrow\uparrow}=\varrho$ and $\Delta\varrho_{\uparrow\downarrow}=0$; if all neutron spins are aligned in one direction and all proton spins in the opposite one (totally polarized AFM spin state), then $\Delta\varrho_{\uparrow\downarrow}=\varrho$ and $\Delta\varrho_{\uparrow\uparrow}=0$. In turn, from Eqs. (6)–(9) one can find the neutron and proton number densities with spin up and spin down as functions of the total density ϱ, isospin excess $\delta\varrho\equiv\alpha\varrho$, and FM and AFM order parameters $\Delta\varrho_{\uparrow\uparrow}$ and $\Delta\varrho_{\uparrow\downarrow}$:

$$\varrho_{n\uparrow}=\tfrac{1}{4}(\varrho+\delta\varrho+\Delta\varrho_{\uparrow\uparrow}+\Delta\varrho_{\uparrow\downarrow}),$$

$$\varrho_{n\downarrow}=\tfrac{1}{4}(\varrho+\delta\varrho-\Delta\varrho_{\uparrow\uparrow}-\Delta\varrho_{\uparrow\downarrow}),$$

$$\varrho_{p\uparrow}=\tfrac{1}{4}(\varrho-\delta\varrho+\Delta\varrho_{\uparrow\uparrow}-\Delta\varrho_{\uparrow\downarrow}),$$

$$\varrho_{p\downarrow}=\tfrac{1}{4}(\varrho-\delta\varrho-\Delta\varrho_{\uparrow\uparrow}+\Delta\varrho_{\uparrow\downarrow}).$$

In order to characterize spin ordering in the neutron and proton subsystems, it is convenient to introduce neutron and proton spin polarization parameters

$$\Pi_n=\frac{\varrho_{n\uparrow}-\varrho_{n\downarrow}}{\varrho_n},\quad \Pi_p=\frac{\varrho_{p\uparrow}-\varrho_{p\downarrow}}{\varrho_p}. \quad (10)$$

The expressions for the spin order parameters $\Delta\varrho_{\uparrow\uparrow}$ and $\Delta\varrho_{\uparrow\downarrow}$ through the spin polarization parameters read

$$\Delta\varrho_{\uparrow\uparrow}=\varrho_n\Pi_n+\varrho_p\Pi_p,\quad \Delta\varrho_{\uparrow\downarrow}=\varrho_n\Pi_n-\varrho_p\Pi_p.$$

To obtain the self–consistent equations, we specify the energy functional of the system in the form

$$E(f)=E_0(f)+E_{int}(f), \quad (11)$$

$$E_0(f)=4\sum_{\mathbf{p}}\varepsilon_0(\mathbf{p})f_{00}(\mathbf{p}),\quad \varepsilon_0(\mathbf{p})=\frac{\mathbf{p}^2}{2m_0},$$

$$E_{int}(f)=2\sum_{\mathbf{p}}\{\tilde{\varepsilon}_{00}(\mathbf{p})f_{00}(\mathbf{p})+\tilde{\varepsilon}_{30}(\mathbf{p})f_{30}(\mathbf{p})+\tilde{\varepsilon}_{03}(\mathbf{p})f_{03}(\mathbf{p})$$
$$+\tilde{\varepsilon}_{33}(\mathbf{p})f_{33}(\mathbf{p})\},$$

$$\tilde{\varepsilon}_{00}(\mathbf{p})=\frac{1}{2\mathcal{V}}\sum_{\mathbf{q}}U_0(\mathbf{k})f_{00}(\mathbf{q}),\quad \mathbf{k}=\frac{\mathbf{p}-\mathbf{q}}{2},$$

$$\tilde{\varepsilon}_{30}(\mathbf{p})=\frac{1}{2\mathcal{V}}\sum_{\mathbf{q}}U_1(\mathbf{k})f_{30}(\mathbf{q}),$$

$$\tilde{\varepsilon}_{03}(\mathbf{p})=\frac{1}{2\mathcal{V}}\sum_{\mathbf{q}}U_2(\mathbf{k})f_{03}(\mathbf{q}),$$

$$\tilde{\varepsilon}_{33}(\mathbf{p})=\frac{1}{2\mathcal{V}}\sum_{\mathbf{q}}U_3(\mathbf{k})f_{33}(\mathbf{q}).$$

Here m_0 is the bare mass of a nucleon, $U_0(\mathbf{k})$, ..., $U_3(\mathbf{k})$ are the normal FL amplitudes, and $\tilde{\varepsilon}_{00}$, $\tilde{\varepsilon}_{30}$, $\tilde{\varepsilon}_{03}$, $\tilde{\varepsilon}_{33}$ are the FL

corrections to the free single particle spectrum. Further we do not take into account the effective tensor forces, which lead to coupling of the momentum and spin degrees of freedom [24–26], and, correspondingly, to anisotropy in the momentum dependence of FL amplitudes with respect to the spin polarization axis. Using Eqs. (1) and (11), we get the self-consistent equations in the form

$$\xi_{00}(\mathbf{p})=\varepsilon_0(\mathbf{p})+\tilde{\varepsilon}_{00}(\mathbf{p})-\mu_{00}^0,\quad \xi_{30}(\mathbf{p})=\tilde{\varepsilon}_{30}(\mathbf{p}), \quad (12)$$

$$\xi_{03}(\mathbf{p})=\tilde{\varepsilon}_{03}(\mathbf{p})-\mu_{03}^0,\quad \xi_{33}(\mathbf{p})=\tilde{\varepsilon}_{33}(\mathbf{p}).$$

To obtain numerical results, we use the Skyrme effective interaction. In the case of Skyrme forces, the normal FL amplitudes read [19]

$$U_0(\mathbf{k})=6t_0+t_3\varrho^\beta+\frac{2}{\hbar^2}[3t_1+t_2(5+4x_2)]\mathbf{k}^2, \quad (13)$$

$$U_1(\mathbf{k})=-2t_0(1-2x_0)-\frac{1}{3}t_3\varrho^\beta(1-2x_3)-\frac{2}{\hbar^2}[t_1(1-2x_1)$$
$$-t_2(1+2x_2)]\mathbf{k}^2$$
$$\equiv a+b\mathbf{k}^2,$$

$$U_2(\mathbf{k})=-2t_0(1+2x_0)-\frac{1}{3}t_3\varrho^\beta(1+2x_3)-\frac{2}{\hbar^2}[t_1(1+2x_1)$$
$$-t_2(1+2x_2)]\mathbf{k}^2,$$

$$U_3(\mathbf{k})=-2t_0-\frac{1}{3}t_3\varrho^\beta-\frac{2}{\hbar^2}(t_1-t_2)\mathbf{k}^2\equiv c+d\mathbf{k}^2,$$

where t_i, x_i, β are the phenomenological constants, characterizing a given parametrization of the Skyrme forces. In the numerical calculations we shall use *SLy*4 and *SLy*5 potentials [27], developed to fit the properties of systems with large isospin asymmetry. With account of the evident form of FL amplitudes and Eqs. (6)–(9), one can obtain

$$\xi_{00}=\frac{p^2}{2m_{00}}-\mu_{00}, \quad (14)$$

$$\xi_{03}=\frac{p^2}{2m_{03}}-\mu_{03}, \quad (15)$$

$$\xi_{30}=\left(a+b\frac{\mathbf{p}^2}{4}\right)\frac{\Delta\varrho_{\uparrow\uparrow}}{8}+\frac{b}{32}\langle\mathbf{q}^2\rangle_{30}, \quad (16)$$

$$\xi_{33}=\left(c+d\frac{\mathbf{p}^2}{4}\right)\frac{\Delta\varrho_{\uparrow\downarrow}}{8}+\frac{d}{32}\langle\mathbf{q}^2\rangle_{33}, \quad (17)$$

where the effective nucleon mass m_{00} and effective isovector mass m_{03} are defined by the formulas

$$\frac{\hbar^2}{2m_{00}} = \frac{\hbar^2}{2m_0} + \frac{\varrho}{16}[3t_1 + t_2(5 + 4x_2)], \qquad (18)$$

$$\frac{\hbar^2}{2m_{03}} = \frac{\alpha\varrho}{16}[t_2(1 + 2x_2) - t_1(1 + 2x_1)], \qquad$$

and the renormalized chemical potentials μ_{00} and μ_{03} should be determined from Eqs. (6) and (7). In Eqs. (16) and (17), $\langle \mathbf{q}^2 \rangle_{30}$ and $\langle \mathbf{q}^2 \rangle_{33}$ are the second order moments of the corresponding distribution functions

$$\langle \mathbf{q}^2 \rangle_{30} = \frac{4}{V} \sum_{\mathbf{q}} \mathbf{q}^2 f_{30}(\mathbf{q}), \qquad (19)$$

$$\langle \mathbf{q}^2 \rangle_{33} = \frac{4}{V} \sum_{\mathbf{q}} \mathbf{q}^2 f_{33}(\mathbf{q}). \qquad (20)$$

Thus, with account of expressions (5) for the distribution functions f, we obtain the self-consistent equations (6)–(9), (19), and (20), for the effective chemical potentials μ_{00}, μ_{03}, FM and AFM spin order parameters $\Delta\varrho_{\uparrow\uparrow}$, $\Delta\varrho_{\uparrow\downarrow}$, and second order moments $\langle \mathbf{q}^2 \rangle_{30}$, $\langle \mathbf{q}^2 \rangle_{33}$. It is easy to see that the self-consistent equations remain invariable under a global flip of spins, when neutrons (protons) with spin up and spin down are interchanged, and under a global flip of isospins, when neutrons and protons with the same spin projection are interchanged.

Let us consider, what differences will be in the case of neutron matter. Neutron matter is an infinite nuclear system, consisting of nucleons of one species, i.e., neutrons, and, hence, the formalism of one-component Fermi liquid should be applied for the description of its properties. Formally neutron matter can be considered as the limiting case of asymmetric nuclear matter, corresponding to the isospin asymmetry $\alpha = 1$. The individual state of a neutron is characterized by momentum $\mathbf{p}$ and spin projection σ. The self-consistent equation has the form of Eq. (2), where all quantities are matrices in the space of $\kappa \equiv (\mathbf{p}, \sigma)$ variables. The normal distribution function and single particle energy can be expanded in the Pauli matrices in spin space,

$$f(\mathbf{p}) = f_0(\mathbf{p})\sigma_0 + f_3(\mathbf{p})\sigma_3, \qquad (21)$$

$$\varepsilon(\mathbf{p}) = \varepsilon_0(\mathbf{p})\sigma_0 + \varepsilon_3(\mathbf{p})\sigma_3.$$

The energy functional of neutron matter is characterized by two normal FL amplitudes $U_0^n(\mathbf{k})$ and $U_1^n(\mathbf{k})$. Applying the same procedure as in Ref. [19], the normal FL amplitudes can be found in terms of the Skyrme force parameters t_i, x_i, β:

$$U_0^n(\mathbf{k}) = 2t_0(1 - x_0) + \frac{t_3}{3}\varrho^\beta(1 - x_3)$$

$$+ \frac{2}{\hbar^2}[t_1(1 - x_1) + 3t_2(1 + x_2)]\mathbf{k}^2, \qquad (22)$$

$$U_1^n(\mathbf{k}) = -2t_0(1 - x_0) - \frac{t_3}{3}\varrho^\beta(1 - x_3)$$

$$+ \frac{2}{\hbar^2}[t_2(1 + x_2) - t_1(1 - x_1)]\mathbf{k}^2$$

$$\equiv a_n + b_n\mathbf{k}^2. \qquad (23)$$

With account of Eqs. (22) and (23), the normalization conditions for the distribution functions can be written in the form

$$\frac{2}{\mathcal{V}} \sum_{\mathbf{p}} f_0(\mathbf{p}) = \varrho, \qquad (24)$$

$$\frac{2}{\mathcal{V}} \sum_{\mathbf{p}} f_3(\mathbf{p}) = \varrho_\uparrow - \varrho_\downarrow \equiv \Delta\varrho_{\uparrow\uparrow}. \qquad (25)$$

Here $\varrho_\uparrow$ and $\varrho_\downarrow$ are the neutron number densities with spin up and spin down and

$$f_0 = \tfrac{1}{2}\{n(\omega_+) + n(\omega_-)\}, \quad \omega_\pm = \xi_0 \pm \xi_3, \qquad (26)$$

$$f_3 = \tfrac{1}{2}\{n(\omega_+) - n(\omega_-)\}, \qquad (27)$$

$$\xi_0 = \frac{p^2}{2m_n} - \mu_n, \qquad (28)$$

$$\xi_3 = \left(a_n + b_n\frac{\mathbf{p}^2}{4}\right)\frac{\Delta\varrho_{\uparrow\uparrow}}{4} + \frac{b_n}{16}\langle \mathbf{q}^2 \rangle_3. \qquad (29)$$

The effective neutron mass m_n is defined by the formula

$$\frac{\hbar^2}{2m_n} = \frac{\hbar^2}{2m_0} + \frac{\varrho}{8}[t_1(1 - x_1) + 3t_2(1 + x_2)], \qquad (30)$$

and the quantity $\langle \mathbf{q}^2 \rangle_3$ in Eq. (29) is the second order moment of the distribution function f_3:

$$\langle \mathbf{q}^2 \rangle_3 = \frac{2}{V} \sum_{\mathbf{q}} \mathbf{q}^2 f_3(\mathbf{q}). \qquad (31)$$

Thus, with account of expressions (26) and (27) for the distribution functions f, we obtain the self-consistent equations (24), (25), and (31) for the effective chemical potential μ_n, spin order parameter $\Delta\varrho_{\uparrow\uparrow}$, and second order moment $\langle \mathbf{q}^2 \rangle_3$.

III. PHASE TRANSITIONS IN STRONGLY ASYMMETRIC NUCLEAR MATTER

Early research on spin polarizability of nuclear matter with the Skyrme effective interaction was based on the calculation of magnetic susceptibility and finding its pole structure [4,5], determining the onset of instability with respect to spin fluctuations. Here we shall find directly solutions of the self-consistent equations for the FM and AFM spin order parameters as functions of density at zero temperature. A

special emphasis will be laid on the study of strongly asymmetric nuclear matter ($\alpha \lesssim 1$), while in symmetric nuclear matter FM spin ordering is thermodynamically more preferable than the AFM one [11].

If all neutron and proton spins are aligned in one direction, then for nontrivial solutions of the self-consistent equations we have

$$\Delta\varrho_{\uparrow\uparrow} = \varrho, \quad \Delta\varrho_{\uparrow\downarrow} = \alpha\varrho, \qquad (32)$$

$$\langle \mathbf{q}^2 \rangle_{30} = \tfrac{3}{10}\varrho k_F^2 [(1+\alpha)^{5/3} + (1-\alpha)^{5/3}],$$

$$\langle \mathbf{q}^2 \rangle_{33} = \tfrac{3}{10}\varrho k_F^2 [(1+\alpha)^{5/3} - (1-\alpha)^{5/3}],$$

where $k_F = (3\pi^2\varrho)^{1/3}$ is the Fermi momentum of totally polarized symmetric nuclear matter. Therefore, for the partial number densities of nucleons with spin up and spin down one can get

$$\varrho_{n\uparrow} = \frac{1+\alpha}{2}\varrho, \quad \varrho_{p\uparrow} = \frac{1-\alpha}{2}\varrho, \quad \varrho_{p\downarrow} = \varrho_{n\downarrow} = 0. \quad (33)$$

If all neutron spins are aligned in one direction and all proton spins in the opposite one, then

$$\Delta\varrho_{\uparrow\uparrow} = \alpha\varrho, \quad \Delta\varrho_{\uparrow\downarrow} = \varrho, \qquad (34)$$

$$\langle \mathbf{q}^2 \rangle_{30} = \tfrac{3}{10}\varrho k_F^2 [(1+\alpha)^{5/3} - (1-\alpha)^{5/3}],$$

$$\langle \mathbf{q}^2 \rangle_{33} = \tfrac{3}{10}\varrho k_F^2 [(1+\alpha)^{5/3} + (1-\alpha)^{5/3}],$$

and, hence,

$$\varrho_{n\uparrow} = \frac{1+\alpha}{2}\varrho, \quad \varrho_{p\downarrow} = \frac{1-\alpha}{2}\varrho, \quad \varrho_{p\uparrow} = \varrho_{n\downarrow} = 0. \quad (35)$$

Now we present the results of the numerical solution of the self-consistent equations with the effective $SLy4$ and $SLy5$ forces for strongly asymmetric nuclear ($\alpha = 0.95, 0.9, 0.8$) and neutron ($\alpha=1$) matter. The neutron and proton spin polarization parameters Π_n and Π_p are shown in Fig. 1 as functions of density at zero temperature. Since in totally polarized state the signs of spin polarizations are opposite ($\Pi_n = 1, \Pi_p = -1$), considering solutions correspond to the case, when spins of neutrons and protons are aligned in the opposite direction. Note that for $SLy4$ and $SLy5$ forces, being relevant for the description of strongly asymmetric nuclear matter, there are no solutions corresponding to the same direction of neutron and proton spins. The reason is that the sign of the multiplier $t_3(-1+2x_3)$ in the density dependent term of the FL amplitude U_1, determining spin-spin correlations, is positive, and, hence, the corresponding term increases with the increase of nuclear matter density, preventing instability with respect to spin fluctuations. Contrarily, the density dependent term $-t_3\varrho^\beta/3$ in the FL amplitude U_3, describing spin-isospin correlations, is negative, leading to spin instability with the oppositely directed spins of neutrons and protons at higher densities.

Another nontrivial feature relates to the density behavior of the spin polarization parameters at large isospin asymme-
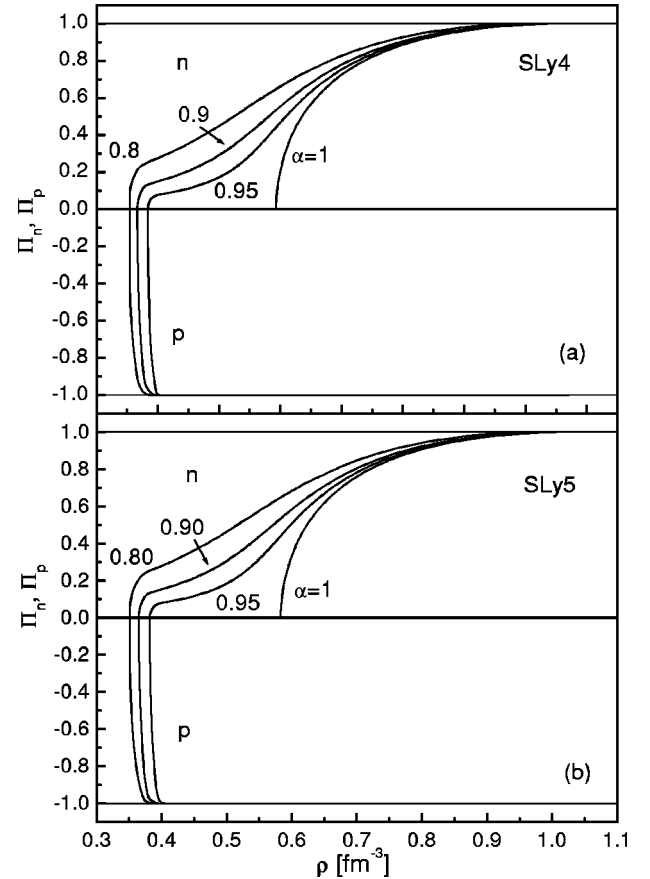

FIG. 1. Neutron and proton spin polarization parameters as functions of density at zero temperature for (a) $SLy4$ force and (b) $SLy5$ force.

try. As seen from Fig. 1, even small admixture of protons leads to the appearance of long tails in the density profiles of the neutron spin polarization parameter near the transition point to a spin ordered state. As a consequence, the spin polarized state is formed much earlier in density than in pure neutron matter. For example, the critical density in neutron matter is $\varrho \approx 0.59$ fm^{-3} for the $SLy4$ potential and $\varrho \approx 0.58$ fm^{-3} for the $SLy5$ potential; in asymmetric nuclear matter with $\alpha=0.95$ the spin polarized state arises at $\varrho \approx 0.38$ fm^{-3} for $SLy4$ and $SLy5$ potentials. Hence, even small quantity of protons strongly favors spin instability of highly asymmetric nuclear matter, leading to the appearance of states with the oppositely directed spins of neutrons and protons. As follows from Fig. 1, protons become totally spin polarized within a very narrow density domain (e.g., if $\alpha=0.95$, full polarization occurs at $\varrho \approx 0.41$ fm^{-3} for $SLy4$ force and at $\varrho \approx 0.40$ fm^{-3} for $SLy5$ force) while the threshold densities for the appearance and saturation of the neutron spin order parameter are substantially different (if $\alpha=0.95$, neutrons become totally polarized at $\varrho \approx 1.05$ fm^{-3} for $SLy4$ force and at $\varrho \approx 1.02$ fm^{-3} for $SLy5$ force).
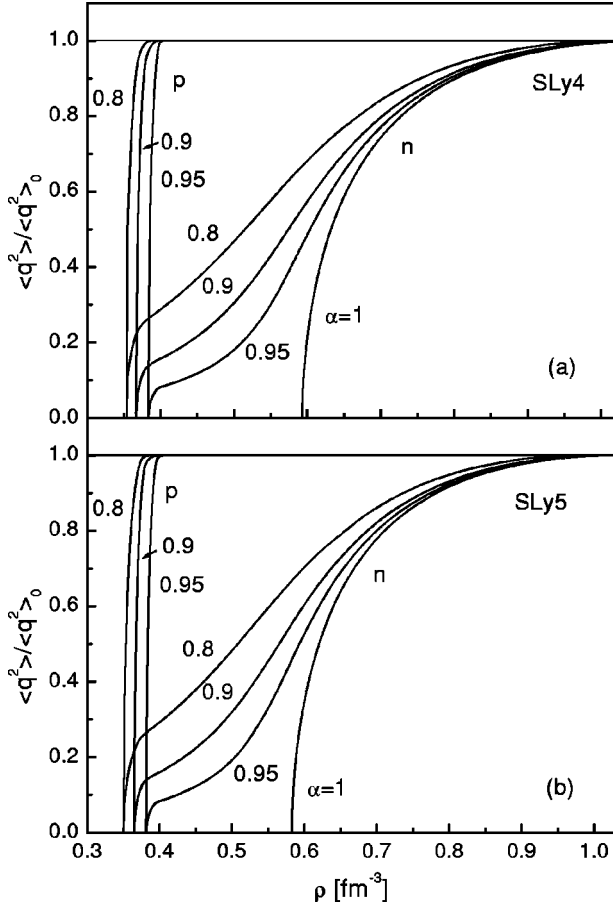
Note that the second order moments

FIG. 2. Same as in Fig. 1, but for the second order moments $\langle \mathbf{q}^2 \rangle_p$ and $\langle \mathbf{q}^2 \rangle_n$, normalized to their values in the totally polarized state.

$$\langle \mathbf{q}^2 \rangle_n \equiv \langle \mathbf{q}^2 \rangle_{n\uparrow} - \langle \mathbf{q}^2 \rangle_{n\downarrow} = \frac{1}{V} \sum_{\mathbf{q}} \mathbf{q}^2 [n(\omega_{+,+}) - n(\omega_{-,+})],$$

(36)

$$\langle \mathbf{q}^2 \rangle_p \equiv \langle \mathbf{q}^2 \rangle_{p\uparrow} - \langle \mathbf{q}^2 \rangle_{p\downarrow} = \frac{1}{V} \sum_{\mathbf{q}} \mathbf{q}^2 [n(\omega_{+,-}) - n(\omega_{-,-})]$$

also characterize spin polarization of the neutron and proton subsystems. If the solutions $\langle \mathbf{q}^2 \rangle_{30}$ and $\langle \mathbf{q}^2 \rangle_{33}$ of the self-consistent equations are known, then

$$\langle \mathbf{q}^2 \rangle_n = \tfrac{1}{2}(\langle \mathbf{q}^2 \rangle_{30} + \langle \mathbf{q}^2 \rangle_{33}),$$

$$\langle \mathbf{q}^2 \rangle_p = \tfrac{1}{2}(\langle \mathbf{q}^2 \rangle_{30} - \langle \mathbf{q}^2 \rangle_{33}).$$

The values of $\langle \mathbf{q}^2 \rangle_n$ and $\langle \mathbf{q}^2 \rangle_p$ for the totally polarized state are

$$\langle \mathbf{q}^2 \rangle_{n0} = \tfrac{3}{10} \varrho k_F^2 (1 + \alpha)^{5/3}, \quad \langle \mathbf{q}^2 \rangle_{p0} = -\tfrac{3}{10} \varrho k_F^2 (1 - \alpha)^{5/3}.$$

In Fig. 2 we plot the density dependence of the second order moments $\langle \mathbf{q}^2 \rangle_n$ and $\langle \mathbf{q}^2 \rangle_p$, normalized to their values in the totally polarized state, for different asymmetries at zero temperature. These quantities behave similarly to the spin
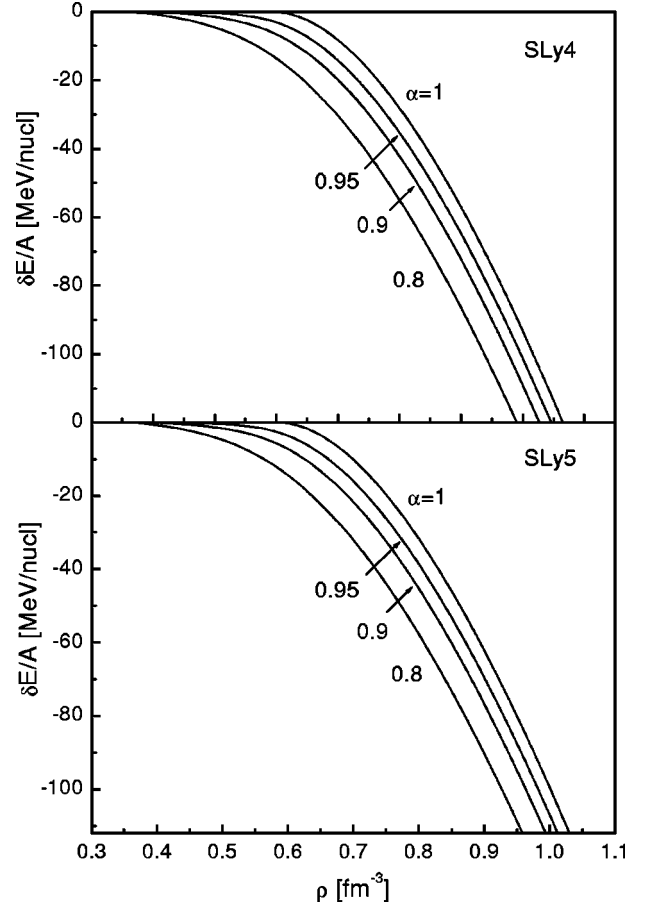

FIG. 3. Total energy per nucleon, measured from its value in the normal state, for the state with the oppositely directed spins of neutrons and protons as a function of density at zero temperature for (a) *SLy*4 force and (b) *SLy*5 force.

polarization parameters in Fig. 1, i.e., there exist long tails in the density profiles of the neutron spin order parameter and the proton spin order parameter is saturated within a very narrow density interval.

To check thermodynamic stability of the spin ordered state with the oppositely directed spins of neutrons and protons, it is necessary to compare the free energies of this state and the normal state. In Fig. 3 the difference between the total energies per nucleon of the spin ordered and normal states is shown as a function of density at zero temperature. One can see that nuclear matter undergoes a phase transition to the state with the oppositely directed spins of neutrons and protons at some critical density, depending on the isospin asymmetry.

IV. DISCUSSION AND CONCLUSIONS

Spin instability is a common feature associated with a large class of Skyrme models, but is not realized in more microscopic calculations. The Skyrme interaction has been successful in describing nuclei and their excited states. In addition, various authors have exploited its applicability to describe bulk matter at densities of relevance to neutron stars [28]. The force parameters are determined empirically by

calculating the ground state in the Hartree-Fock approximation and by fitting the observed ground state properties of nuclei and nuclear matter at the saturation density. In particular, the interaction parameters, describing spin-spin and spin-isospin correlations, are constrained from the data on isoscalar [29,30] and isovector (giant Gamow-Teller) [31,32] spin-flip resonances.

In a microscopic approach, one starts with the bare interaction and obtains an effective particle-hole interaction by solving iteratively the Bethe-Goldstone equation. In contrast with the Skyrme models, calculations with realistic NN potentials predict more repulsive total energy per particle for a polarized state comparing to the nonpolarized one for all relevant densities, and, hence, give no indication of a phase transition to a spin ordered state. It must be emphasized that the interaction in the spin- and isospin-dependent channels is a crucial ingredient in calculating spin properties of isospin asymmetric nuclear matter and different behavior at high densities of the interaction amplitudes, describing spin-spin and spin-isospin correlations, lays behind this divergence in calculations with the effective and realistic potentials.

In this study as a potential of NN interaction we chose $SLy4$ and $SLy5$ Skyrme effective forces, which were constrained originally to reproduce the results of microscopic neutron matter calculations (pressure versus density curve) [27]. Besides, in a recent publication [28] it was shown that the density dependence of the nuclear symmetry energy, calculated up to densities $\varrho \lesssim 3\varrho_0$ with $SLy4$ and $SLy5$ effective forces (together with some other sets of parameters among the total 87 Skyrme force parametrizations checked) gives the neutron star models in a broad agreement with the observables, such as the minimum rotation period, gravitational mass-radius relation, the binding energy, released in supernova collapse, etc. This is an important check for using these parametrizations in the high density region of strongly asymmetric nuclear matter. However, it is necessary to note that the spin-dependent part of the Skyrme interaction at densities of relevance to neutron stars still remains to be constrained. Probably, these constraints will be obtained from the data on the time decay of magnetic field of isolated neutron stars [33]. In spite of this shortcoming, $SLy4$ and $SLy5$ effective forces hold one of the most competing Skyrme parametrizations at present time for description of isospin asymmetric

nuclear matter at high density, while a Fermi liquid approach with Skyrme effective forces provides a consistent and transparent framework for studying spin instabilities in a nucleon system.

In summary, we have considered the possibility of phase transitions into spin ordered states of strongly asymmetric nuclear matter within the Fermi liquid formalism, where NN interaction is described by the Skyrme effective forces ($SLy4$ and $SLy5$ potentials). In contrast with the previous considerations, where the possibility of formation of FM spin polarized states was studied on the base of calculation of magnetic susceptibility, we obtain the self-consistent equations for the FM and AFM spin order parameters and solve them in the case of zero temperature. It has been shown in the model with $SLy4$ and $SLy5$ effective forces that strongly asymmetric nuclear matter undergoes a phase transition to the spin polarized state with the oppositely directed spins of neutrons and protons, while the state with the same direction of the neutron and proton spins does not appear. An important peculiarity of this phase transition is the existence of long tails in the density profile of the neutron spin polarization parameter near the transition point. This means that even small admixture of protons to neutron matter leads to a considerable shift of the critical density of spin instability in the direction of low densities. In the model with $SLy4$ effective interaction this displacement is from the critical density $\varrho \approx 3.7\varrho_0$ for neutron matter to $\varrho \approx 2.4\varrho_0$ for asymmetric nuclear matter at the isospin asymmetry $\alpha=0.95$, i.e., for 2.5% of protons only. As a result, the state with the oppositely directed spins of neutrons and protons appears, where protons become totally polarized in a very narrow density domain. This picture is different from the case of symmetric nuclear matter, where the FM spin configuration is thermodynamically more preferable than the AFM one [11]. Obtained results may be of importance for the description of thermal and magnetic evolution of pulsars, whose core represents strongly asymmetric nuclear matter.

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